Grating Lobe Suppression for Distributed Digital Subarrays Using Virtual Filling

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***Abstract*—A receiving array processing method is presented that synthesizes the antenna response of a contiguous array from the outputs of physically separated arrays of subarrays. It involves filling the gaps between the subarrays with "virtual" elements, thus forming a contiguous array. Therefore, the synthesized pattern has no grating lobes, and amplitude tapering for sidelobe control can be applied. The contiguous array's response can be duplicated in a number of directions limited by the total number of elements. The directions of interest would normally be those of the desired signals (main beam) and interference or clutter (low sidelobes). The directions of arrival of the signals of interest must first be determined before synthesizing the contiguous array response. The modified matrix pencil method has been extended to handle the problem of multiple subarrays for both single and multiple snapshots. The performance of the synthesis method is examined as a function of signal-to-noise ratio (SNR) per element and various array parameters.**

***Index Terms*—Distributed subarray antennas, grating lobe suppression, matrix pencil method, virtual elements.**

I. Introduction

COMPLETE digital control of amplitude and phase at the element level of an array allows great flexibility in beamforming. Modern radar and communications systems are incorporating phased arrays with wider bandwidths, allowing for the possibility that several systems on the same platform can share arrays. A system that incorporates distributed digital subarrays (DDSAs) working cooperatively as a single array (thus forming an array of subarrays) can potentially increase the output signal-to-noise ratio (SNR) and provide better spatial resolution compared to using the arrays individually. However, even if the individual array patterns have no grating lobes, conventional beamforming with periodic subarrays will have an output response with grating lobes, which is unacceptable for most applications.

Many methods have been employed to reduce the grating lobes, but all have their limitations and disadvantages. A common approach is to place subarray nulls at grating lobe locations using overlapping subarrays [1], but this severely limits the array geometry. Another approach is to rotate or tilt the subarrays, thereby reducing the periodicity [2]. The grating-lobe level varies as $20\log(1/N)$, where N is the number of subarrays. To be effective, this method requires a large number of subarrays. Random or fractal element spacings within the subarrays and randomizing the number of elements between subarrays have been used [3]. Again, large numbers of elements and subarrays are needed for truly random behavior, and only modest grating lobe suppression is achieved (for a 128-element linear array, the improvement is about 6 dB) [4]. Multiplicative beamforming has also been applied to suppress grating lobes [5], but the resultant gain loss (average of 6 dB loss) is the main drawback of this method [6].

Multiple signals that impinge on an array can be either desired (e.g., radar target return) or undesired (e.g., interference or clutter). If the subarrays are widely separated, then closely spaced grating lobes occur, and there will be many angles where an undesired signal has a large response when the main beam is scanned.

In this letter, we propose virtual filling of the gaps between the subarrays to eliminate the grating lobes on receiving so that the response of a single large contiguous array is synthesized. Therefore, no grating lobes will appear as long as element spacing within all subarrays is less than one half of the wavelength. Furthermore, amplitude tapering can be applied to the synthesized array to reduce interference and clutter. The output response for the synthetic array mimics the response of a contiguous array so that the mainbeam is in the direction of the desired signal and the interference is in a low sidelobe.

The virtual approach was recently suggested in [7] to fill gaps in the array matrices for super-resolution direction-of-arrival (DOA) estimation. They use minimum weighted norm (MWN) and super spatially variant apodization (Super-SVA) for virtual filling, which requires more computational power and large numbers of snapshots. Also, degradation in performance was observed for coherent signals. In [8] and [9], virtual elements were used within a single array, but only single source and noiseless cases were considered. Here, a method is presented to estimate the virtual element weights from the in-phase (I) and quadrature (Q) baseband signals received by the real elements. When multiple signals are incident, the information needed to reconstruct the contiguous array response is the DOAs, magnitudes, and phases.

In Section II, the DDSA model is introduced. Because DOA estimation is crucial to synthesizing the virtual element weights, Section III discusses how some modified "single snapshot" DOA algorithms perform with regard to this problem. A multiple snapshot algorithm is also described in Section III. In Section IV, we present formulas for estimating signal amplitudes and phases based on the DOAs. The dependence on element-level SNR of the synthesized array response is formulated. Section V contains a summary and conclusions.

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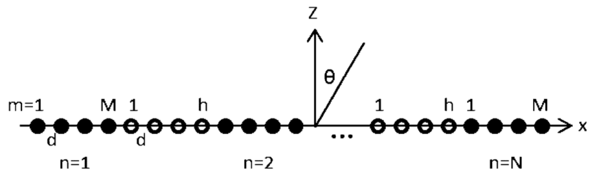

Fig. 1. Linear distributed digital subarray model. Black filled dots are real elements, and nonfilled dots are virtual elements.

II. DDSA Model

For simplicity, we consider a linear array of N identical subarrays. Each subarray contains M elements that are equally spaced d along the x-axis as shown in Fig. 1. The gap between subarrays is $D = h\,d$, where h is an integer greater than zero (i.e., the gap is an integer multiple of the element spacing as shown in Fig. 1).

If the entire array is centered at the origin, the location of element m in subarray n is

$$\begin{aligned} x(n,m) &= \left[m - \frac{NM+(N-1)h+1}{2} + (n-1)(M+h)\right] d \\ &\equiv P(n,m)d \qquad (n=1,2,\ldots,N;\ m=1,2,\ldots,M). \end{aligned} \tag{1}$$

If there are $K \leq NM$ signals incident on the array from angles θ_s $(s = 1,2,\ldots,K)$ with complex voltages $V_s e^{j\phi_s}$ the element outputs can be expressed in phasor form ($e^{j\omega t}$ time dependence) as

$$\begin{aligned} A(n,m) &= \sum_{s=1}^{K} V_s \exp(-jkP(n,m)d\sin\theta_s + j\phi_s) \\ &\equiv I(n,m) + jQ(n,m) \end{aligned} \tag{2}$$

where $k = 2\pi/\lambda$ (λ is wavelength).

III. DOA Estimation and Effect of Noise

For DOA estimation, only real elements are used. From measurement of the I and Q at the real elements, the signal parameters θ_s and $V_s e^{j\phi_s}$ can be estimated. Thermal noise is accounted for by adding a complex noise to the $A(n,m)$ in (2). The noise leads to an error in the parameter estimates, which in turn results in a distortion of the synthesized antenna response. Numerous DOA estimation algorithms are available, but the matrix pencil (MP) method performs particularly well for single-snapshot noisy data. It utilizes singular value decomposition (SVD) to divide the matrix space into signal and noise subspaces. By discarding the eigenvector corresponding to the noise signal, the noise effect can be reduced and hence the estimation accuracy can be improved. In principal, MP requires only a single snapshot, but it can be extended to multiple snapshots, thus resulting in a lower root mean square error (RMSE) [10].

We propose an extension of the MP method that is tailored to the DDSA by arranging the Hankel matrices of each subarray from top to bottom sequentially. Let $\mathbf{Y}_n$ be the Hankel matrix for subarray n

$$\mathbf{Y}_n = \begin{pmatrix} A(n,1) & \cdots & A(n,L) \\ \vdots & \ddots & \vdots \\ A(n,M-L+1) & \cdots & A(n,M) \end{pmatrix}_{[(M-L+1)]\times L} \tag{3}$$

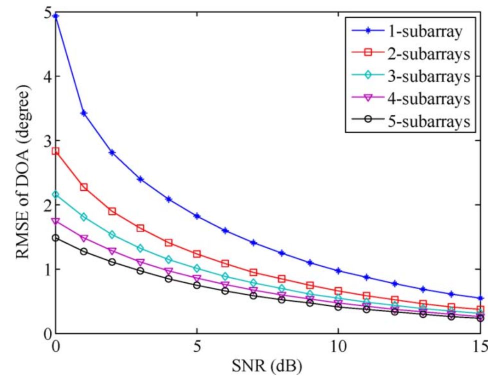

Fig. 2. RMSE of DOA for one signal versus SNR per element for various numbers of subarrays. Each subarray has eight elements spaced 0.42λ, and gaps are equal to the subarray size.

so that for a single snapshot (SS)

$$\mathbf{Y}_{\text{SS}} = \begin{pmatrix} \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_N \end{pmatrix}_{[(M-L+1)N]\times L}. \tag{4}$$

L is the pencil parameter that is selected between $M/3$ and $M/2$ for best noise reduction [11].

The multiple-snapshot MP can be considered as a concatenation of multiple columns of single-snapshot MP. If $\mathbf{Y}_{n,p}$ is the Hankel matrix for snapshot p $(p = 1,2,\ldots,P)$ of subarray n

$$\mathbf{Y}_{n,p} = \begin{pmatrix} A_p(n,1) & \cdots & A_p(n,L) \\ \vdots & \ddots & \vdots \\ A_p(n,M-L+1) & \cdots & A_p(n,M) \end{pmatrix}_{(M-L+1)\times L} \tag{5}$$

then for multiple snapshots (MS)

$$\mathbf{Y}_{\text{MS}} = \begin{pmatrix} \mathbf{Y}_{1,1} & \cdots & \mathbf{Y}_{1,P} \\ \vdots & \ddots & \vdots \\ \mathbf{Y}_{N,1} & \cdots & \mathbf{Y}_{N,P} \end{pmatrix}_{[(M-L+1)N]\times(LP)}. \tag{6}$$

Note that computational time increases with the number of snapshots. Some advantages of using multiple snapshots are to stabilize the DOA estimation for a small number (one or two) of subarrays with low SNR (0–4 dB). The RMSE of DOA estimation tends to decrease as the number of snapshots increases. However, in terms of DOA estimation, the average of multiple single snapshots will provide more accurate results compared to the multiple snapshot case as in (6).

After the matrices for the DDSA are formed, the standard MP procedure for finding the DOAs of signals in the noisy environment is applied [12].

The improved performance of the angle estimates from the modified matrix pencil method was verified using a Monte Carlo simulation with 100 trials. First, a signal is incident from 30° with phase $\pi/5$ radian onto a DDSA composed of five eight-element linear arrays with an element spacing of 0.42λ. The spacing between subarrays is 3.36λ. The advantage of using the modified matrix pencil method can be observed in Fig. 2. It can be seen that for one subarray (i.e., small NM) at low SNR,

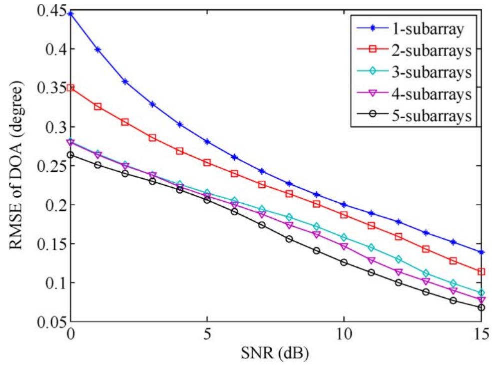

Fig. 3. RMSE of DOA for signal 1 versus SNR per element for various numbers of subarrays. Each subarray has 80 elements spaced 0.42λ, and gaps are equal to the subarray size.

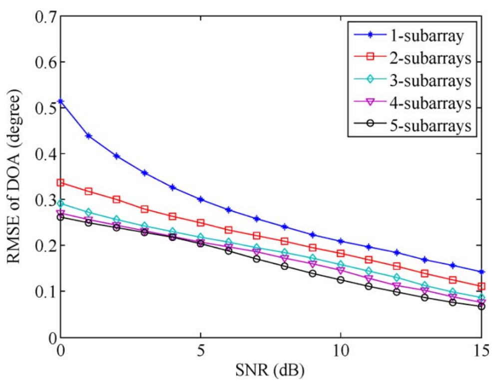

Fig. 4. RMSE of DOA for signal 2 versus SNR per element for various numbers of subarrays. Each subarray has 80 elements spaced 0.42λ, and gaps are equal to the subarray size.

the RMSE of the DOA is high. As expected, employing more subarrays makes the estimates much more accurate.

Next, we increase the number of elements in the DDSAs from 8 to 80. The gaps are now $80d = 33.6\lambda$ long. Also, the number of signals is increased to two: one from 30° and a second from 31°. The phases of the signals are $\pi/5$ and $-4\pi/5$, respectively. From the curves in Figs. 3 and 4, it can be seen that the increasing the number DDSAs provides much smaller RMSE even at low SNR (0 dB) for the two signals. If the fast Fourier transform (FFT) method [13] were used at the subarray level, the beamwidth would be too wide to resolve the two closely spaced signals. Just as for the single-target case, using more subarrays in the processing results in much more stable and accurate results, which is crucial for effective "filling."

IV. SUBARRAY FILLING METHOD

The complex signal at the output of the elements can be written in terms of the estimated angles as

$$\begin{aligned} A(n,1) &= \sum_{s=1}^{K} V_s \exp(-jkP(n,1)d\sin\theta_s)\exp(j\phi_s) \\ &\vdots \\ A(n,M) &= \sum_{s=1}^{K} V_s \exp(-jkP(n,M)d\sin\theta_s)\exp(j\phi_s). \end{aligned} \tag{7}$$

Casting these in matrix form gives

$$\mathbf{E}_n \mathbf{V} = \mathbf{A}_n \tag{8}$$

where

$$\mathbf{E}_n = \begin{pmatrix} e^{-jkP(n,1)d\sin\theta_1} & \cdots & e^{-jkP(n,1)d\sin\theta_K} \\ \vdots & \ddots & \vdots \\ e^{-jkP(n,M)d\sin\theta_1} & \cdots & e^{-jkP(n,M)d\sin\theta_K} \end{pmatrix}_{M\times K} \tag{9}$$

$$\mathbf{V} = \begin{pmatrix} V_1 e^{j\phi_1} \\ \vdots \\ V_K e^{j\phi_K} \end{pmatrix}_{K\times 1} \quad \text{and} \quad \mathbf{A}_n = \begin{pmatrix} A(n,1) \\ \vdots \\ A(n,M) \end{pmatrix}_{M\times 1}. \tag{10}$$

To estimate signal magnitudes and phases, a least-squares method can be used that employs all subarray element outputs [11]. They are assembled column-wise and solved to obtain estimates of the complex signals $\hat{\mathbf{V}}$

$$\begin{pmatrix} \hat{\mathbf{V}}_1 \\ \vdots \\ \hat{\mathbf{V}}_K \end{pmatrix}_{K\times 1} = \begin{pmatrix} \mathbf{E}_1 \\ \vdots \\ \mathbf{E}_N \end{pmatrix}^{-1}_{NM\times K} \begin{pmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_N \end{pmatrix}_{NM\times 1} \tag{11}$$

Note that adding subarrays increases the total number of elements and thus the number of signals that can be handled.

Now the estimated signal magnitudes $\hat{V}_s$ and phases $\hat{\phi}_s$ can be used to create virtual complex data to "fill" the gaps between subarrays. Since an incident plane wave is assumed, an amplitude equal to the real elements is used, along with a linear phase based on the known or estimated DOA. The location of virtual element r in gap i (between subarrays i and $i+1$) is

$$\begin{aligned} x(i,r) &= \left(r - \frac{(N-2)M + (N-1)h + 1}{2} + (i-1)(M+h)\right) d \\ &\equiv Z(i,r)d \qquad (r = 1,2,\ldots,h;\ i = 1,2,\ldots,N-1) \end{aligned} \tag{12}$$

and the complex data for filling is given by the same formula as for the real data in (2)

$$B(i,r) = \sum_{s=1}^{K} \hat{V}_s \exp(-jkZ(i,r)d\sin\theta_s)\exp(j\hat{\phi}_s). \tag{13}$$

Combining the real and virtual data gives the response of the synthesized array

$$\begin{aligned} \mathbf{F} = [(\mathbf{A}_1)_{1\times M}(\mathbf{B}_1)_{1\times h} \cdots \\ (\mathbf{B}_{N-1})_{1\times h}(\mathbf{A}_N)_{1\times M}]_{1\times[NM+(N-1)h]} \end{aligned} \tag{14}$$

where the $\mathbf{B}$ partitions are compoed of the terms given by (13). $\mathbf{F}$ represents the complex outputs of the synthesized array in the K signal directions. Multiplying by the desired beamforming weights and summing give the array response.

Consider a five-subarray DDSA with 30 elements in each subarray and an element spacing of 0.42λ. The subarray length is 12.6λ. The gaps are also (arbitrarily) set to 12.6λ. One unit amplitude signal is incident from 0° with a phase of $\pi/5$. A second interference signal is coming in at 2.3° with a phase $-4\pi/5$. The pattern is shown in Fig. 5 as the weights are changed to scan the main beam in a region of direction cosine

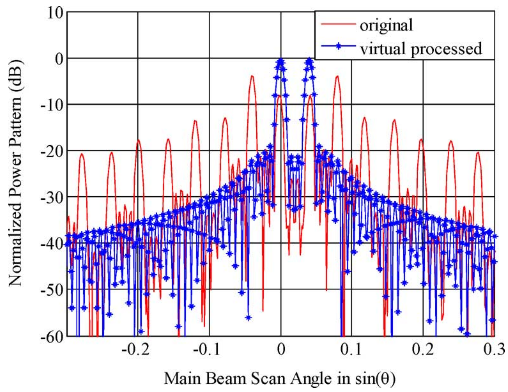

Fig. 5. Comparison of original and synthesized antenna responses of five subarrays each with 30 elements, for signals of equal power (noiseless) incident from 0° and 2.3°.

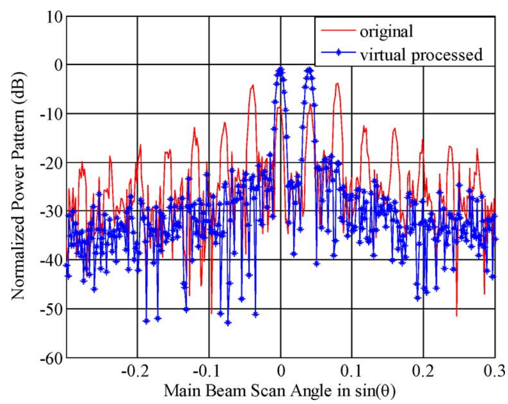

Fig. 6. Comparison of original and synthesized antenna response after virtual filling for an element level SNR of 6 dB with 20 dB sidelobe tapering.

TABLE I
EXACT AND ESTIMATED SIGNAL PARAMETERS FOR VIRTUAL PROCESSING

Parameter	Actual		Estimated (mean)		Estimated (variance)	
	Signal 1	Signal 2	Signal 1	Signal 2	Signal 1	Signal 2
DOA (degrees)	0	2.3	0.0356	2.2965	0.1808	0.2878
Amplitude (Volts)	1	1	0.9630	0.9533	0.002	0.0034
Phase (radians)	0.6283	2.5133	0.6371	2.5045	0.0018	0.002

space $(\sin\theta)$. A 20-dB Taylor amplitude distribution is applied. As can be seen, the high response of the interfering signal that occurs at grating lobe locations has been eliminated. The synthesized array response in the direction of both signals is the same as that of a contiguous array.

Fig. 6 shows the average synthesized array response for the same case shown in Fig. 5, but with an SNR per element of 6 dB (single snapshot). Table I summarizes the results of a Monte Carlo simulation of 30 trials (equivalent to 30 single snapshots) using (4) for the five-subarray two-signal case used to generate Fig. 6.

The effectiveness of the virtual approach relies on having accurate angle estimates for the signals of interest, which in turn requires a high effective array SNR (i.e., large NM if the SNR per element is low; high SNR if the total number of elements NM is small). The SNR per element can be increased by adding a low noise amplifier or increasing the gain of the array element. It is also possible to improve the DOA estimates with multiple snapshots or by averaging multiple single snapshots.

V. SUMMARY AND CONCLUSION

Filling of the gaps between arrays with virtual elements for the purpose of receiving processing allows a synthesized antenna response that duplicates a contiguous array in a number of directions that is limited by the total number of elements used in the processing. We have neglected the effects of errors and assumed that common time and frequency references are available to all subarrays.

As a first step, the signal amplitudes, phases, and DOAs must be extracted from the real element I and Q samples. The extracted signal parameters are used to generate I and Q data that would be provided by virtual elements filling the gaps between subarrays. Low sidelobes and interference rejection were demonstrated for the virtual processed DDSA.

The matrix pencil method was found to be well suited for this application. The MP technique was extended to handle multiple subarrays, for either single or multiple snapshots. Multiple snapshots provide improved stability in a low-SNR situation. This method can deal with coherent and noncoherent signals and requires fewer snapshots for accurate DOA estimation.

The proposed filling method allows suppression of undesired signals that would normally occur at grating lobe angles. It was shown that, for high SNR, the response approaches that of a contiguous array of the same extent.

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