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A Resistive Sheet Approximation for Mesh Reflector Antennas

DAVID C. JENN, MEMBER, IEEE, A. PRATA, JR., WILLARD V. T. RUSCH, FELLOW, IEEE, AND M. R. BARCLAY

***Abstract*— A simplified method of estimating the equivalent surface resistance of a reflecting mesh is presented. The equivalent resistance is obtained from the approximate mesh reflection coefficients, which are based on the method of averaged boundary conditions. This resistance approximation allows for an integral equation solution for the mesh reflector that is a simple extension of the perfectly conducting reflector. Paraboloid radiation patterns using physical optics in conjunction with the reflection coefficients are compared to an *E*-field integral equation solution for a resistive surface. The agreement is excellent for low-to-moderate resistance values, even in the sidelobe regions.**

I. Introduction

Reflectors with mesh surfaces are often used in applications requiring lightweight deployable antennas. Although a rigorous solution of the electromagnetic mesh problem is feasible [1], it is difficult to incorporate within the framework of a reflector scattering program because of its complexity and excessive time requirements. Approximate formulas were derived by Astrakhan [2] based on the technique of averaged boundary conditions. The formulas give the plane wave reflection coefficients of the mesh for perpendicular and parallel polarizations as a function of the grid geometry, incidence direction, and the electrical contact properties of the wires. They apply to cases where the mesh wire radius is much less than the grid separations, and the grid separations are much less than a wavelength.

While the Astrakhan formulas are suited to radiation pattern calculations using the physical optics (PO) approximation, a surface impedance is convenient for an integral equation solution. If an equivalent surface resistance can be defined for a given mesh geometry, then a method of moments (MM) solution for the mesh currents is a simple extension of the solution for a perfectly conducting reflector [3].

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D. C. Jenn is with the Ground Systems Group, Hughes Aircraft Company, P. O. Box 3310, Ms 66/D141, Fullerton, CA 92634.
A. Prata, Jr., W. V. T. Rusch, and M. R. Barclay are with the Electrical Engineering Department, University of Southern California, Los Angeles, CA 90089.
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II. Method of Estimating the Surface Resistance

An impedance boundary condition relates the tangential components of the electric and magnetic fields at the surface of the scatterer. For a resistive sheet, which is simply an imperfect electrical conductor, there are no magnetic currents and the discontinuities in the fields across the sheet satisfy the following conditions [4]:

$$\hat{n} \times (E^+ - E^-) = 0 \tag{1}$$

$$\hat{n} \times (H^+ - H^-) = J \tag{2}$$

with

$$\hat{n} \times (\hat{n} \times E_p) = -n_s J, \; p = + \text{ or } - \tag{3}$$

where $\hat{n}$ is a unit vector normal to the surface pointing toward the + side, J is the total current on the sheet, and η_s is the surface resistance in units of ohms per square. Because it is inversely proportional to the conductivity of the sheet, $\eta_s = 0$ is equivalent to a perfect conductor, and when $\eta_s = \infty$ the sheet does not exist [4].

For reflector antenna applications the mesh geometry is chosen so that it is a good reflector and η_s will be close to zero. Then the equivalent resistive sheet is nearly opaque, and the reflection coefficient is given approximately by the standard plane wave formula for an interface between air and a medium of impedance η_s

$$R = \frac{\eta - \eta_s}{\eta + \eta_s} \tag{4}$$

where $\eta = 377\,\Omega$. For higher resistances a more accurate value of the reflected field is given by considering the sheet to be in parallel with the free space on the back side, in which case the reflection coefficient is [5]

$$R = \frac{-\eta}{2\eta_s + \eta}. \tag{5}$$

Since both η and η_s are real, a purely real estimate of R will be needed for use in either (4) or (5). Once R is supplied, the equivalent sheet resistance can be solved for yielding

$$\frac{\eta_s}{\eta} = \frac{1-R}{1+R} \tag{6}$$

from (4), or

$$\frac{\eta_s}{\eta} = \frac{1-R}{2R} \tag{7}$$

from (5). For η_s (and therefore R) to be of use in an MM solution, it must be independent of angle because it occurs as a load impedance that is added to the perfect conductor MM impedance matrix [6].

To obtain a resistance from the Astrakhan formulas that is independent of angle, consider the special case of a square grid with perfect wire contact. Using the notation defined in [2], a Cartesian coordinate system is established on the mesh surface with the z axis as the surface normal and the x and y axes coincident with the two (orthogonal) wire axes. The direction of a plane wave impinging

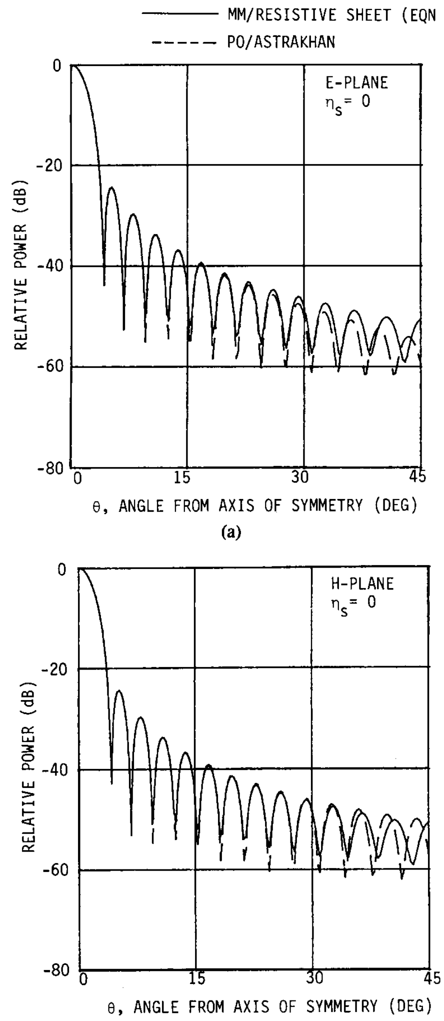

Fig. 1. Comparison of the methods of analysis for a perfectly conducting paraboloid. (a) *E*-plane. (b) *H*-plane. ($D = 20\lambda$, $f/D = 0.4$, $\cos\theta$ feed.)

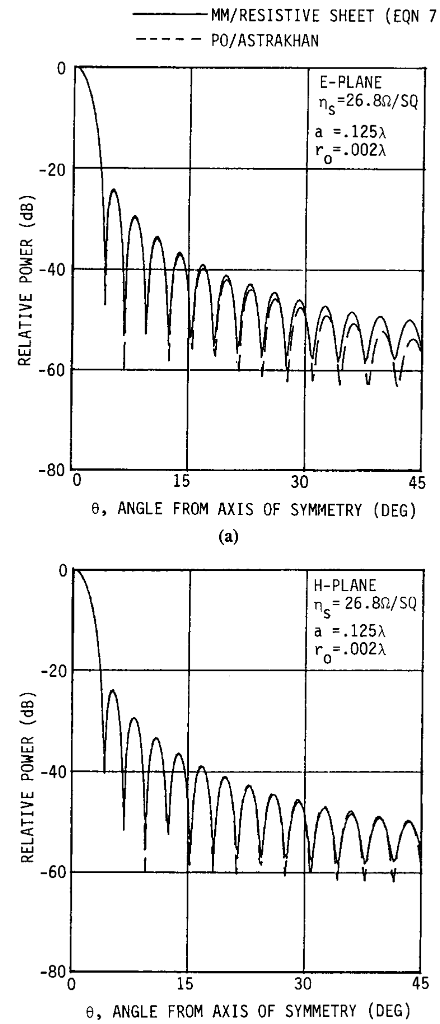

Fig. 2. Comparison of the methods of analysis for a paraboloidal reflector with a mesh surface: (a) *E*-plane. (b) *H*-plane. ($D = 20\lambda$, $f/D = 0.4$, $\cos\theta$ feed.)

on the mesh is specified by the spherical polar angles (θ, ϕ). For a square grid the dependence on the azimuth incidence angle (ϕ) drops out, and for most reflector antennas the angle of incidence with the local mesh surface normal (θ) will be small. Therefore, at near normal angles the parallel and perpendicular reflection coefficients of [2] are approximately

$$R^e_{\|} = -R^h_{\perp} = \left\{1 - j\frac{2a}{\lambda}\ln\left(\frac{a}{2\pi r_0}\right)\right\}^{-1}. \tag{8}$$

In (8), a is the grid separation in both mesh planes, r_0 is the radius of the wires, and λ the wavelength. Thus, an approximate value of R for use in (6) or (7) is

$$R = |R^e_{\|}| = |R^h_{\perp}|. \tag{9}$$

III. Radiation Pattern Calculations

The radiation patterns of a prime focus paraboloid with a mesh surface were calculated by two techniques. The first uses the physical optics approximation in conjunction with the Astrakhan formulas [7]. The second method is based on an MM solution of the *E*-field integral equation (EFIE) for a body of revolution as reported in [8]. The calculation of the load impedance elements to account for the surface resistance is similar to that described in [6] and [9]. Fig. 1 compares the PO and MM results for a perfectly conducting 20 λ paraboloid with $f/D = 0.4$ and a $\cos\theta$ shaped feed in both principal planes. The agreement is good out to about 40° or so, at which point the discrepancy can be attributed to the reflector edge. MM correctly accounts for the change in the current near the edge, whereas PO does not.

Fig. 2 compares the principal plane patterns of a mesh reflector of the same geometry with mesh separations of 0.125 λ and a wire radius of 0.002 λ. This corresponds to a surface resistance of 26.8 ohms per square. The agreement between MM and PO is better in the sense that the locations of the peaks and nulls of the sidelobes are coincident. The improvement is probably due to a diminishing of the edge effect because electrically the reflector edge is "softer" for the mesh than it is for the perfect conductor. The main beam gain values are within 0.02 dB of each other, but for larger mesh separations the

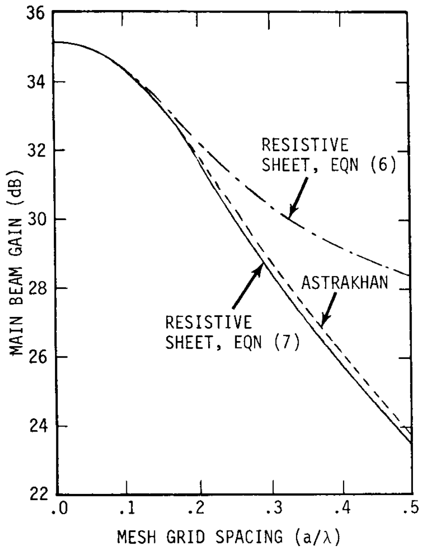

Fig. 3. Gain of a 20λ paraboloid as a function of mesh grid size. ($f/D = 0.4$, $\cos\theta$ feed, $r_0 = 0.002\lambda$.)

agreement is not as good, as shown in Fig. 3. The advantage of (6) over (7) for higher resistances is also evident in Fig. 3.

IV. Conclusion

For certain grid geometries and electrical properties, a mesh can be modeled accurately as a thin resistive sheet. The close agreement in antenna gains calculated by the two methods described here begins to break down when the surface resistance is greater than about 50 ohms per square. It is expected that significant discrepancies at wider angles would occur at even lower values [10].

It is conceivable that an equivalent sheet resistance could be obtained for rectangular grids or ones with imperfect contact at the junctions. Difficulty occurs for these cases because the ϕ dependence does not drop out of the Astrakhan formulas as it does for the square grid. It would be necessary to define an angle (or perhaps an range of angles) at which the Astrakhan formulas would be evaluated. Since the dependence of the reflection coefficients on angle is much stronger, it is expected that the overall agreement would not be as good.

This method can be extended to more complex surfaces that are comprised of both mesh and perfect conductor portions. In addition to reflector antennas this includes mesh screens used for antenna ground planes and electromagnetic interference suppression. It could also be used to choose a resistive sheet that could be substituted for mesh in the laboratory.

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Improvement of the Numerical Solution of Dielectric Bodies with High Permittivity

AHMED A. KISHK, MEMBER, IEEE, AND
LOTFOLLAH SHAFAI, FELLOW, IEEE

Abstract—**A method for improving the accuracy of the numerical solution of dielectric bodies is presented. Its utilization makes the matrix size independent of the relative dielectric constant and reduces its size. It also improves the accuracy of the solution of the Müller formulation when the dielectric constant is high. The root mean square (rms) errors are calculated for dielectric spheres by comparing the numerical solution with the exact solution using Mei series. The surface current distributions are presented in magnitude and phase. The bistatic radar cross sections of the sphere and finite cylinder are presented using different formulations.**

I. Introduction

In the past few years, a number of different methods have been developed to compute the scattering from homogeneous dielectrics and recently, dielectric resonators. These methods are the T-matrix [1], [2], unimoment [3], Fredholm integral equation approach [4], and the method of moments for the surface integral formulations [5], [6]. Each of these methods has certain limitations in its implementation. The method of moments is the most efficient method for solving the surface integral equations of homogeneous bodies and is easily adaptable to different integral equations. However, the solution accuracy deteriorates as the dielectric constant increases [5]. In [6], different integral equation formulations were used to overcome this

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A. A. Kishk is with the Department of Electrical Engineering, University of Mississippi, University, MS 38677.
L. Shafai is with the Department of Electrical Engineering, University of Manitoba, Winnipeg, MB, R3T 2N2 Canada.
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