The Effect of Crowd Density on the Expected Number of Casualties in a Suicide Attack

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Abstract: Utilizing elementary geometric and probability considerations, we estimate the effect of crowd blocking in suicide bombing events. It is shown that the effect is quite significant. Beyond a certain threshold, the expected number of casualties decreases with the number of people in the arena. The numerical results of our model are consistent with casualty data from suicide bombing events in Israel. Some operational insights are discussed. © 2004 Wiley Periodicals, Inc. Naval Research Logistics 52: 22–29, 2005.

Keywords: suicide bomber; crowd blocking; fragmentation

1. INTRODUCTION

Terror events that involve suicide bombers (SB) have become a major concern to governments, particularly in Israel, but also elsewhere in the world. In the past few years, there were over 100 suicide bomb events in Israel alone that ended with the attackers blowing up others, along with themselves. Typical suicide bomb events occur in relatively small, crowded areas, henceforth called *arenas,* such as restaurants, buses, and bus stations.

One would expect that the number of casualties would increase with the density of the crowd, although at a decreasing rate, and would decrease with the size of the arena for a fixed number of people. The reason for the first assertion is that a higher density of people increases the probability that a random fragment of the bomb hits a person, and therefore it also increases the expected number of casualties. The second assertion seems reasonable, since the effect of the bomb decreases with distance, and larger arenas imply longer average distance from the SB to a randomly selected person.

In this note, we show that this is not necessarily the case. *Crowd blocking* has a significant effect on the expected number of casualties. Crowd blocking occurs when some persons are shielded from the fragments of the bomb by other persons who stand between them and the SB. The effect of crowd blocking is modeled and quantified in this note. It is shown numerically that the expected number of casualties is a unimodal function of both the crowd density and the size of the arena. We also obtain estimates for the expected number of casualties that conform to data regarding casualties in SB attacks in Israel [4]. The model presented here was motivated by related research reported in [3], where the crowd density in the arena is represented by a continuous parameter. Here we present a discrete model, which we believe better captures the physical aspects of the SB situation.

There is an abundance of publications on wound ballistics. They include books (e.g., [2,6]) and reports—mostly by the U.S. Army Ballistics Research Laboratories (e.g., [1]) and the International Wound Ballistics Association [5]. However, most of these publications address the clinical aspects associated with the physical and biological effects that result from the impact of kinetic energy projectiles on humans. These aspects are beyond the scope of this paper which focuses on the effect of mutual blocking. Some technical data in [2] (e.g., Table 19, p. 113) are used in the next section to determine some assumptions of the model.

2. THE SITUATION

A typical SB carries an explosive charge (EC) mounted on a belt, which is concealed under a coat or a large shirt. The EC contains explosive and small pieces of metal, such as screws and nails. We assume that the SB attempts to position himself as close to the center of the crowd as possible. Then he sets off the bomb, and the metal fragments disperse in a beam spray that hit people in the vicinity. We consider only bomb fragment injuries (which are the most severe ones) and not blast or blast-related injuries (e.g., burns or cuts from shattered glass). Also, due to the relatively small amount of explosive

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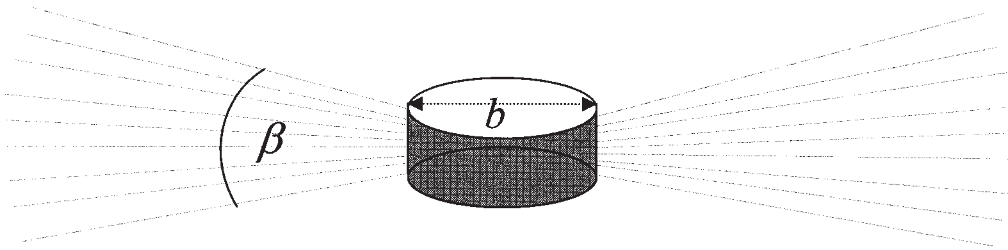

Figure 1. A suicide belt and its spray beam.

(3–4 kg) we assume, in our base model, that the energy of the fragments is such that a fragment may injure, at most, one person [2,8]. Secondary injuries are typically negligible. It is shown, however, that the model can be modified to also account for secondary hits, which may injure, with a certain probability, persons that stand immediately behind an exposed person. The number of *effective fragments*—those which are potentially effective—is roughly one half the number of fragments on the belt. The other half is wasted on the SB himself. We assume that the explosive and fragments are uniformly distributed on the belt, which is depicted in our model as a full circle. See Figure 1.

3. THE PHYSICAL MODEL

Let

N = number of effective fragments in the beam spray,
R = range from the belt of the SB,
b = diameter of the suicide belt (=the average "diameter" of a person's body),
β = dispersion angle of the effective beam spray,
$P_H(R)$ = probability that an exposed person at range R is hit.

The density of fragments at a range R is

$$\sigma_R = \frac{N}{4\pi R^2 \sin\frac{\beta}{2}}. \qquad (1)$$

The denominator in (1) is the total area of dispersion at range R (see Appendix A).

A person is represented in the model as a cylinder of diameter b and height cb, $c > 1$. Its exposed area to the fragments A is approximately a rectangle of width b and height that is determined by the dispersion angle β (see Fig. 2). That is,

$$A = b\ \mathrm{Min}\left\{2R\tan\frac{\beta}{2}, cb\right\}. \qquad (2)$$

Assuming uniform and independent dispersion of fragments, the probability that a person at range R is hit by at least one fragment is

$$P_H(R) = 1 - \left(1 - \frac{A}{4\pi R^2 \sin\frac{\beta}{2}}\right)^N. \qquad (3)$$

From (1) it follows that

$$P_H(R) = 1 - \left(1 - \frac{A}{\frac{N}{\sigma_R}}\right)^N \simeq 1 - e^{-A\sigma_R}. \qquad (4)$$

We assume that the effectiveness of the explosive and the size of the arena are such that air resistance and gravitation has no significant effect on the trajectory and energy of the fragments. A fragment does not get any more harmless with distance.

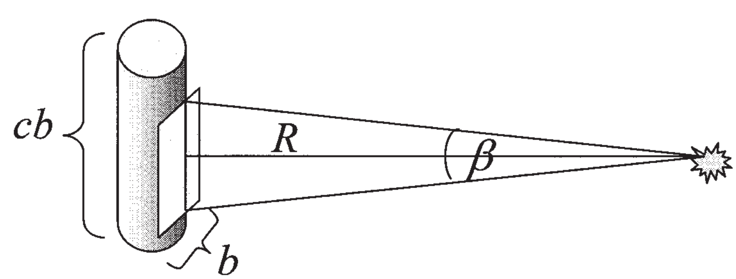

Figure 2. The exposed area.

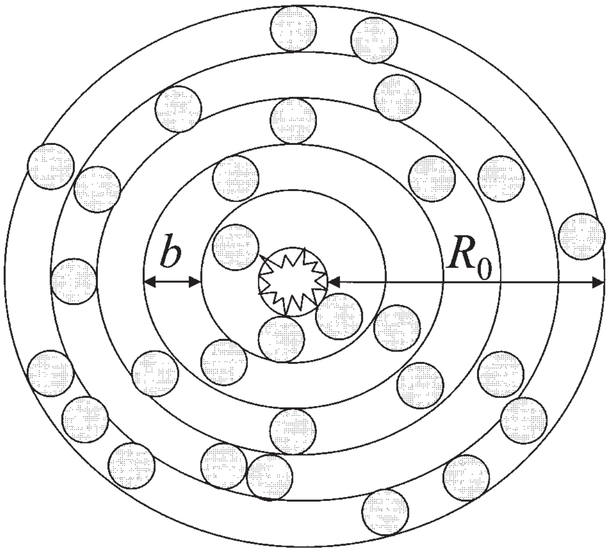

Figure 3. The arena.

Some typical SB arenas, such as restaurants, may be depicted as a circular area around the SB. Let R_0 denote the distance between the SB (the EC belt) and the boundary of the arena. We assume that the crowd is distributed randomly and uniformly in the arena.

For modeling purposes, it is convenient to view the circular arena as a sequence of M concentric rings of width b. Each person occupies a round "slot" of diameter b in a certain ring. In particular, the SB is located in the central slot (see Fig. 3). The maximum possible number of circles (slots) a_m in the mth ring is

$$a_m = \frac{\pi}{\arcsin \frac{1}{2m}}, \qquad m = 1, \ldots, M, \tag{5}$$

which is obviously independent of b. From now on we take $b = 1$. See Appendix B.

It can be verified (at least for $M \le 200$, which is much more rings than we need for the SB scenario) that a_m is integer only for $m = 1$ ($a_1 = 6$). Assume that in each ring we pack, with possible small overlaps, $k_m = \lceil a_m \rceil$ slots. Define the *overlap factor* of ring m by

$$d_m = \frac{k_m - a_m}{k_m}. \tag{6}$$

For example, the overlap factors for $m = 1, 2, 3, 4, 5, 6$ are 0, .04, .01, .04, .02, .01, respectively. For higher values of M the d_m gets even smaller since the nominator in (6) is bounded by 1 and the denominator is (strictly) monotone increasing in m. See Figure 4 for $m = 1, 2$. Thus, the overlaps are marginal and they can be removed by slightly increasing the width of a ring. From now on we assume that the mth ring in the arena contains k_m slots.

Let

$$K(m) = \sum_{n=1}^{m} k_n, \qquad m = 1, \ldots, M. \tag{7}$$

$K(M)$ is the maximum possible number of people in the arena (excluding the SB). For example, if $M = 10$, then $K(M) = 6 + 13 + 19 + 26 + 32 + 38 + 44 + 51 + 57 + 63 = 349$.

Since we assume random homogeneous mixing in the arena, the probability distribution of the number of people l_m in the mth ring is hypergeometric with parameters $K(M)$ and k_m. That is, the probability that l_m out of L people in the arena are in the mth ring is

$$\Pr(l_m) = \frac{\binom{k_m}{l_m}\binom{K(M) - k_m}{L - l_m}}{\binom{K(M)}{L}}, \tag{8}$$

and the expected number of people in the mth ring is

$$\mu_m = \frac{k_m}{K(M)} L. \tag{9}$$

4. THE EFFECT OF CROWD BLOCKING

The number of casualties depends on the spatial distribution of people in the arena. Some people may become

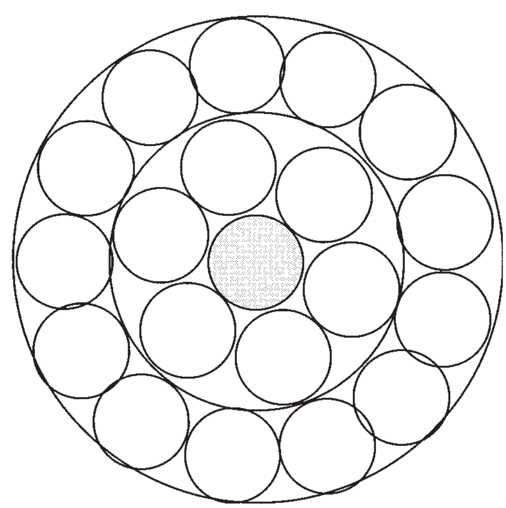

Figure 4. Number of slots, $k_1 = 6$, $k_2 = 13$.

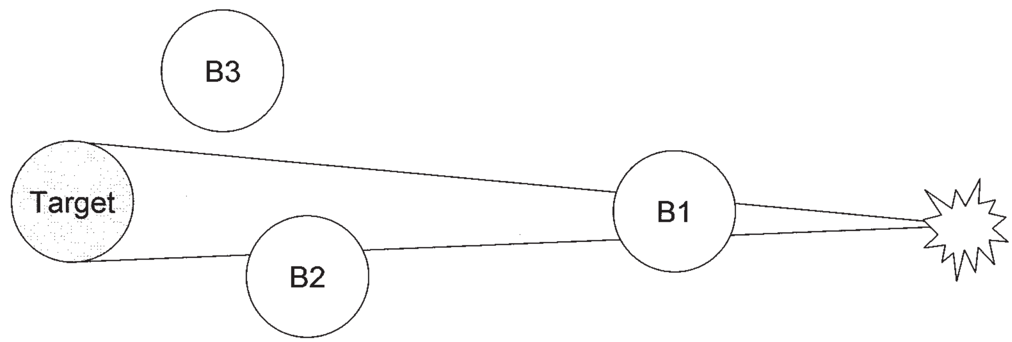

Figure 5. Partial and complete blocking.

human shields to others by blocking the fragments. As shown in Figure 5, B1 totally shields (blocks) the target (T), B2 partially shields it, and B3 provides no shield at all. We estimate the effect of blocking by a 0–1 damage function, similar to the widely used *cookie-cutter* function in combat modeling and firing theory [8]. A target (person) is protected against a fragment that is moving its way if and only if its center is shielded from the beam spray. In other words, the target is safe if and only if there exists at least one person that blocks the line-of-sight from the center of the SB to the center of the target. B1 in Figure 6 provides total shield for the target, while B2 provides none. Instead of describing the situation in terms of persons, we will do it by looking at the slots in the various rings. Thus, a person (T) in ring m is protected if and only if there is at least one slot in rings $1, \ldots, m-1$ such that (a) it intersects the line-of-sight SB − T, and (b) it is occupied by a person.

Since the slots in each ring are packed, there exists in each ring $1, \ldots, m-1$ a slot that intersects the line-of-sight SB − T. It follows that T is safe if and only if at least one of these $m-1$ slots is occupied by a person. Denote the probability of the complement of this event—the probability that the target is exposed and vulnerable to the explosion—by $\alpha(m)$. It can be shown (see Appendix C) that

$$\alpha(m) = \begin{cases} \prod_{l=1}^{L-1} \left(1 - \frac{m-1}{K(M)-l}\right) & \text{if } L < K(M) - m + 2, \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

It is easily seen that $1 = \alpha(1) \geq \alpha(2) \geq, \ldots \geq \alpha(\mathrm{M})$.

The expected number of casualties in the mth ring is

$$E_m = \mu_m \times \alpha(m) \times P_H(m), \quad (11)$$

where

$$\begin{aligned} P_H(m) &= 1 - e^{-A\sigma_m} \\ &= 1 - e^{-(N\,\mathrm{Min}\{2m\tan(\beta/2),c\}/(4\pi\sin(\beta/2)m^2)} = 1 - e^{-D(m)}. \end{aligned} \quad (12)$$

The total expected number of casualties is

$$E(M) = \sum_{m=1}^{M} E_m. \quad (13)$$

Next, we relax the assumption of complete crowd blocking and assume that, in certain cases, a person whose path to the bomber is obstructed by another may be subject to injury too. Specifically, assume that secondary injuries may occur independently, with probability q, to persons located right behind exposed persons. Using the formula for total probability, it is easily seen that the probability that a target in ring m stands right behind an exposed target in ring $m-1$ is $\alpha(m-1) - \alpha(m)$, $m = 2, \ldots, M$. Therefore, the expected number $\hat{E}_m$ of casualties in ring m is

$$\hat{E}_m = \begin{cases} E_m & m = 1, \\ \mu_m[(1-q)\alpha(m) + q\alpha(m-1)]P_H(m) & \text{otherwise.} \end{cases} \quad (14)$$

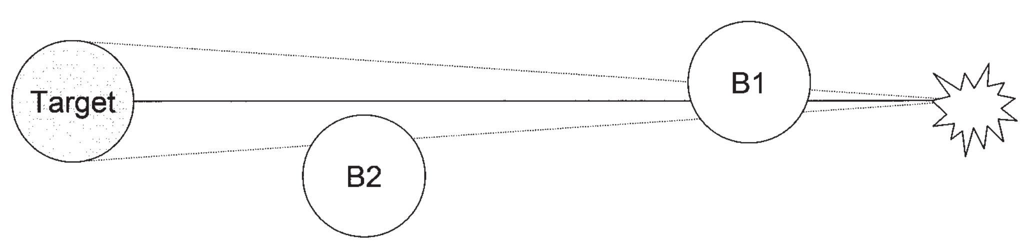

Figure 6. Crowd blocking.

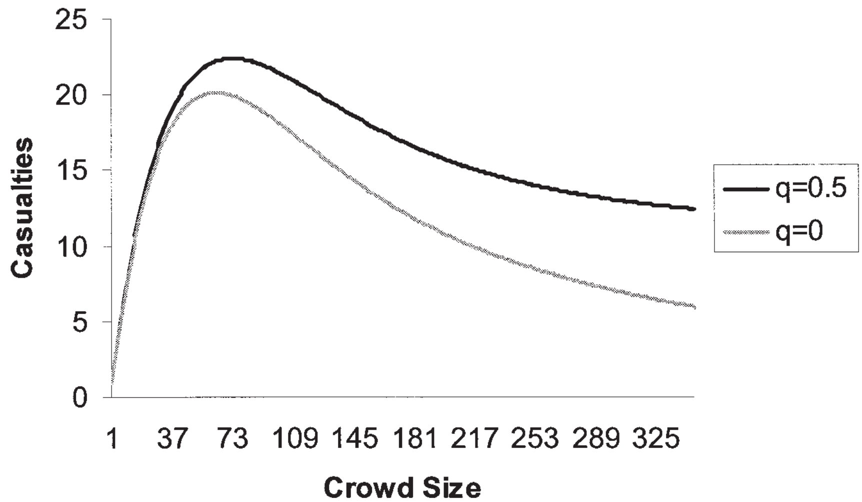

Figure 7. Expected number of casualties, $M = 10$, $\beta = 10°$.

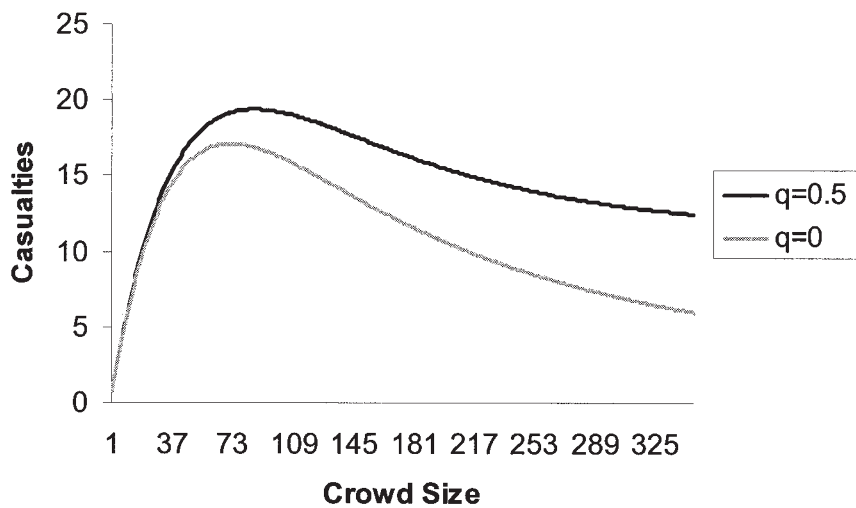

Figure 8. Expected number of casualties, $M = 10$, $\beta = 60°$.

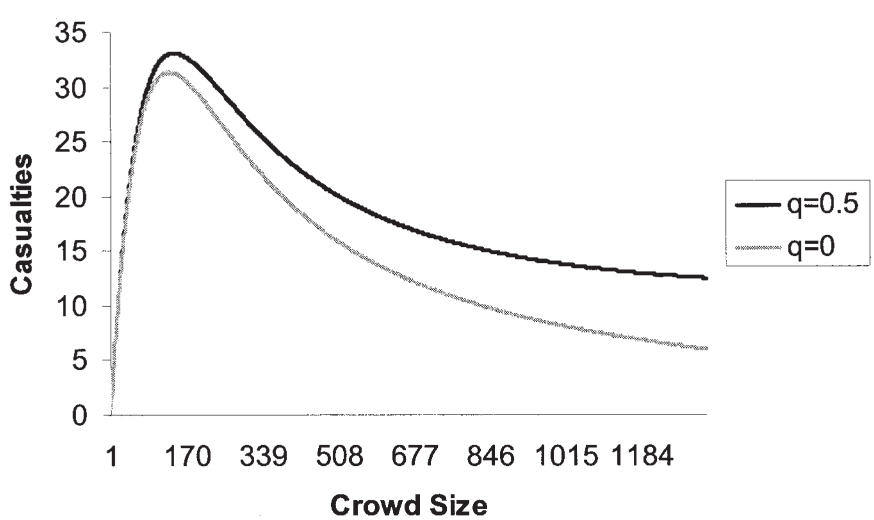

Figure 9. Expected number of casualties, $M = 20$, $\beta = 10°$.

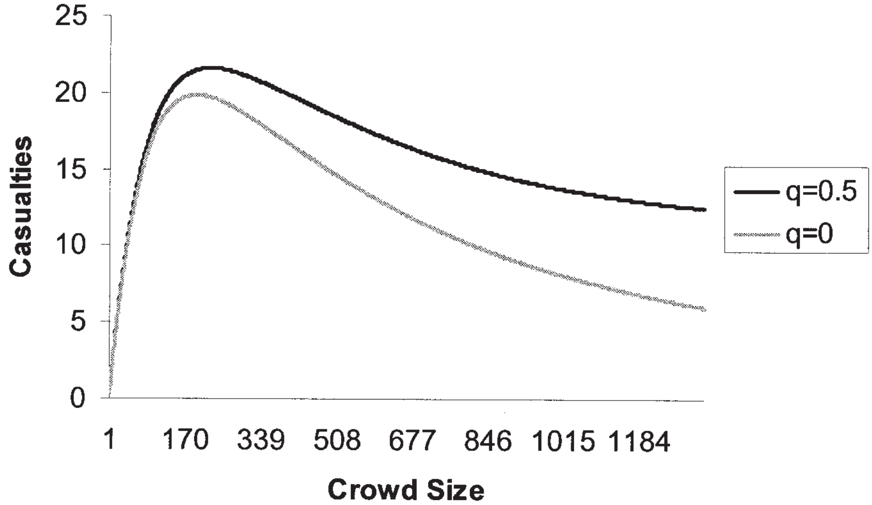

Figure 10. Expected number of casualties, $M = 20$, $\beta = 60°$.

5. NUMERICAL RESULTS AND CONCLUSIONS

Figures 7–10 plot the values of E(M) as functions of the crowd size L for the cases of perfect blocking ($q = 0$) and partial blocking ($q = 0.5$). We examine two arena sizes, $M = 10, 20$, and two values of the spray beam angle, $\beta =$ 10° and 60°. We assume that $N = 100$ fragments, and on average the height of a person is 3.5 times his width ($c =$ 3.5). For $M = 10$ ($K(M) = 349$), see Figures 7 and 8. For $M = 20$ ($K(M) = 1328$), see Figures 9 and 10.

Figures 7–10 demonstrate that the effectiveness of the suicide bomb does not necessarily increase with the size of the crowd in the arena. Beyond a certain threshold, the expected number of casualties gets smaller. This phenomenon is attributed to crowd blocking, which becomes more significant as the density of the crowd increases. Note that in the case of perfect blocking ($q = 0$) the expected number of casualties decreases to 6. This result is true in general since the effect of the explosion is limited to the first ring (with $k_1 = 6$) when the arena is fully crowded. The effectiveness of the explosion also depends on the size of the spray beam angle. When this beam is narrow, the effect is stronger than when it is wider. Note also that the effect of secondary injuries is significant only when the arena becomes crowded. In a low-density arena only direct hits of the fragments affect the number of casualties.

Figure 11 depicts the effect of the size of the arena on the expected number of casualties. We assume a crowd of $L =$ 100, and we vary the size of the arena between $M = 6$ (the

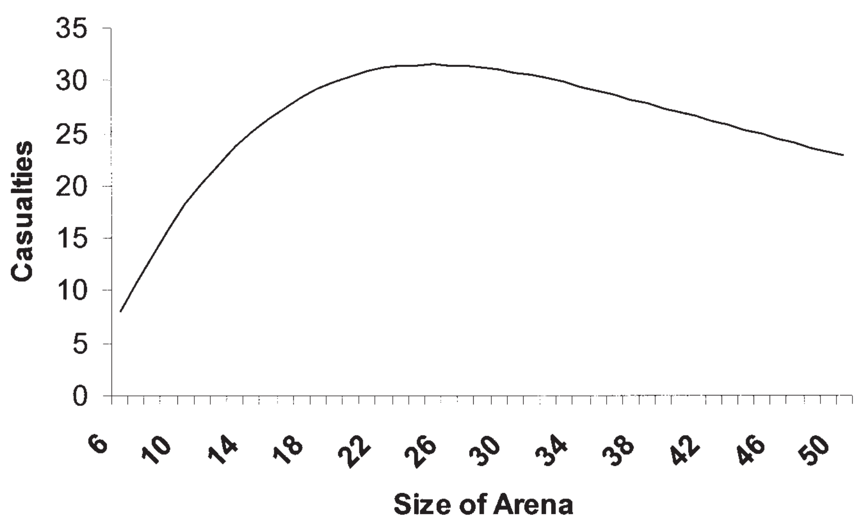

Figure 11. Expected number of casualties as a function of the arena size, $\beta = 10°$.

Table 1. Casualties of SB events in Israel—2003 (source:[4]]).

Date	Location	Killed	Injured
4 October 03	Haifa	21	60
8 September 03	Jerusalem	8	30
8 September 03	Zrifin Junction	9	20
19 August 03	Jerusalem	24	102
12 August 03	Ariel	2	4
12 August 03	Rosh Ha A'yn	1	9
19 June 03	Sde Trumot	1	0
11 June 03	Jerusalem	17	50
19 May 03	Afula	3	47
18 May 03	Jerusalem	7	20
30 April 03	Tel Aviv	3	60
30 March 03	Netanya	0	58
5 March 03	Haifa	17	40
5 January 03	Tel Aviv*	23	100

*Two simultaneous SB events.

minimum-size arena that can contain 100 people) and $M = 50$. We assume perfect blocking.

From Figure 11 we see that the size of the arena affects the damage in a nonmonotone way. For a certain crowd ($L = 100$ in our example), there is a capacity ($M = 25$) for which the damage is maximal.

While the results shown in Figures 7–11 seem to be consistent with data regarding SB events (see Table 1), a rigorous statistical analysis to confirm the numerical results of the model is difficult, if not impossible. First, there is no reliable record on the size of the crowd L in the arena at the time of the event. This number can be estimated, at best, based on interviews of eyewitnesses. Second, while fragments are the main cause for fatalities in a SB event, some victims may have been killed by blast effects. Also, although some of the recorded injuries are related to mental shock and secondary injuries, such as cuts and bruises from debris, many of them may have been caused directly by the fragments. Third, the number of effective fragments in a suicide belt can also be only estimated. Fourth, the position of the SB in the arena affects the results, too. Incidents with relatively few casualties were typically consequences of a partial successful interdiction of the SB, where a guard identified the SB at the door, and as a result, the latter blew himself up outside the arena.

If we assume that the number of fatalities plus 20% of the injuries is a reasonable estimate for the number of casualties of direct hits by fragments, then the 4 October 2003 SB event (see Table 1) that occurred at the Maxim restaurant in Haifa, Israel may confirm our model, e.g., the results in Figures 9 and 11. The capacity of this restaurant (in the sense of Fig. 3) is around 1000. According to the owner, there where around 100 patrons in the restaurant at the time of the event. The SB, a young woman from the West Bank, was sitting close to the center of the hall. Based on our

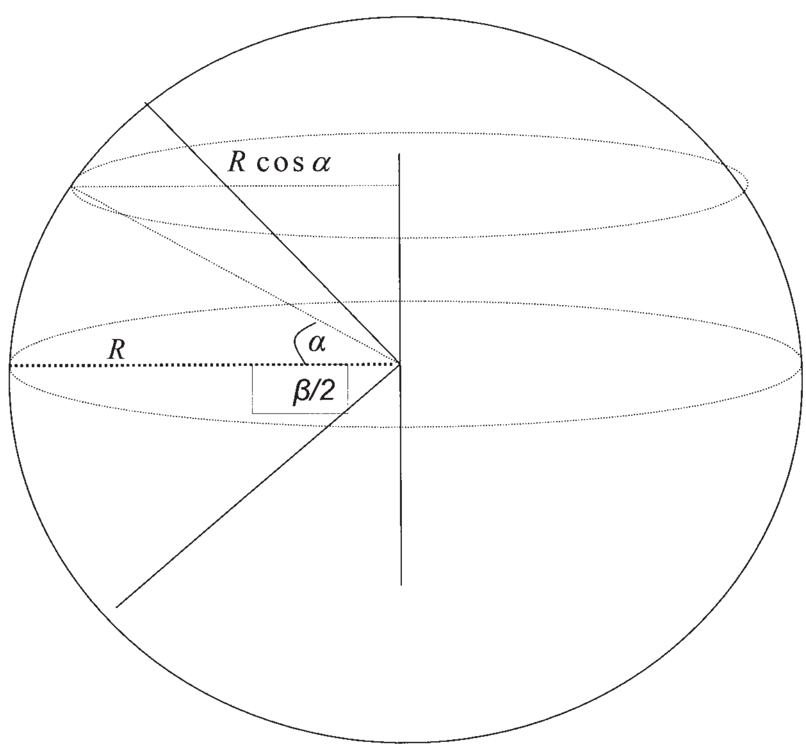

Figure 12. The dispersion area.

assumption above, the number of direct-hit casualties in this event was $21 + (.2)60 = 33$ (see Table 1), $L = 100$ and $M = 18$ ($K(M) = 1084$). Assuming a narrow spray beam (10°) and $N = 100$ fragments, our model projects 29 casualties for $q = 0$ and 31 casualties for $q = 0.5$.

We may conclude that, from the SB point of view, there is an optimal density of people in an arena for which the effect of the suicide bomb is maximized. It seems that at least in the Maxim restaurant event, the SB operated, unfortunately, very close to "optimality." One possible operational conclusion from this analysis is that if a SB is detected in a very dense crowd, dispersion of the crowd as a result of an alarm can actually increase the number of casualties. While, by running away, individuals who are close to the SB may increase their survivability, many others will become vulnerable. Perhaps the best policy would be to reduce, as much as possible, the exposed area to the fragments. This can be done, for example, by instructing the crowd to lie down, in a direction parallel to the beam spray (away from the SB), and cover their head. This action would reduce the average exposed area, in particular when β is relatively small.

APPENDIX A

The beam spray is distributed vertically with angle between $-\beta/2$ and $+\beta/2$ from the horizontal at the point of explosion—the center of the sphere in Figure 12. An arc of size ds at range R is $ds = R\,d\alpha$. The surface area of a complete circumferential strip at angle a is $2\pi R \cos \alpha R\, d\alpha = 2\pi R^2 \cos\alpha\, d\alpha$. The total area of dispersion at range R is therefore

$$2\int_0^{\beta/2} 2\pi R^2 \cos\alpha\, d\alpha = 4\pi R^2 \int_0^{\beta/2} \cos\alpha\, d\alpha = 4\pi R^2 (\sin\alpha]_0^{\beta/2}$$

$$= 4\pi R^2 \sin\frac{\beta}{2}.$$

APPENDIX B

Consider Figure 13. The number of circles that can fit into the mth ring, a_m, is determined by the angle α_m between the line that connects the POE with the center of the circle and its tangent. That is, $a_m = 2\pi/2\alpha_m$. But $\sin\alpha_m = (b/2)/mb = 1/2m$; therefore, $\alpha_m = \arcsin(1/2m)$, and the result follows.

APPENDIX C

Consider a certain person in ring m. Because of the uniform and independent distribution of people in the arena, the probability that the first of the other $L - 1$ persons *will not* occupy a blocking slot is that he chooses one of $K(M) - m$ empty slots out of the $K(M) - 1$ available. The second person can choose from $K(M) - m - 1$ empty slots out of $K(M) - 2$ available, and so on.

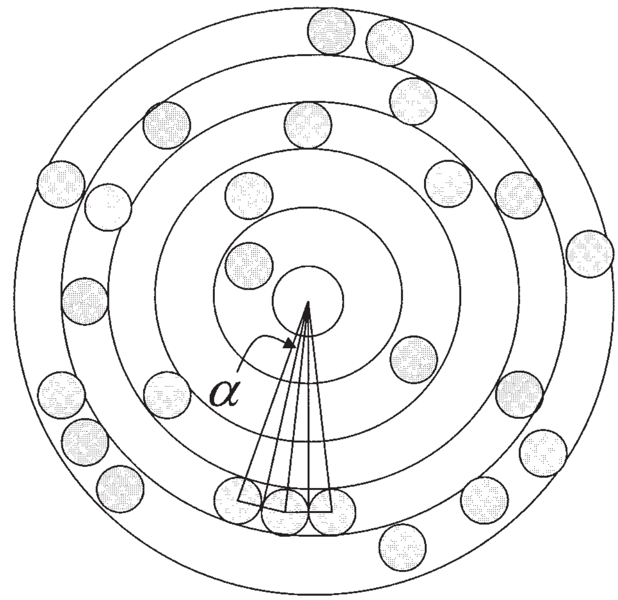

Figure 13. Slots in ARENA rings.

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