Basic Theorems and Concepts
(Chapter 2)

EC4630 Radar and Laser Cross Section

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Prof. D. Jenn
jenn@nps.edu

Introduction

Review/introduce basic concepts and tools for use in RCS analysis. Many are familiar from antenna analysis:

Theorems:

- Uniqueness
- Reciprocity
- Duality
- Superposition
- Similitude

Principles:

- Method of images and boundary conditions
- Equivalence
  - Surface
  - Volume
- Magnetic currents
- Equivalent currents
- Huygen's

Tools:

- Radiation integrals (Stratton-Chu integrals)
- Far-field approximations
- Physical optics approximation
- Arrays of scatterers
- Null field hypothesis
- Surface impedance
- Impedance boundary conditions
- Discontinuity boundary conditions
- Surface waves

Review and Notation* (1)

Electrical properties of a medium are specified by its constitutive parameters:

- permeability, $\mu = \mu_o \mu_r$ (for free space, $\mu \equiv \mu_o = 4\pi \times 10^{-7}$ H/m)
- permittivity, $\varepsilon = \varepsilon_o \varepsilon_r$ (for free space, $\varepsilon \equiv \varepsilon_o = 8.85 \times 10^{-12}$ F/m)
- conductivity, σ (for a metal, $\sigma \sim 10^7$ S/m)

Electric and magnetic field intensities:* $\vec{E}(x,y,z,t)$ V/m and $\vec{H}(x,y,z,t)$ A/m

- vector functions of location in space and time, e.g., in Cartesian coordinates

$$\vec{E}(x,y,z,t) = \hat{x}E_x(x,y,z,t) + \hat{y}E_y(x,y,z,t) + \hat{z}E_z(x,y,z,t)$$

- similar expressions for other coordinates systems
- the fields arise from current $\vec{J}$ and charge ρ_v on the source ($\vec{J}$ is the volume
- current density in A/m^2 and ρ_v is volume charge density in C/m^3)

Electromagnetic fields are completely described by Maxwell's equations (shown here in differential form):

$$(1)\ \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (2)\ \nabla \times \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (3)\ \nabla \cdot \vec{H} = 0 \quad (4)\ \nabla \cdot \vec{E} = \rho_v / \varepsilon$$

*The same symbols are used for the time and frequency domains. The argument is explicitly written when needed for clarity. See Appendix A in the book for a complete review.

Review and Notation (2)

The wave equations are derived from Maxwell's equations:

$$\nabla^2 \vec{E} - \frac{1}{u_p^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad \nabla^2 \vec{H} - \frac{1}{u_p^2} \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

The phase velocity is $u_p = \omega\sqrt{\mu\varepsilon}$ (in free space $u_p = c = 2.998 \times 10^8$ m/s)

The simplest solutions to the wave equations are plane waves. For example, a plane wave propagating in the z direction is:

$$\vec{E}(z,t) = \hat{x}\, E_o e^{-\alpha z} \cos(\omega t - \beta z)$$

- $\alpha =$ attenuation constant (Np/m); $\beta = 2\pi / \lambda =$ phase constant (rad/m)
- $\lambda =$ wavelength; $\omega = 2\pi f$ (rad/sec); $f =$ frequency (Hz); $f = \dfrac{u_p}{\lambda}$

Features of this plane wave:

- propagating in the $+z$ direction
- x polarized (direction of electric field vector is $\hat{x}$)
- peak amplitude of the wave is E_o

Review and Notation (3)

Phasor notation: A time-harmonic plane wave is represented by the phasor $\vec{E}(z)$

$$\vec{E}(z,t) = \mathrm{Re}\left\{\hat{x}\, E_o e^{-(\alpha + j\beta)z} e^{j\omega t}\right\} = \mathrm{Re}\left\{\vec{E}(z) e^{j\omega t}\right\}$$

$\vec{E}(z)$ is the phasor representation;

$\vec{E}(z,t)$ is the instantaneous quantity

$\mathrm{Re}\{\cdot\}$ is the real operator (i.e., “take the real part of”)

$j = \sqrt{-1}$

Since the time dependence varies as $e^{j\omega t}$, the time derivatives in Maxwell’s equations are replaced by $\partial / \partial t \equiv j\omega$:

$$(1)\ \nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad (3)\ \nabla \cdot \vec{H} = 0$$

$$(2)\ \nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E} \qquad (4)\ \nabla \cdot \vec{E} = \rho_v / \varepsilon$$

The wave equations are derived from Maxwell’s equations:

$$\nabla^2 \vec{E} - \gamma^2 \vec{E} = 0$$

$$\nabla^2 \vec{H} - \gamma^2 \vec{H} = 0$$

where $\gamma = \alpha + j\beta$ is the propagation constant.

Review and Notation (4)

Plane and spherical waves belong to a class called transverse electromagnetic (TEM). They have the following features:

1. $\vec{E}$, $\vec{H}$ and the direction of propagation $\hat{k}$ are mutually orthogonal
2. $\vec{E}$ and $\vec{H}$ are related by the intrinsic impedance of the medium

$$\eta = \sqrt{\frac{\mu}{(\varepsilon - j\sigma/\omega)}} \quad \Rightarrow \quad \eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \approx 377\ \Omega \text{ for free space}$$

The above relationships are expressed in the vector equation $\vec{H} = \dfrac{\hat{k} \times \vec{E}}{\eta}$

The time-averaged power propagating in the plane wave is given by the Poynting vector:

$$\vec{W} = \tfrac{1}{2}\operatorname{Re}\left\{\vec{E} \times \vec{H}^*\right\} \ \ \text{W/m}^2$$

For a plane wave: $\vec{W}(z) = \dfrac{1}{2}\dfrac{|E_o|^2}{\eta}\hat{z}$

For a spherical wave: $\vec{W}(R) = \dfrac{1}{2\eta}\dfrac{|E_o|^2}{R^2}\hat{R}$ (R is the distance from the source, inverse square law for power spreading)

Review and Notation (5)

Materials have ohmic and dielectric losses that cause attenuation of a wave as it propagates through the medium. Energy is extracted from the wave. The attenuation constant determines the rate of decay of the wave. In general:

$$\alpha = \omega\left\{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]\right\}^{1/2} \qquad \beta = \omega\left\{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]\right\}^{1/2}$$

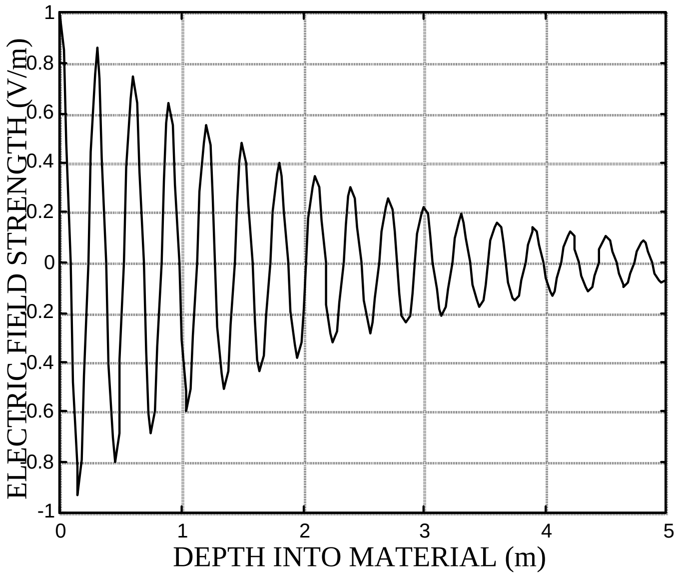
For lossless media $\sigma = 0 \quad \rightarrow \quad \alpha = 0$. Traditionally, for lossless cases, k is used rather than β. For good conductors ($\sigma/\omega\varepsilon >> 1$), $\alpha \approx \sqrt{\pi\mu f\sigma}$, and the wave decays rapidly with distance into the material.

<u>Skin depth</u>, δ_s is the distance the wave travels in the material for the amplitude to decay by a factor of $1/e$

$$\left|e^{-\alpha\delta_s}\right| = e^{-1}$$

or

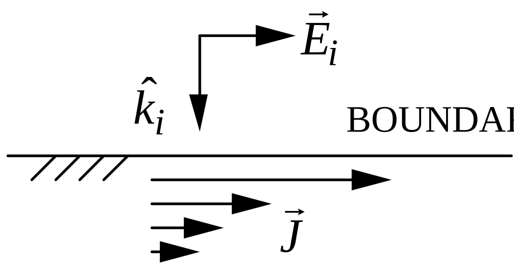
$$\delta_s = \frac{1}{\alpha}$$



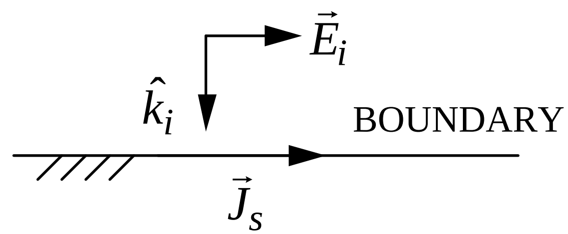
Review and Notation (6)

For good conductors at high frequencies the current is concentrated near the surface. The current can be approximated by an infinitely thin current sheet, or <u>surface current</u>, $\vec{J}_s$ A/m and <u>surface charge</u>, ρ_s C/m

Current in a good conductor



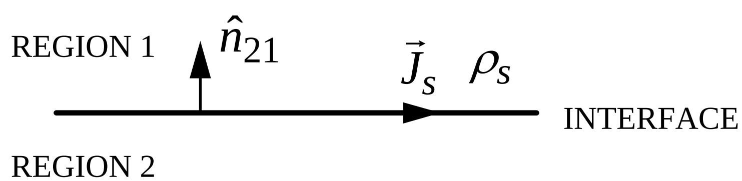
Surface current approximation



At an interface between two media the boundary conditions must be satisfied:

$$(1)\ \hat{n}_{21} \times (\vec{E}_1 - \vec{E}_2) = 0 \qquad (3)\ \hat{n}_{21} \cdot (\vec{E}_1 - \vec{E}_2) = \rho_s / \varepsilon$$

$$(2)\ \hat{n}_{21} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s \qquad (4)\ \hat{n}_{21} \cdot (\vec{H}_1 - \vec{H}_2) = 0$$



Magnetic Current

To our knowledge, free magnetic charges do not exist. For convenience, fictitious magnetic charge and current are frequently included in Maxwell's equations.

$\vec{J}_m$ = volume magnetic current density (V/m^2)
$\vec{J}_{ms}$ = surface magnetic current density (V/m)
ρ_{mv} = volume magnetic charge density (C/m^3)
ρ_{ms} = surface magnetic charge density (C/m^2)

Time-harmonic form of Maxwell's equations with magnetic charge and current:

$$(1)\ \nabla\times\vec{E} = -j\omega\mu\vec{H} - \vec{J}_m \qquad (3)\ \nabla\cdot\vec{B} = \rho_{mv}$$
$$(2)\ \nabla\times\vec{H} = \vec{J} + j\omega\varepsilon\vec{E} \qquad (4)\ \nabla\cdot\vec{E} = \rho_v/\varepsilon$$

Boundary conditions at interfaces with magnetic surface charge and current:

$$(1)\ \hat{n}_{21}\times(\vec{E}_1-\vec{E}_2) = -\vec{J}_{ms} \qquad (3)\ \hat{n}_{21}\cdot(\vec{D}_1-\vec{D}_2) = \rho_s$$
$$(2)\ \hat{n}_{21}\times(\vec{H}_1-\vec{H}_2) = \vec{J}_s \qquad (4)\ \hat{n}_{21}\cdot(\vec{B}_1-\vec{B}_2) = \rho_{ms}$$

Basic Theorems Summarized

Uniqueness Theorem: For a given problem, a solution to Maxwell's equations that satisfies the boundary conditions is unique.

- This theorem allows us to construct equivalent problems to the original one that are more easily solved. The equivalent problems are generally valid under some limited conditions (e.g. a limited region of space).
- An important fact arising from the uniqueness theorem is that a harmonic field $(\vec{E},\vec{H})$ in a source free dissipative region *V* is determined uniquely by the tangential components of $\vec{E}$ or $\vec{H}$ on the closed surface *S* that bounds the volume *V*.

Reciprocity Theorem: In general, the response of a system to a source is unchanged if the source and observer are interchanged.

- As applied to antennas constructed of linear isotropic materials and reciprocal devices, the transmitting and receiving patterns are identical.

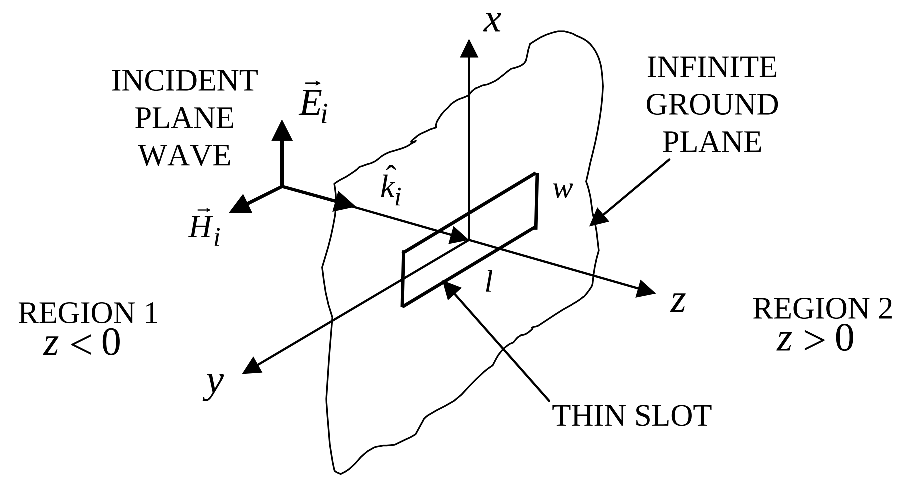
Superposition Theorem: For a linear medium, Maxwell's equations are linear. The total fields due to multiple sources turned on simultaneously is equal to the sum of the fields when energized separately.

- Superposition applies to the complex vector electric and magnetic field intensities $(\vec{E},\vec{H})$, not to the magnitudes $(|\vec{E}|,|\vec{H}|)$ or power $(\sim|\vec{E}|^2,\sim|\vec{H}|^2)$!

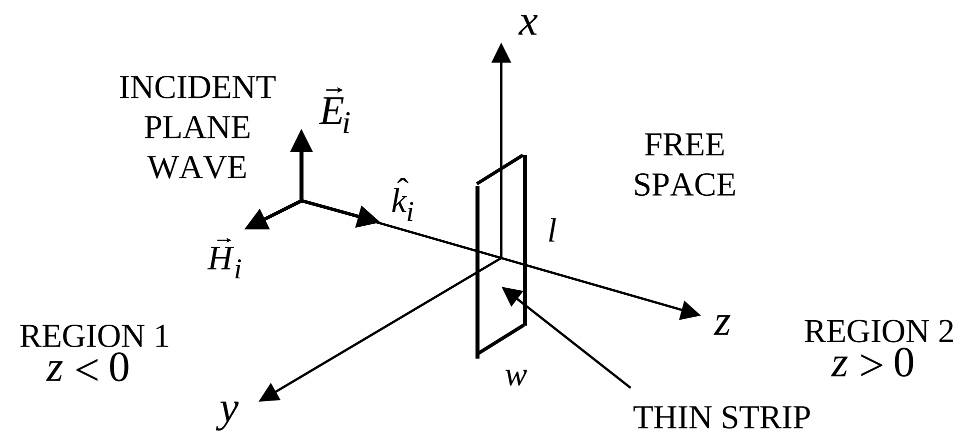
Basic Theorems Summarized

Duality Theorem: There are symmetries between electric and magnetic quantities in Maxwell's equations and the boundary conditions. When a problem is formulated, a dual problem can be obtained by substituting the quantities given in Table 2.1 of the book.

Electric	Magnetic
$\vec{E}$	$\vec{H}$
$\vec{H}$	$-\vec{E}$
$\vec{J}$	$\vec{J}_m$
ε	μ
μ	ε
η	$1/\eta$



Example 2.1 illustrates Babinet's principle, which deals with the fundamental dual structures of slots and strips of the same shape. When the polarization is reversed, the scattering patterns are identical.



Theorem of Similitude

This theorem is commonly referred to as frequency scaling. Scaled models are more convenient to use for development and testing. Let the full scale length and time variables be (x, y, z, t) and the corresponding scaled variables be (x', y', z', t').

Scale factors for field intensities: $\alpha = \vec{E} / \vec{E}', \quad \beta = \vec{H} / \vec{H}'$
Scale factor for length: $p = x / x' = y / y' = z / z'$
Scale factor for time: $q = t / t'$

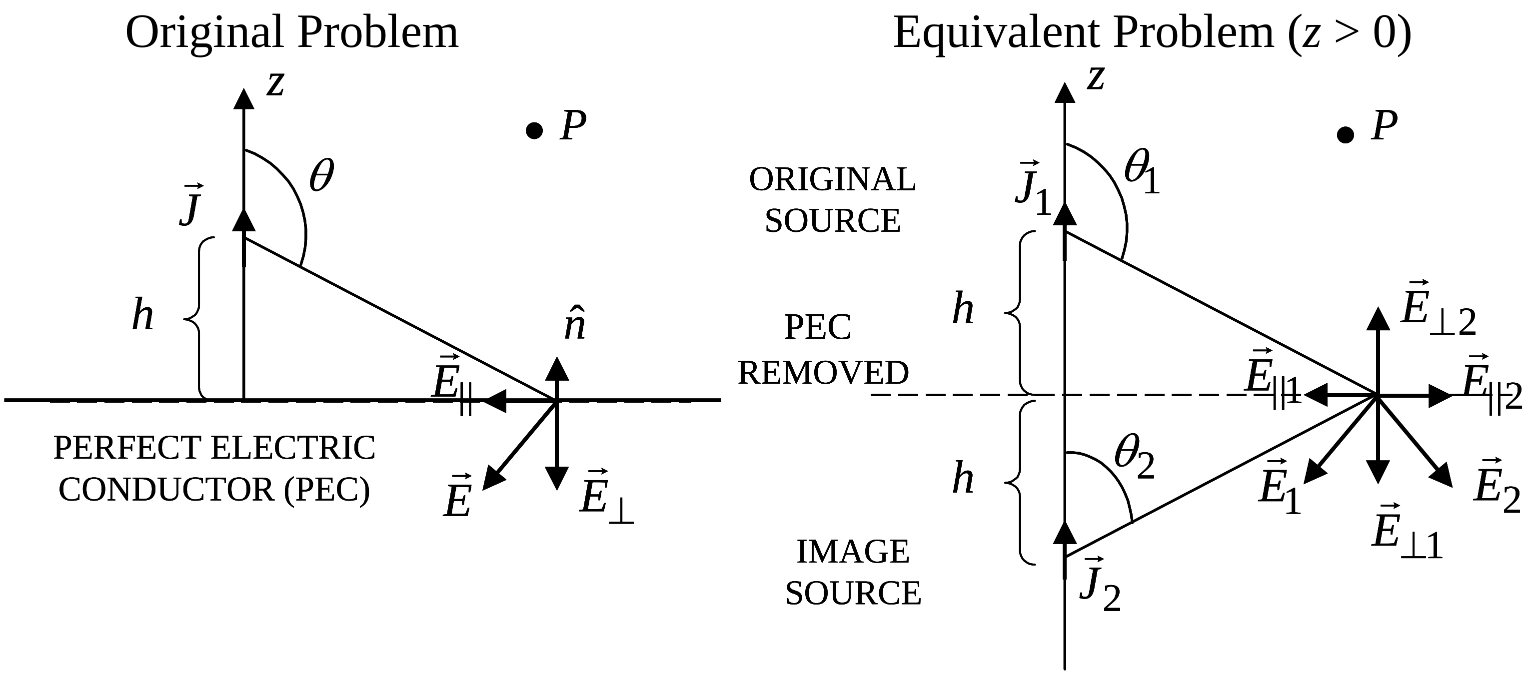
The special case of $p = q$ and $\alpha = \beta$ is called the geometrical model. It allows the same materials to be used in both the full and scaled models. The scaling of quantities is shown in Table 2.2.

Quantity	General case	Geometrical model
Time	$t' = (t/q)$	$t' = (t/p)$
Length	$L' = (L/p)$	$L' = (L/p)$
Wavelength	$\lambda' = (\lambda/q)$	$\lambda' = (\lambda/p)$
Frequency	$f' = qf$	$f' = pf$
Permittivity	$\epsilon' = (p\alpha\epsilon/\beta q)$	$\epsilon' = \epsilon$
Permeability	$\mu' = (p\beta\mu/\alpha q)$	$\mu' = \mu$
Conductivity	$\sigma' = (p\alpha\sigma/\beta)$	$\sigma' = p\sigma$
Current density	$\vec{J}' = (p\vec{J}/\beta)$	
Power density	$\vec{W}' = (\vec{W}/\alpha\beta)$	
Phase velocity	$u'_p = (qu_p/p)$	$u'_p = u_p$
Antenna gain	$G' = G$	
Propagation constant	$\gamma' = p\gamma$	
Impedance	$\eta' = (\beta\eta/\alpha)$	$\eta' = \eta$
Radar cross section	$\sigma' = (\sigma/p^2)$	$\sigma' = (\sigma/p^2)$

Method of Images

The method of images provides a means of creating equivalent problems that are more easily solved than the original problem.

Example: A differential electric current element over an infinite flat PEC. Find the electric and magnetic field intensities at *P*.



Remove the ground plane and solve the equivalent problem in a homogeneous medium.

Image Currents

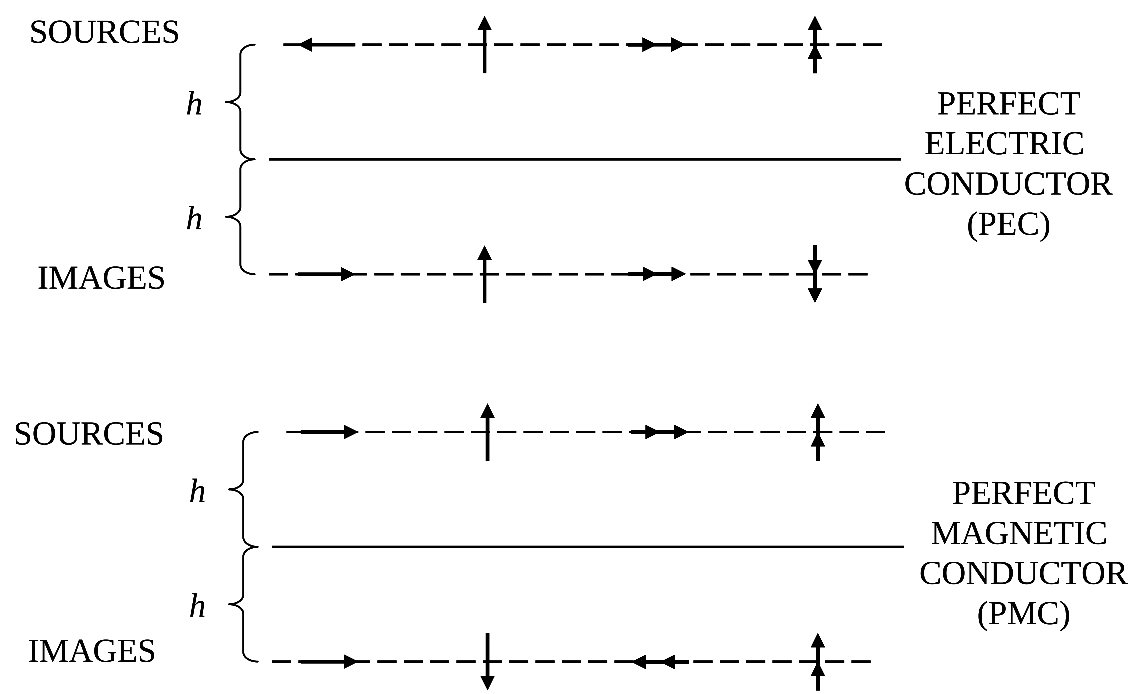
We can usually approximate a curved surface locally by a flat surface. An arbitrary current vector can be decomposed into components parallel (horizontal) and perpendicular to the surface (vertical).

The image equivalents for electric and magnetic current elements are shown in the diagram

$\vec{J} =$ single tipped arrows
$\vec{J}_m =$ double tipped arrows

Perfect electric conductor (PEC): $\vec{E}_{\tan} = 0$

Perfect magnetic conductor (PMC): $\vec{H}_{\tan} = 0$



(Note error in book Figure 2.6)

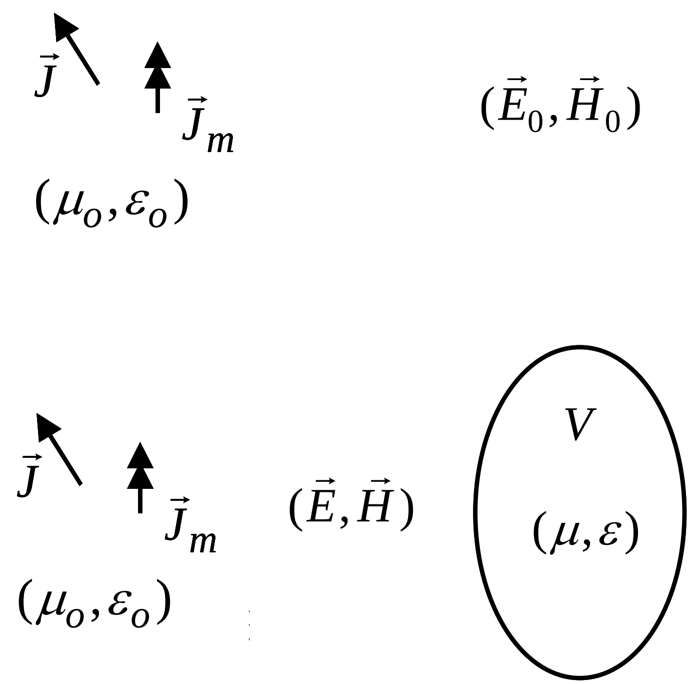
Equivalence Principles

<u>Equivalence principles</u> are another means of generating equivalent problems that can be easier to solve than the original. Sometimes the equivalent problem is not any easier to solve, but it involves different quantities, or has a different integral or differential form that is more suited to a numerical solution.

<u>Volume Equivalence</u>:

- Sources in free space with no object present set up fields $(\vec{E}_o, \vec{H}_o)$
- With the object (μ, ε) present the fields $(\vec{E}, \vec{H})$ exist.
- The scattered fields are the differences $(\vec{E}_s, \vec{H}_s) = (\vec{E} - \vec{E}_o, \vec{H} - \vec{H}_o)$
- Equivalent currents in V that would set up the same scattered fields are:

$$\vec{J}_{eq} = j(\varepsilon - \varepsilon_o)\vec{E}$$
$$\vec{J}_{meq} = j(\mu - \mu_o)\vec{H}$$



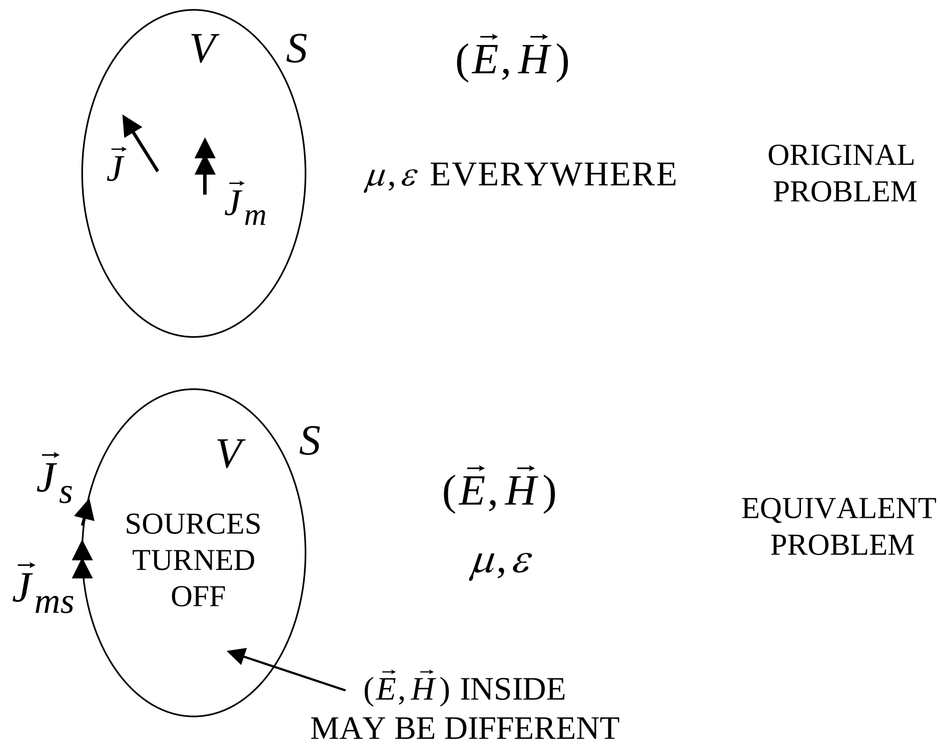
Equivalence Principles

Surface Equivalence:
We are interested only in the fields outside of S. An equivalent problem outside is to remove the sources and replace them with equivalent currents flowing on S:

$$\vec{J}_{seq} = \hat{n} \times \vec{H}$$
$$\vec{J}_{mseq} = -\hat{n} \times \vec{E}$$

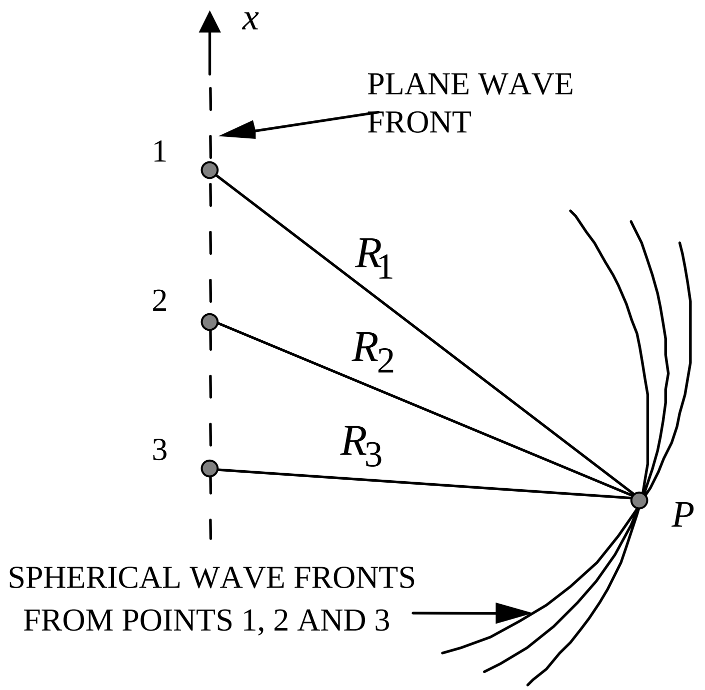
- Must keep the sources and media for which we keep the fields (outside in this case).
- Many solutions give the same fields outside. Usually we select the one that gives zero fields inside (i.e., Love's Equivalence).

- The currents radiate in an unbounded homogeneous medium.



Huygen's Principle

Each point on a wavefront can be considered as a new source of secondary spherical waves and that a new wave front can be constructed from the envelope of these secondary waves



For the plane wavefront shown:

$$\left|\vec{E}(P)\right| \sim \left|\sum_{n=-\infty}^{n=\infty}\left(\frac{e^{-jkR_n}}{R_n}\right)\right| \to \left|\int_{-\infty}^{\infty}\left(\frac{e^{-jkR(x')}}{R(x')}\right)dx'\right|$$

Huygen's principle can be used to determine the effect of obstructions on a wave. If part of a wavefront is blocked, the integral limits are modified appropriately. For example, for an aperture of length $2a$ (in the x direction) in a PEC ground plane:

$$\left|\vec{E}(P)\right| \sim \left|\int_{-a}^{a}\left(\frac{e^{-jkR(x')}}{R(x')}\right)dx'\right|$$

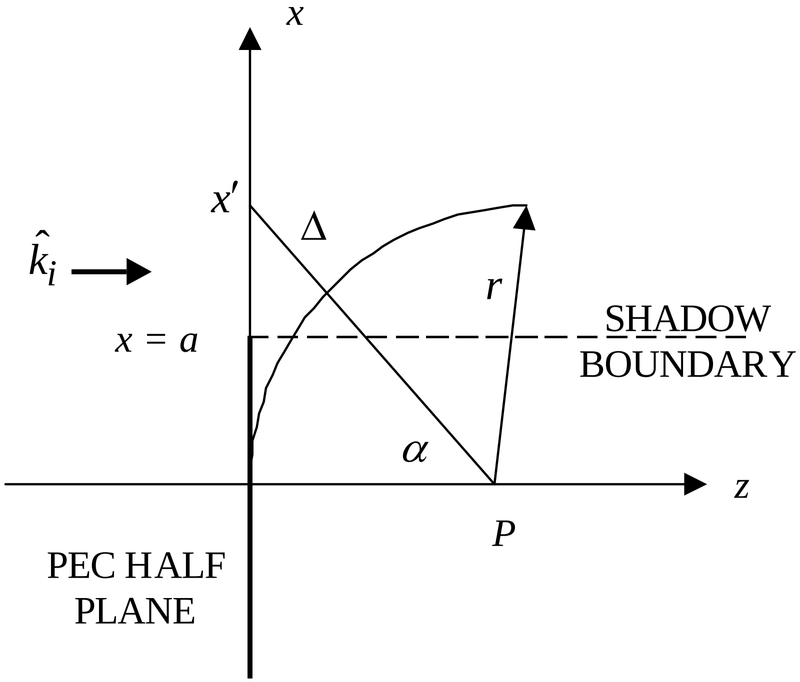
Knife Edge Diffraction

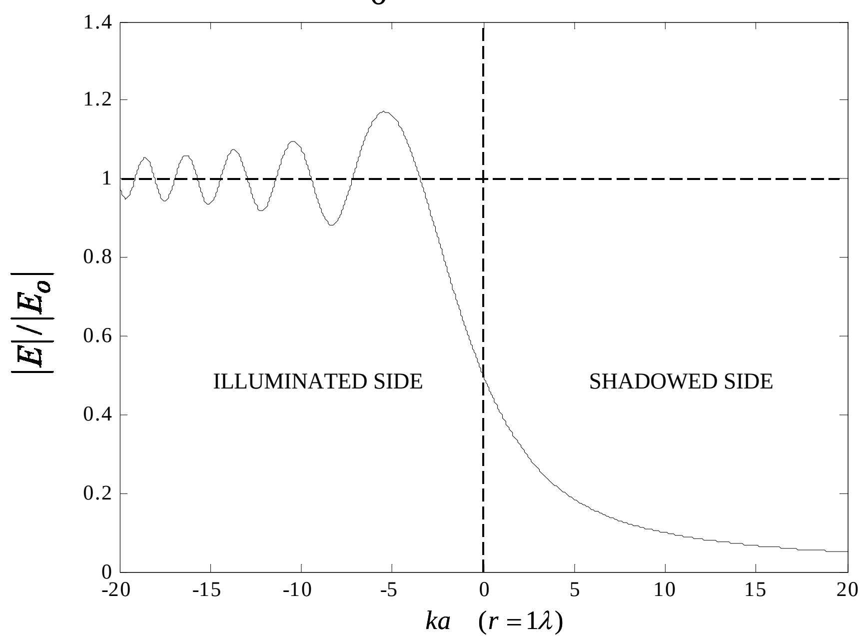
Example 2.8: Diffraction by a knife edge. The observation point P is in the x=0 plane. See details in the book. The final closed form result, in terms of $\xi = 2/(r\lambda)$, is:

$$\frac{|E(P)|}{|E_o|} = \left| \frac{e^{-jkr}}{\xi r} \left[\frac{1+j}{2} - C(\xi a) - jS(\xi a) \right] \right|$$

$$C(\alpha) = \int_0^\alpha \cos(\pi\tau^2/2)\, d\tau$$

$$S(\alpha) = \int_0^\alpha \sin(\pi\tau^2/2)\, d\tau$$



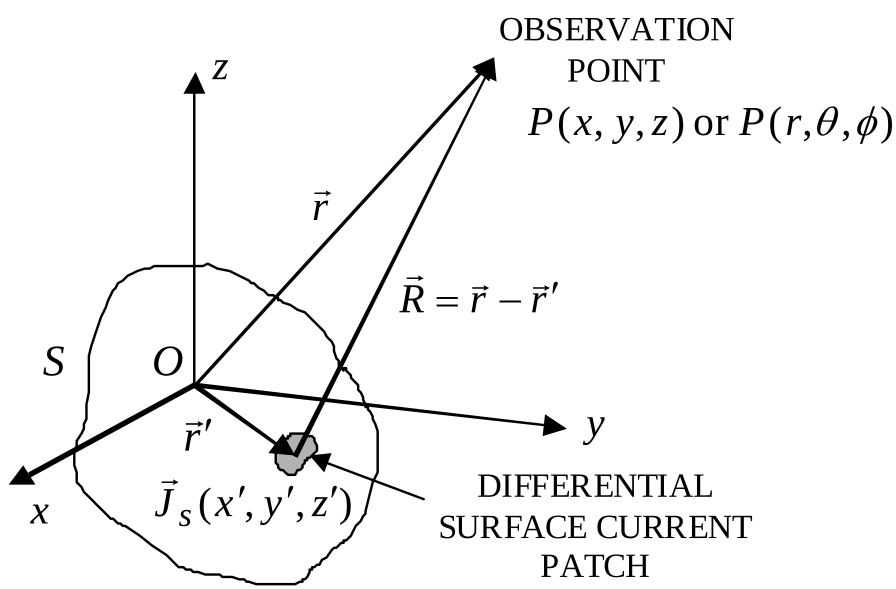


Radiation Integrals

Also known as the Stratton-Chu integrals, they give the electric and magnetic field intensities at an observation point P for a known distribution of electric and magnetic currents, $\vec{J}$ and $\vec{J}_m$. The geometry of a current distribution is shown in the figure. The currents can be volume currents $\left(\vec{J}, \vec{J}_m\right)$ or surface currents $\left(\vec{J}_s, \vec{J}_{ms}\right)$. An electric surface current is shown. Note that:

- Primed quantities are associated with the source point.
- Unprimed quantities are associated with the observation point, P.

For example, the currents are specified at source points so we write $\vec{J}_s(x', y', z')$, or more concisely, $\vec{J}_s(\vec{r}\,')$, since $\vec{r}\,' = x'\hat{x} + y'\hat{y} + z'\hat{z}$.



Radiation Integrals

The electric vector potential at the observation point is

$$\vec{A}(\vec{r}) = \mu \iint_S \vec{J}_s(\vec{r}\,')G(\vec{r},\vec{r}\,')ds'$$

where the Green's function is

$$G(\vec{r},\vec{r}\,') = \frac{\exp(-jkR)}{4\pi R}$$

with $\vec{R} = \vec{r} - \vec{r}\,' = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$ and $R = |\vec{R}|$. The dual quantity is the magnetic vector potential at the observation point

$$\vec{F}(\vec{r}) = \varepsilon \iint_S \vec{J}_{ms}(\vec{r}\,')G(\vec{r},\vec{r}\,')ds'$$

For volume distributions the integrals are in three dimensions. The fields at the observation point are:

$$\vec{E}(r) = -j\omega\vec{A}(\vec{r}) - \frac{j}{\omega\mu\varepsilon}\nabla\left(\nabla\bullet\vec{A}(\vec{r})\right) - \frac{1}{\varepsilon}\nabla\times\vec{F}(\vec{r})$$

$$\vec{H}(r) = -j\omega\vec{F}(\vec{r}) - \frac{j}{\omega\mu\varepsilon}\nabla\left(\nabla\bullet\vec{F}(\vec{r})\right) + \frac{1}{\mu}\nabla\times\vec{A}(\vec{r})$$

Note that the ∇ operators involve derivatives with respect to the unprimed (observation) coordinates.

Far Zone Radiation Integrals

Notes on the radiation integrals:

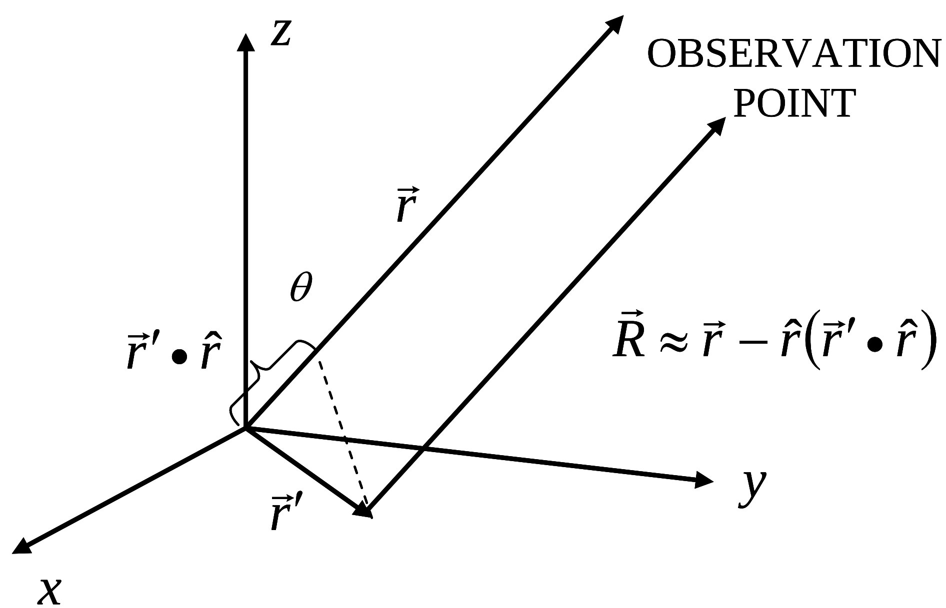
- They give the fields from known currents.
- The currents must radiate in an unbounded homogeneous medium.
- The observation point can be anywhere, even within the distribution of currents.

In reality, the currents of interest are those on flowing on antennas and scatterers, and the observation points are in the far zone of the current distribution. In this case we can make the approximation of parallel rays:

$$R = \left|\vec{r} - \vec{r}'\right| \approx r - \hat{r} \bullet \vec{r}'$$
$$\hat{r} = \hat{x}u + \hat{y}v + \hat{z}w$$

(u,v,w) are the <u>direction cosines</u>

$$u = \cos\alpha_x = \sin\theta\cos\phi$$
$$v = \cos\alpha_y = \sin\theta\sin\phi$$
$$w = \cos\alpha_z = \cos\theta$$



Far Zone Radiation Integrals

Use the approximation in the radiation integral to get:

$$\vec{E}(\vec{r}) = \frac{-j\overbrace{\omega\mu}^{=k\eta}}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\iiint_V \left[\vec{J}(\vec{r}\,') - \hat{r}\left(\vec{J}(\vec{r}\,')\bullet\hat{r}\right) + \frac{\vec{J}_m(\vec{r}\,')\times\hat{r}}{\eta}\right] e^{jk\overbrace{\vec{r}\,'\bullet\hat{r}}^{\equiv g}} dv'$$

or, in terms of the spherical components (e.g., $E_\theta = \vec{E}\bullet\hat{\theta}$)

$$E_\theta(P) = \frac{-jk\eta}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\iiint_V \left[\vec{J}(\vec{r}\,')\bullet\hat{\theta} + \frac{\vec{J}_m(\vec{r}\,')\bullet\hat{\phi}}{\eta}\right] e^{jkg} dv'$$

$$E_\phi(P) = \frac{-jk\eta}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\iiint_V \left[\vec{J}(\vec{r}\,')\bullet\hat{\phi} - \frac{\vec{J}_m(\vec{r}\,')\bullet\hat{\theta}}{\eta}\right] e^{jkg} dv'$$

Note that we have defined $g = \vec{r}\,'\bullet\hat{r} = x'u + y'v + z'w$. The total field at the observation point is $\vec{E}(P) = E_\theta(P)\hat{\theta} + E_\phi(P)\hat{\phi}$. Since in the far field the wave is spherical (TEM) the magnetic field can be found from

$$\vec{H}(P) = \frac{\hat{r}\times\vec{E}(P)}{\eta}$$

Physical Optics Approximation

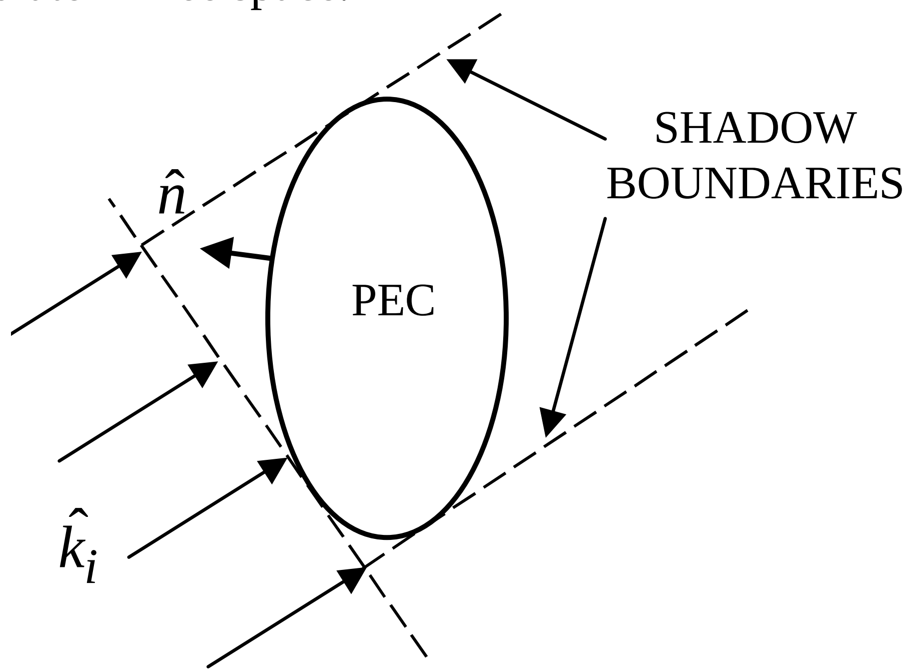
The <u>physical optics</u> (PO) approximation gives an estimate of the current that is induced on a body by an incident field arriving from the direction $\hat{k}_i$.

- The approximation becomes more accurate as the wavelength approaches zero, i.e., at high frequencies.
- Therefore, this is called a <u>high frequency approximation</u>.
- The induced current is used in the radiation integral to get the scattered field.
- The object is removed and the currents radiate in free space.

On a PEC the current is approximately

$$\vec{J}_s(r') = \begin{cases} 2\hat{n} \times \vec{H}_i(r'), & \text{illuminated part} \\ 0, & \text{in the shadow} \end{cases}$$

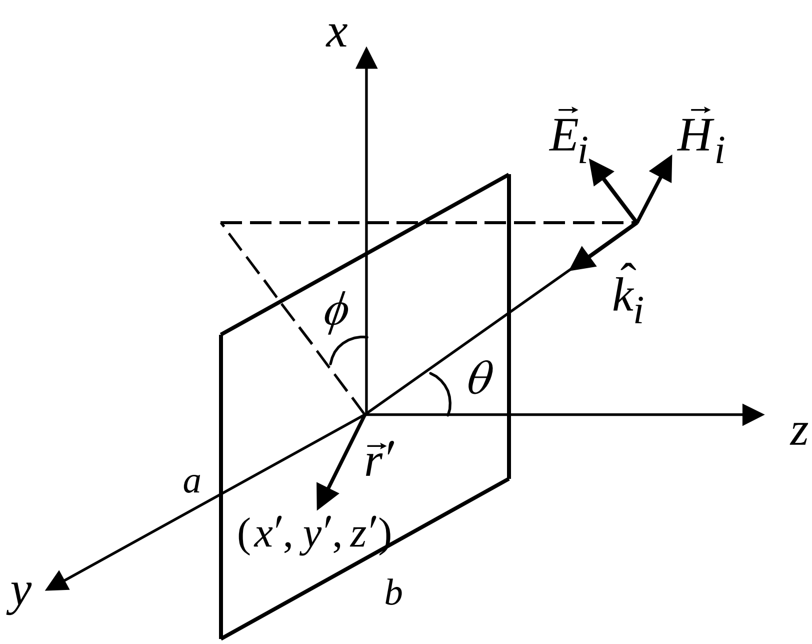
- The current abruptly goes to zero at a shadow boundary



RCS of a PEC Plate

PO is used to calculate the RCS of a rectangular PEC plate in the *x*-*y* plane. Consider an arbitrary polarization comprised of TM and TE components $\vec{E}_i = \left(E_{0\theta}\hat{\theta} + E_{0\phi}\hat{\phi}\right)e^{-j\hat{k}_i \bullet \vec{r}}$ and the observation point at $P(x,y,z) = P(r,\theta,\phi)$.

Then $\vec{k}_i = -k\hat{r}_i = -k\left(\hat{x}u + \hat{y}v + \hat{z}w\right)$



The point at which the field is being evaluated is a source point, so we use

$$\vec{r}' = \hat{x}x' + \hat{y}y' + \hat{z}z'$$

The PO current is obtained from:

$$\vec{H}_i = \frac{\hat{k}_i \times \vec{E}_i}{\eta_o} = \frac{-\hat{r}_i \times \left(E_{0\theta}\hat{\theta} + E_{0\phi}\hat{\phi}\right)e^{-j\hat{k}_i \bullet \vec{r}'}}{\eta_o}$$

$$= -\left(E_{0\theta}\hat{\phi} - E_{0\phi}\hat{\theta}\right)\frac{e^{-j\hat{k}_i \bullet \vec{r}'}}{\eta_o}$$

$$\vec{J}_s = 2\hat{n} \times \vec{H}_i = -2\hat{z} \times \left(E_{0\theta}\hat{\phi} - E_{0\phi}\hat{\theta}\right)\frac{e^{-j\hat{k}_i \bullet \vec{r}'}}{\eta_o}$$

RCS of a PEC Plate

The cross products are

$$\hat{z}\times\hat{\phi} = -\hat{x}\cos\phi - \hat{y}\sin\phi$$
$$\hat{z}\times\hat{\theta} = -\hat{x}\cos\theta\sin\phi + \hat{y}\cos\theta\cos\phi$$

Since the current expression is at a source point, we use primed coordinates. Define

$$h \equiv \hat{k}_i \bullet \vec{r}\,' = x'u + y'v + z'w$$

- Note this is similar to $g = \hat{k}\bullet\vec{r}\,'$
- $\hat{k}_i$ is in the direction of the incident wave; $\hat{k}$ is in the direction we will be evaluating the scattered field (pointed toward *P*).
- For monostatic RCS they are related: $\hat{k}_i = -\hat{k}$
- The plate is in the *x*-*y* plane: $z' = 0$

Final expression for the PO current is

$$\vec{J}_s = -\frac{2e^{jkh}}{\eta_o}\left[\hat{x}\underbrace{\left(E_{0\theta}\cos\phi - E_{0\phi}\cos\theta\sin\phi\right)}_{\equiv J_{sx}} + \hat{y}\underbrace{\left(E_{0\theta}\sin\phi + E_{0\phi}\cos\theta\cos\phi\right)}_{\equiv J_{sy}}\right]$$

Monostatic RCS of a PEC Plate

The θ component of the scattered field is:

$$E_\theta(P) = -\frac{jk\eta_o}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\iint_{S_{\mathrm{illum}}} \frac{2e^{jkh}}{\eta_o}\left(J_{sx}\hat{x} + J_{sy}\hat{y}\right)\bullet\hat{\theta}\, e^{jkg}\, dx'dy'$$

S_{illum} is the illuminated area of the plate. For an infinitely thin plate it is the entire area. In general:

$h \equiv \hat{k}_i \bullet \vec{r}' = x'u_i + y'v_i + z'w_i$ where (u_i, v_i, w_i) are determined from the angle of incidence

$g = \hat{k} \bullet \vec{r}' = x'u + y'v + z'w$ where (u, v, w) are determined from the angle of observation

- For monostatic: $u = u_i, v = v_i, w = w_i$ $(\theta = \theta_i, \phi = \phi_i)$ and $g = h$

$$E_\theta(P) = -\frac{jk}{2\pi}\left(\frac{e^{-jkr}}{r}\right)\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left(J_{sx}\hat{x} + J_{sy}\hat{y}\right)\bullet\hat{\theta}\, e^{j2kg}\, dx'dy'$$

Monostatic RCS of a PEC Plate

Consider TM incidence $\left(E_{0\theta}=1, E_{0\phi}=0\right)$ which gives

$$J_{sx}=\cos\phi,\ J_{sy}=\sin\phi$$

$$\left(J_{sx}\hat{x}+J_{sy}\hat{y}\right)\bullet\hat{\theta}=\cos\theta\cos^2\phi+\cos\theta\sin^2\phi=\cos\theta$$

$$E_\theta(P)=-\frac{jk}{2\pi r}e^{-jkr}\cos\theta\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}e^{j2(x'u+y'v)}dx'dy'$$

$$=-\frac{jk}{2\pi r}e^{-jkr}\cos\theta\underbrace{\int_{-a/2}^{a/2}e^{j2x'u}dx'}_{a\,\mathrm{sinc}(kau)}\underbrace{\int_{-b/2}^{b/2}e^{j2y'v}dy'}_{b\,\mathrm{sinc}(kbv)}$$

$$=-\frac{jk\overbrace{ab}^{=A}}{2\pi r}e^{-jkr}\cos\theta\,\mathrm{sinc}(kau)\,\mathrm{sinc}(kbv)$$

Note the definition of "sinc" used here is $\mathrm{sinc}(\alpha)\equiv\sin(\alpha)/\alpha$.

Monostatic RCS of a PEC Plate

Substitute into the definition of RCS:

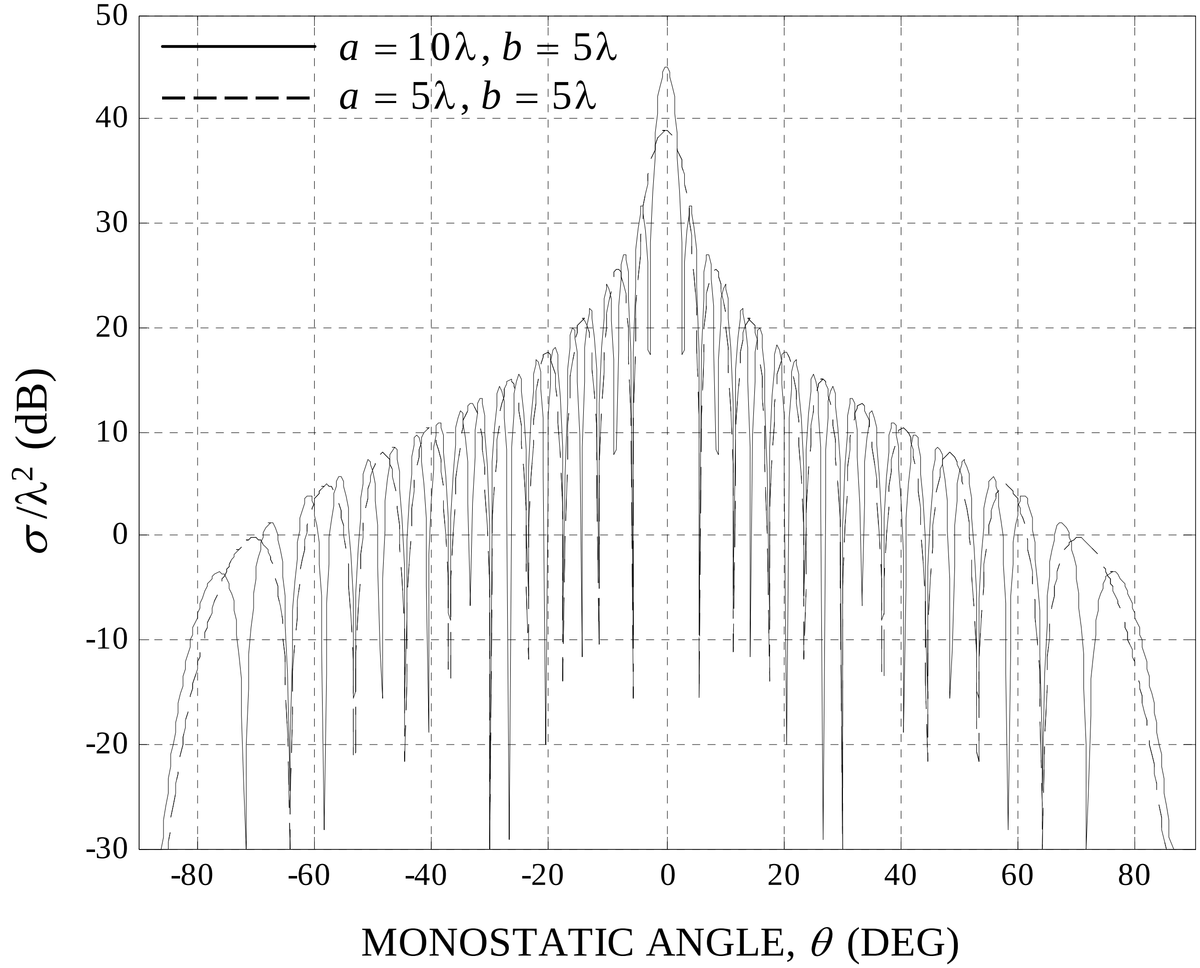
$$\sigma = \lim_{r\to\infty} 4\pi r^2 \frac{\left|E_\theta(P)\right|^2}{\left|E_{0\theta}\right|^2}$$
$$= 4\pi r^2 \frac{k^2 A^2}{4\pi^2 r^2} \cos^2\theta \left|\operatorname{sinc}(kau)\operatorname{sinc}(kbv) e^{-jkr}\right|^2$$
$$= \frac{4\pi A^2}{\lambda^2} \cos^2\theta \operatorname{sinc}^2(kau)\operatorname{sinc}^2(kbv)$$

Properties:

- Peak value at normal incidence ($\theta = 0^\circ$) is $\dfrac{4\pi A^2}{\lambda^2}$.
- Scatter pattern is separable in x,y or u,v because the plate edges align with Cartesian axes.
- Similarity to aperture radiation: $\operatorname{sinc}(kau)$ versus $\operatorname{sinc}(kau/2)$. RCS has "round trip" phase difference, hence the factor of 2 difference.

RCS of Rectangular Plates

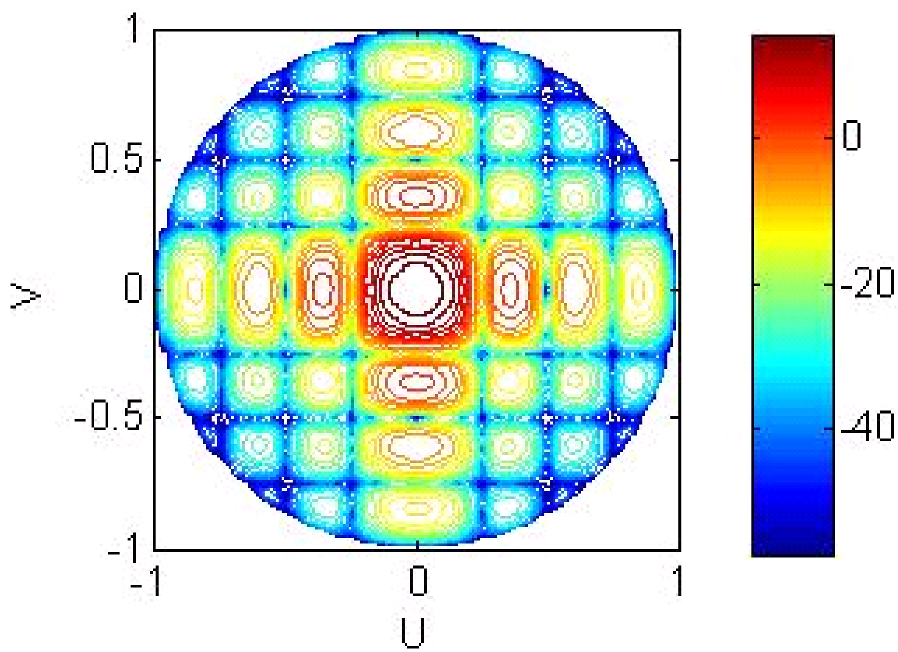
TM polarization, $\phi = 0^\circ$



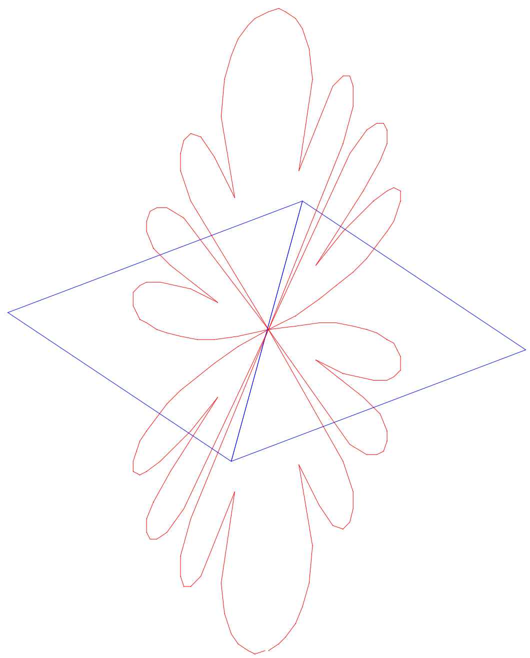
2-D RCS of Rectangular Plates

Example: 1 m by 1 m plate at 600 MHz, TM polarization. Computed by *POFACETS*. *POFACETS* sums the scattering from collections of triangles, thus the plate is represented as two triangles

- Plot in direction cosine space



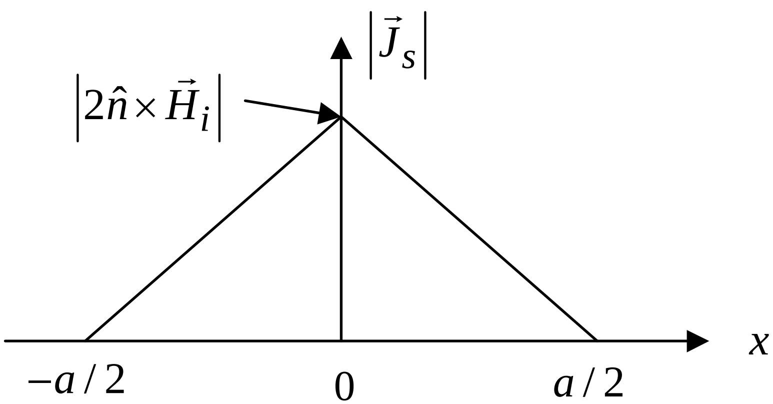
- Polar plot in 3-d showing plate orientation



Tapered Current Distributions

In principle RCS sidelobes can be controlled by tapering the amplitude distribution across the plate, similar to what is done for an antenna aperture.

Example 2.5: Linear tapering of the current amplitude distribution in the *x* direction. The current remains constant in the *y* direction. To simplify the formulas, assume the incidence and observation points are in the *x*-*z* plane ($\phi = 0^\circ$) so we can observe the effect of the taper on the sidelobes.



TM polarized wave incident:

$$\vec{E}_i = E_{0\theta} e^{-j\hat{k}_i \bullet \vec{r}'}\hat{\theta}$$

$$\vec{H}_i = -\frac{E_{0\theta}}{\eta_o} e^{-j\hat{k}_i \bullet \vec{r}'}\hat{\phi} = -H_o e^{-j\hat{k}_i \bullet \vec{r}'}\hat{\phi}$$

$$\vec{J}_s = 2\hat{z}\times\vec{H}_i\left(1-2\left|x'\right|/a\right)$$

$$= -\hat{\rho}2H_o\left(1-2\left|x'\right|/a\right)e^{-jkh}$$

Note that since we are doing the monostatic case $h = g = \hat{k}\bullet\vec{r}' = x'u + y'v + z'w$. In the $\phi = 0^\circ$ plane $\hat{\rho} = \hat{x}$.

Tapered Current Distributions

The scattered field is

$$E_\theta(P) = -\frac{jk}{2\pi r} e^{-jkr} \cos\theta \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} e^{j2(x'u+y'v)} H_o \left(1 - 2|x'|/a\right) dx'dy'$$

$$= -\frac{jkH_o}{2\pi r} e^{-jkr} \cos\theta \underbrace{\int_{-a/2}^{a/2} \left(1 - 2|x'|/a\right) e^{j2x'u} dx'}_{(a/2)\mathrm{sinc}^2(kau/2)} \underbrace{\int_{-b/2}^{b/2} e^{j2y'v} dy'}_{b\,\mathrm{sinc}(kbv)}$$

The RCS is

$$\sigma = \lim_{r\to\infty} 4\pi r^2 \frac{|E_\theta(P)|^2}{|E_{0\theta}|^2} = 4\pi r^2 \left| \frac{jk}{2\pi r} e^{-jkr} (ab/2) \cos\theta \, \mathrm{sinc}^2(kau/2) \, \mathrm{sinc}(kbv) \right|^2$$

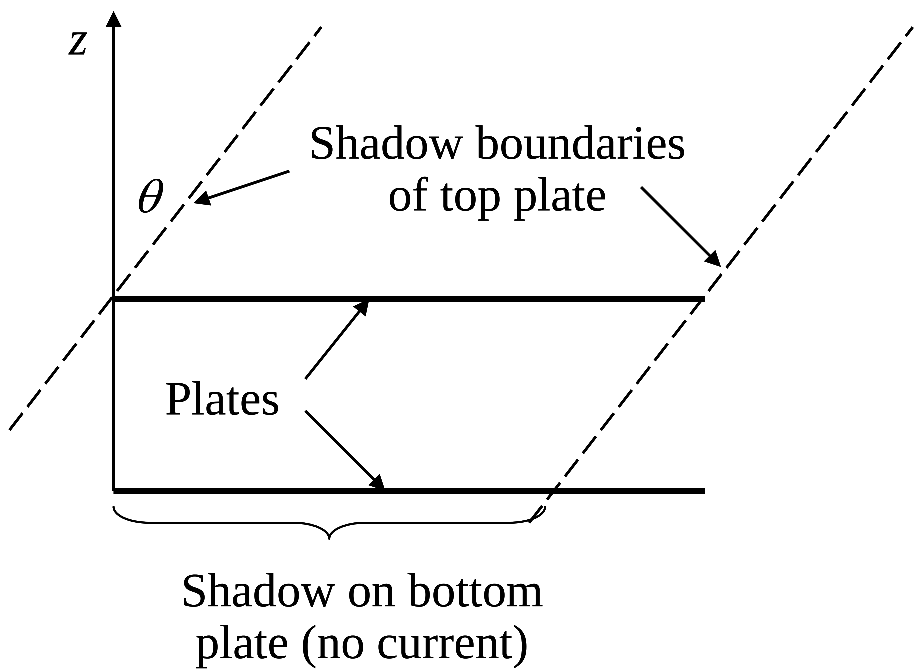
$$= \frac{\pi A^2}{\lambda^2} \cos^2\theta \, \mathrm{sinc}^4(kau/2) \, \mathrm{sinc}^2(kbv)$$

Comments:

- The peak value at normal incidence is 6 dB lower than that of a PEC plate.
- The scatter pattern in the taper plane has lower sidelobes but wider beamwidth.
- To taper the current we must attenuate the incident field $\vec{H}_i$ along the plate.

Null Field Hypothesis

The null-field hypothesis states that: there is no surface current induced in a shadow. We have encountered this already in the PO approximation. We can apply this to multiple structures as shown below. Areas of the surface in shadows do not contribute to the radiation integral. Note that we are neglecting any multiple reflections between the plates that might occur.



For multiple surfaces:

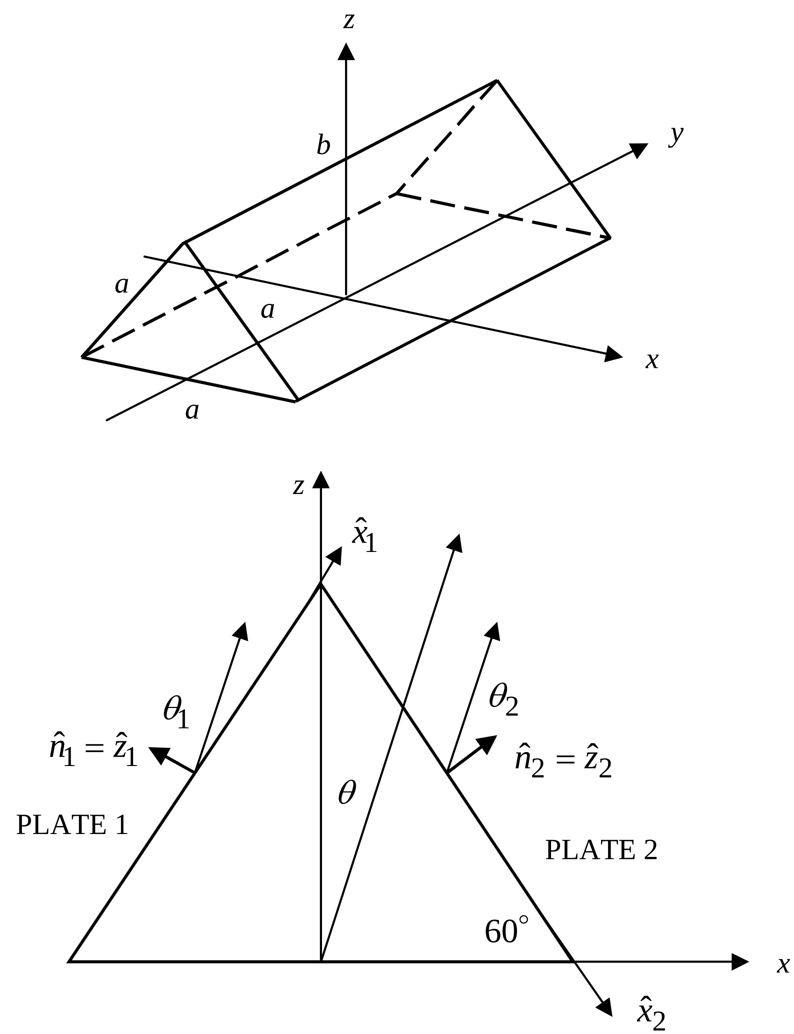
- Find the current on each surface
- Find the scattered field from the individual currents
- Apply superposition (complex vector addition)
- A common phase reference must be used in the vector summation
- Care must be taken in accounting for polarization (e.g., TM polarization for one surface may not be TM for another)

Tent Structure

Example 2.7: Find the RCS of the structure in the $\phi = 0^\circ$ plane, $|\theta| \le 90^\circ$ for TM polarization.

Approach:

- Note that TM_z is also TM_{z_1} and TM_{z_2}
- We can use the previous result for $E_\theta(P)$ of a rectangular plate
- Centers of the plates are displaced from the origin so a phase shift must be added
- Illumination depends on angle of incidence
- Symmetry exists – only need to solve for $0^\circ \le \theta \le 90^\circ$



Tent Structure

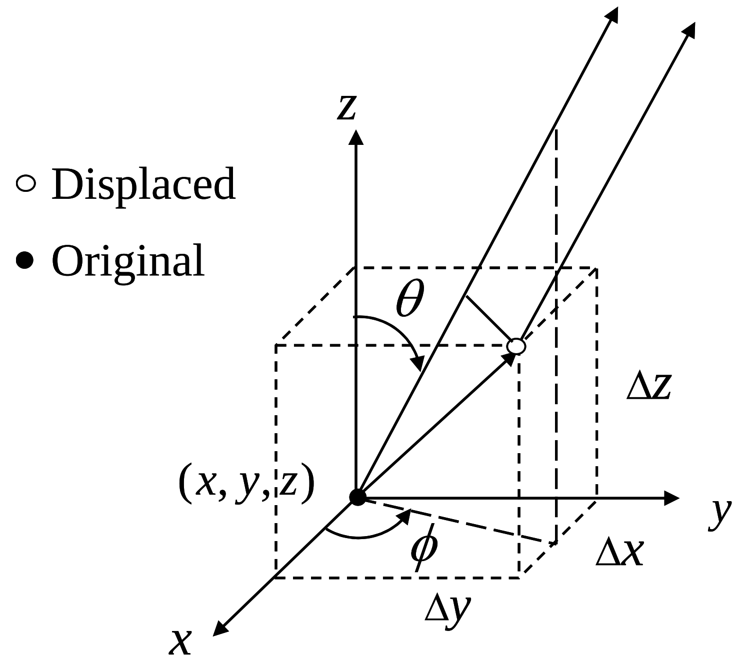
Observe that displacement of a scatterer from the point (x, y, z) to the point $(x+\Delta x, y+\Delta y, z+\Delta z)$ introduces a <u>one way</u> phase shift

$$\exp[\,jk(u\Delta x + v\Delta y + w\Delta z)]$$

where (u, v, w) are the direction cosines.

The effect on the monostatic scattered field is the two-way phase shift:

$$\exp[\,j2k(u\Delta x + v\Delta y + w\Delta z)]$$



If a rectangular plate is displaced from the origin, the far scattered field becomes:

$$E_\theta(P) = \frac{jkA}{2\pi r} e^{-jkr} \cos\theta \,\mathrm{sinc}(kau)\,\mathrm{sinc}(kbv) e^{j2k(u\Delta x + v\Delta y + w\Delta z)}$$

Note that the direction cosines and vector component θ are still defined in terms of the local plate coordinates. If they change (i.e. plate is rotated) then they must be modified.

Tent Structure

For the individual plates (in the $\phi_1 = \phi_2 = \phi = 0$ planes):

$$\theta_1 = \theta + 60^\circ \qquad \theta_2 = \theta - 60^\circ$$

$$\hat{x}_1 = \hat{x}\cos 60^\circ + \hat{z}\sin 60^\circ \qquad \hat{x}_2 = \hat{x}\cos(-60^\circ) + \hat{z}\sin(-60^\circ)$$

$$u_1 = \sin\theta_1 = u\cos 60^\circ + w\sin 60^\circ \qquad u_2 = \sin\theta_2 = u\cos(-60^\circ) + w\sin(-60^\circ)$$

Total scattered field is the sum

$$E_\theta(\theta) = E_{1\theta}(\theta)e^{jka(-u/2+0.866w)} + E_{2\theta}(\theta)e^{jka(u/2+0.866w)}$$

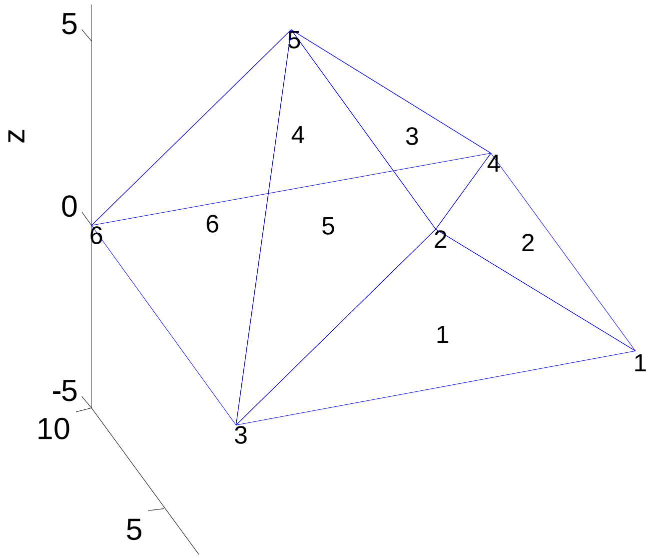
Contribution from plate 1: $E_{1\theta}(\theta) = \begin{cases} \dfrac{jkA_1}{2\pi r}e^{-jkr}\cos\theta_1 \,\mathrm{sinc}(kau_1), & |\theta_1| < 90^\circ \\ 0, & \text{else} \end{cases}$

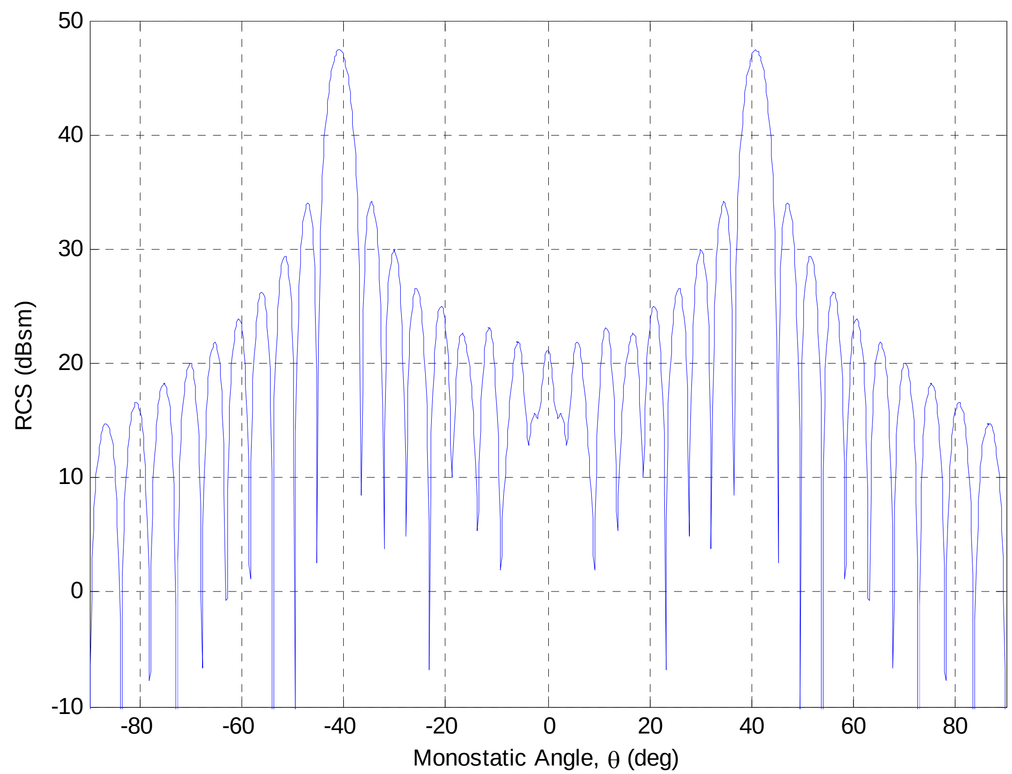
Contribution from plate 2: $E_{2\theta}(\theta) = \begin{cases} \dfrac{jkA_2}{2\pi r}e^{-jkr}\cos\theta_2 \,\mathrm{sinc}(kau_2), & |\theta_2| < 90^\circ \\ 0, & \text{else} \end{cases}$

RCS for unit plane wave incidence: $\sigma_{\theta\theta} = 4\pi r^2 \left|E_\theta(\theta)\right|^2$

Tent Structure RCS

POFACETS model and pattern

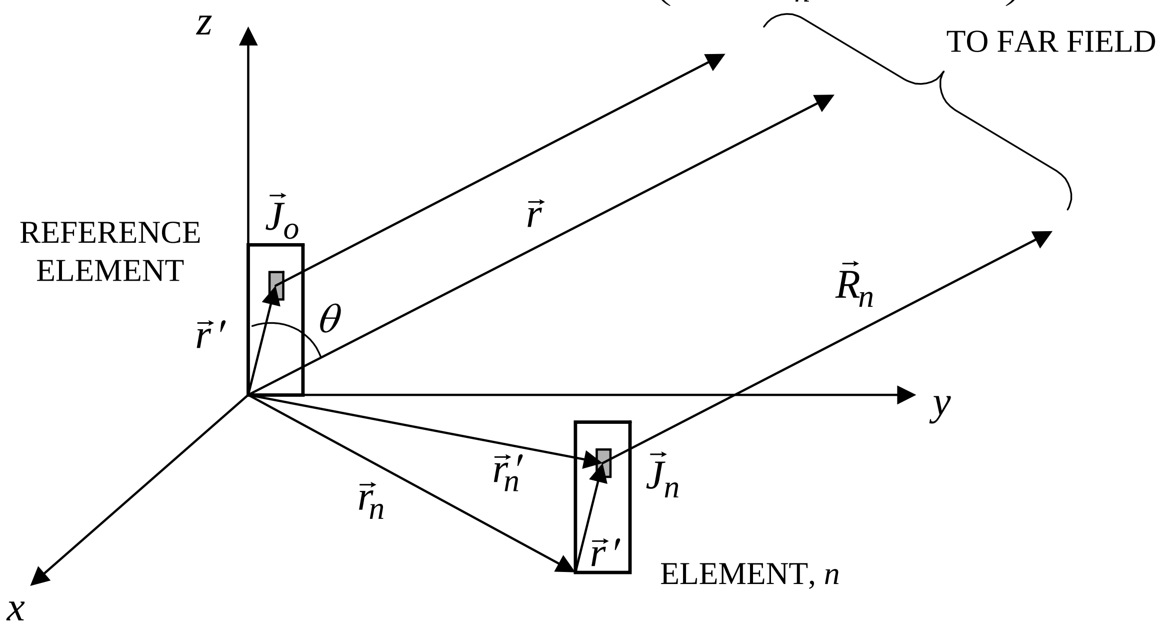




Arrays of Scatterers

If the current on a scattering element is known, then we simply sum the results of the individual radiation integrals (if all quantities are defined in a common coordinate system). If there are N elements, and element n has current density $\vec{J}_n$ in region V_n the far-field is

$$\vec{E}(x,y,z) = -\frac{jk\eta_o}{4\pi r} e^{-jkr} \sum_{n=1}^{N} \left(e^{jkg_n} \iiint_{V_n} \vec{J}_n e^{jkg} dv' \right)$$ (discard any radial component)



where:

- $(x_n, y_n, z_n) =$ location of element n
- $g_n = \hat{r} \bullet \vec{r}_n = ux_n + vy_n + wz_n$
- $g = \hat{r} \bullet \vec{r}' = ux' + vy' + wz'$

Identical Scatterers

Consider the case of identical scatterers. The current density for each scatterer is the same except for a complex scale factor, A_n. Let one element be located at the origin ($\vec{J}_0$) so that $\vec{J}_n = A_n \vec{J}_0$. Then

$$\vec{E}(x,y,z) = \underbrace{\left[\sum_{n=1}^{N} A_n e^{j2kg_n} \right]}_{=AF\text{, ARRAY FACTOR}} \underbrace{\left[-\frac{jk\eta_o}{4\pi r} e^{-jkr} \iiint_{V_0} \vec{J}_0 e^{jkg} dv' \right]}_{=EF\text{, ELEMENT FACTOR}}$$

This is an example of the principle of pattern multiplication.

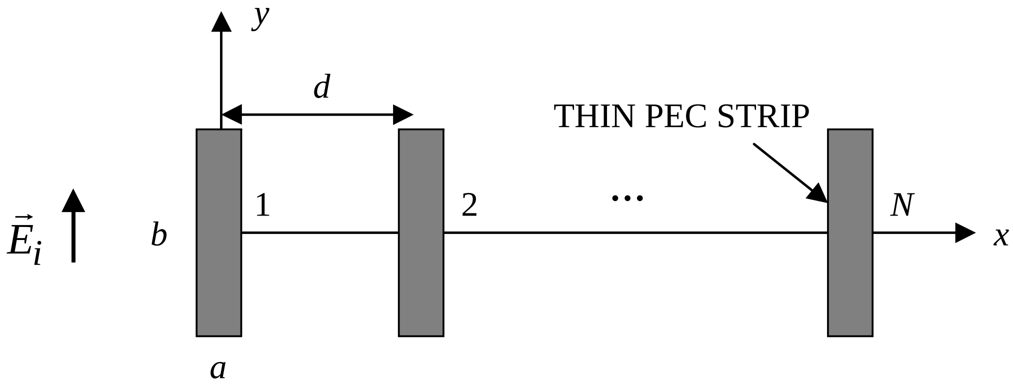
Comments:

- For plane wave incidence $A_n = 1$.
- The array factor depends only on the geometry.
- Round trip phase difference occurs for monostatic RCS ("2" in the exponent).
- The element factor depends only on the current distribution on the element.
- We have neglected the differences in current distribution due to mutual coupling variations between elements.

Periodic arrays

An important class of problems involve periodic arrays of scattering elements, which lead to Bragg scattering (or Bragg diffraction).

Example 2.9: A linear array of thin strips. N strips spaced d along the x axis. Consider TE incidence ($E_{0\phi} = 1$) in the $\phi = 0$ plane. The element factor (from Example 2.4):



$$EF = \frac{jkA}{2\pi r} e^{-jkr} \cos\theta \times \underbrace{\mathrm{sinc}(kau)}_{\approx 1 \text{ (thin strip)}} \underbrace{\mathrm{sinc}(kbv)}_{=1\,(v=0)}$$

Array factor:

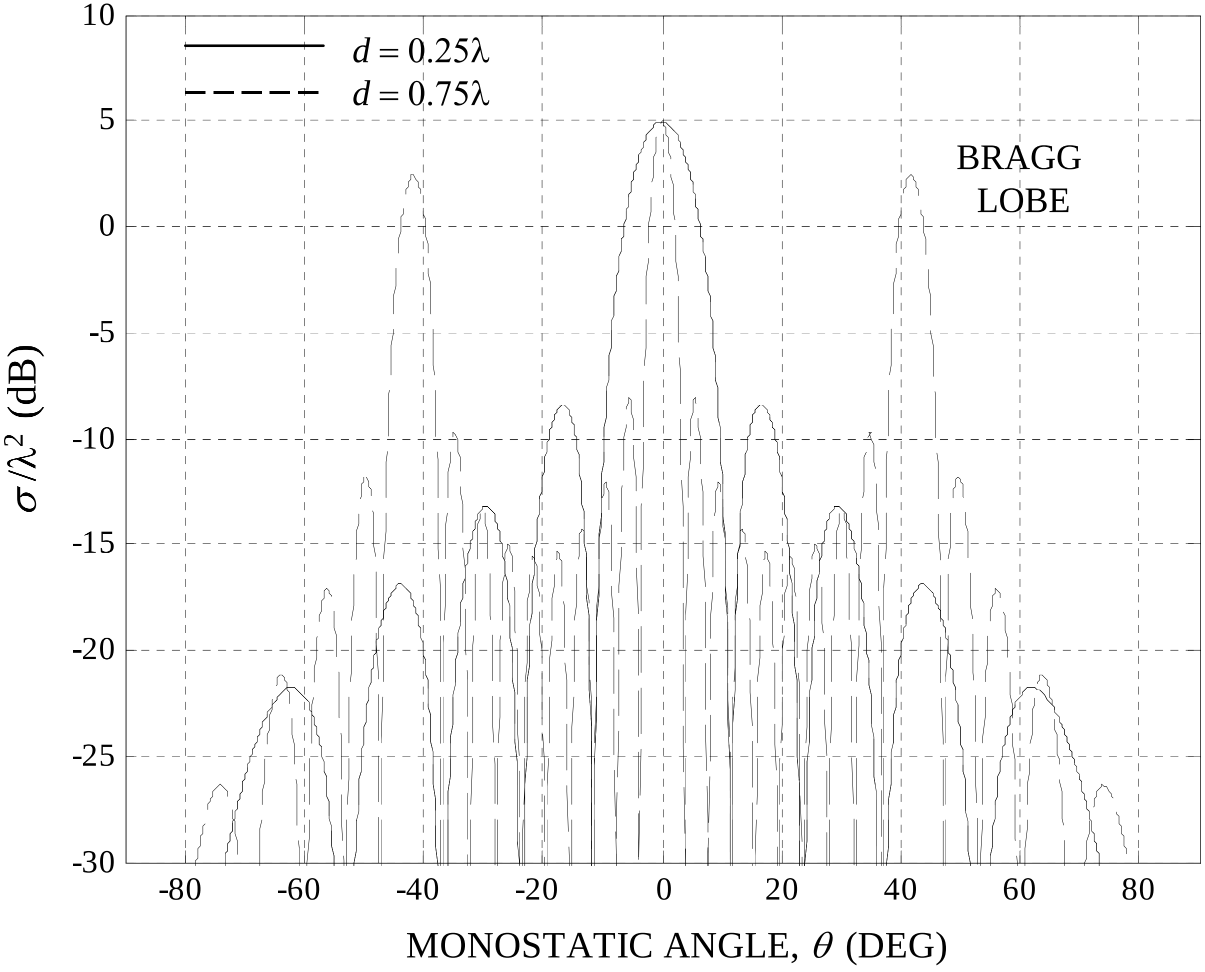
$$AF = \sum_{n=1}^{N} e^{j2kg_n} = \sum_{n=1}^{N} \left(e^{j2kdu}\right)^{n-1} = e^{j(N-1)kdu} \frac{\sin(Nkdu)}{\sin(kdu)}$$

$$\vec{r}_n = \hat{x}(n-1)d$$
$$\hat{r} = u\hat{x} + w\hat{z} \quad (v = 0,\ u = \sin\theta),$$
$$g_n = \vec{r}_n \bullet \hat{r} = u(n-1)d$$

$$\sigma_{\phi\phi} = \frac{4\pi A^2}{\lambda^2} \cos^2\theta \left[\frac{\sin(Nkdu)}{\sin(kdu)}\right]^2$$

Bragg Scattering

Sample results for N = 10. Bragg lobes are analogous to array grating lobes.

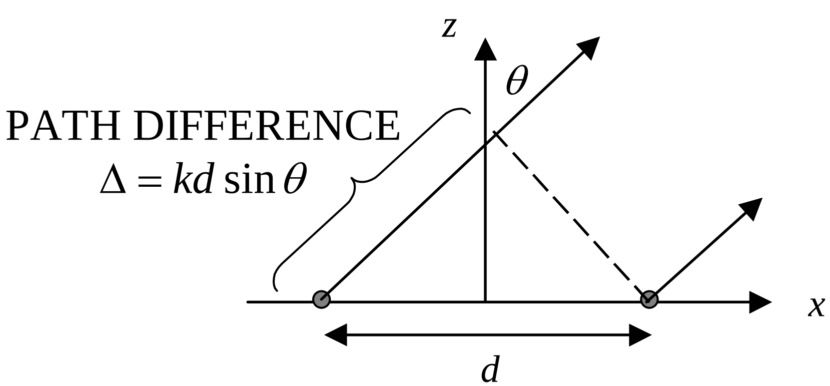


Bragg Scattering

Secondary maxima occur when the denominator is zero, but the numerator is not:

$$kdu = kd\sin\theta_m = m\pi \quad (m = \pm 1, \pm 2, \ldots) \quad \rightarrow \quad \theta_m = \sin^{-1}\left(\frac{m\lambda}{2d}\right)$$

Values of m that give $|\sin\theta_m| \le 1$ correspond to the visible region. Bragg lobes are due to the round trip path phase difference between adjacent elements being an integer multiple of 2π.



For monostatic RCS the first Bragg lobe occurs at $\theta = 90^\circ$ with a spacing of $d = \lambda/2$.

Bragg lobes are a form of aliasing:

- An array of discrete scatterers is "sampling" the incident wavefront
- Nyquist sampling requires twice the frequency (or higher)
- Twice the frequency is a half the wavelength (or less)

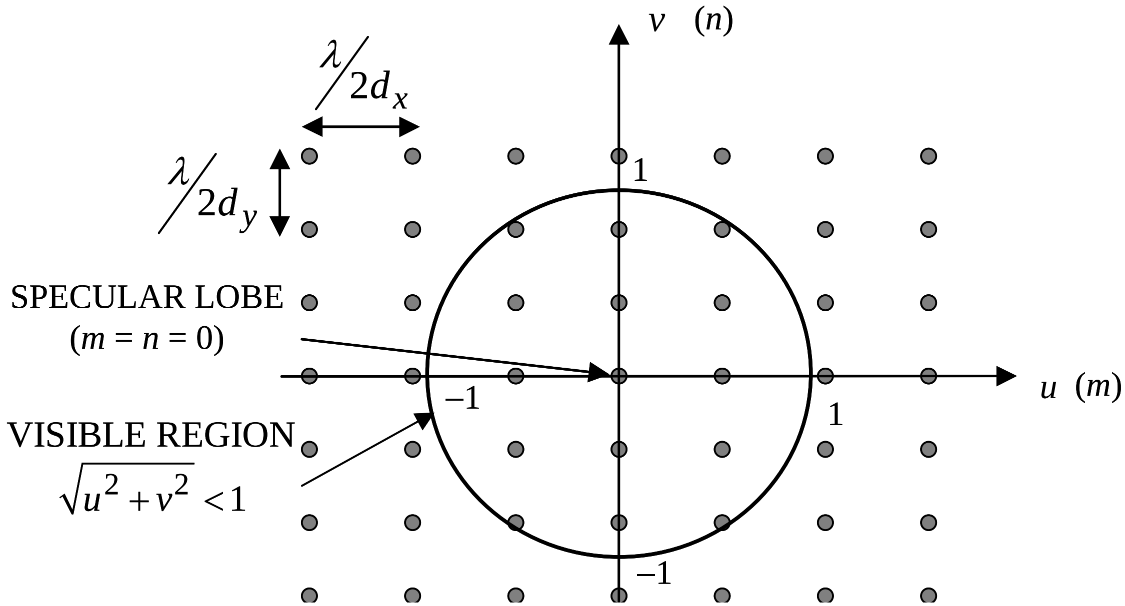
Bragg Diagram

Extend the array factor to a rectangular grid in two dimensions. The RCS will vary as:

$$\sigma_{\phi\phi} \sim \left[\frac{\sin(N_x k d_x u)}{\sin(k d_x u)}\right]^2 \left[\frac{\sin(N_y k d_y v)}{\sin(k d_y v)}\right]^2$$

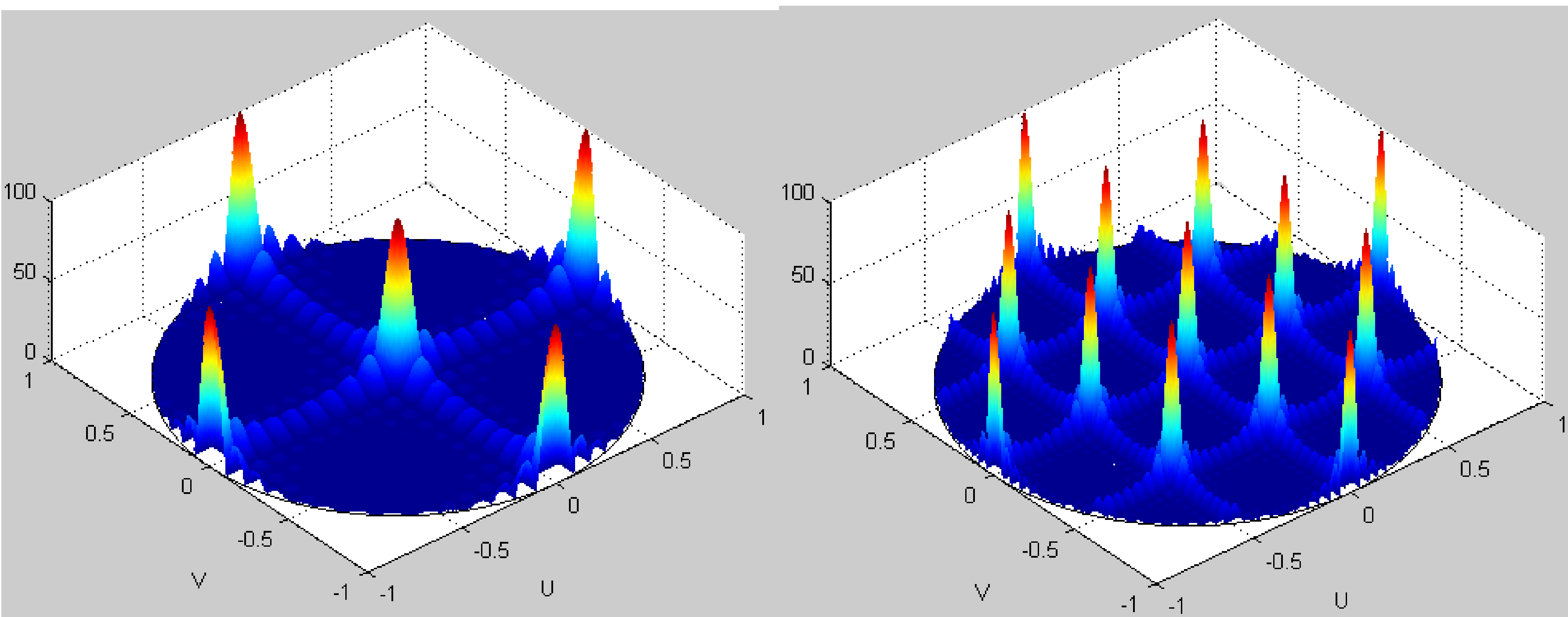
which have Bragg lobes at the direction cosines

$$u_m = \sin\theta_m \cos\phi_m = \frac{m\lambda}{2d_x} \quad \text{and} \quad v_n = \sin\theta_\ell \sin\phi_\ell = \frac{n\lambda}{2d_y} \quad (m, n = \pm 1, \pm 2, \ldots)$$



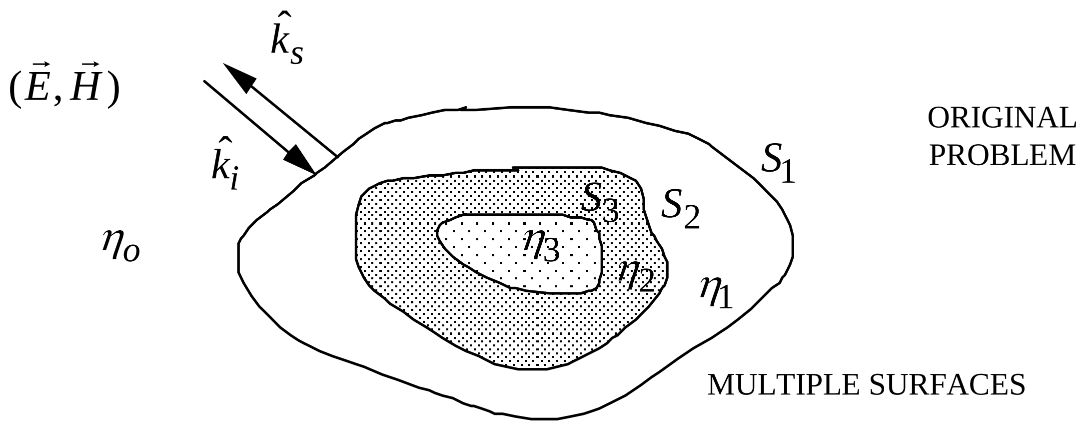
Sample Scattering Patterns

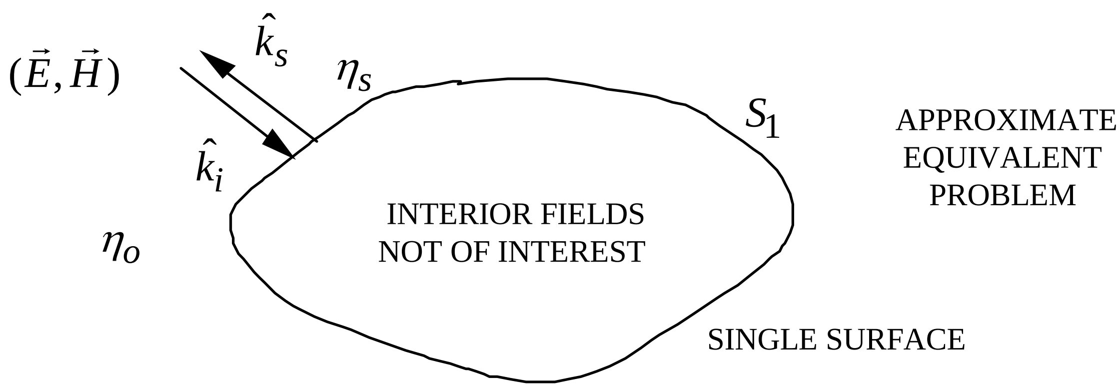
Example 2.10: Array factor plotted for $N_x = N_y = 40$, $d_x = d_y = 0.5\lambda$ (left) and $d_x = d_y = \lambda$ (right)



Surface Impedance

Surface impedance is a convenient (usually approximate) means of simplifying scattering calculations for complex targets. It is related to the surface equivalence principle. For a complex target bounded by surface *S*, we can get the exterior fields from the surface currents on *S*.





Rather than work with the currents, we define a <u>surface impedance</u> η_s, which relates the tangential field components on the surface

$$\eta_s = \frac{E_{\tan}}{H_{\tan}}$$

Normalized to free space:

$$Z_s = \frac{\eta_s}{\eta_o}$$

Surface Impedance

Special cases:

$$\eta_s = 0 \quad \rightarrow \quad E_{\tan} = 0 \text{ is a PEC}$$
$$\eta_s = \infty \quad \rightarrow \quad H_{\tan} = 0 \text{ is a PMC}$$

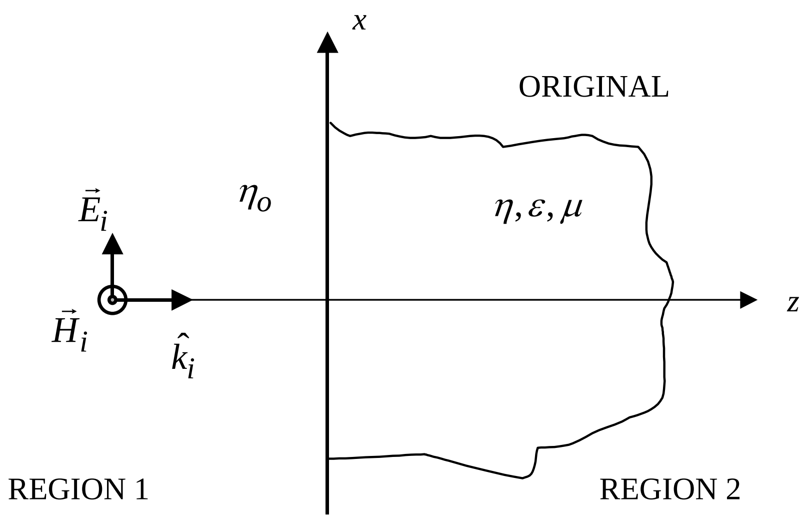
Vector relationships:

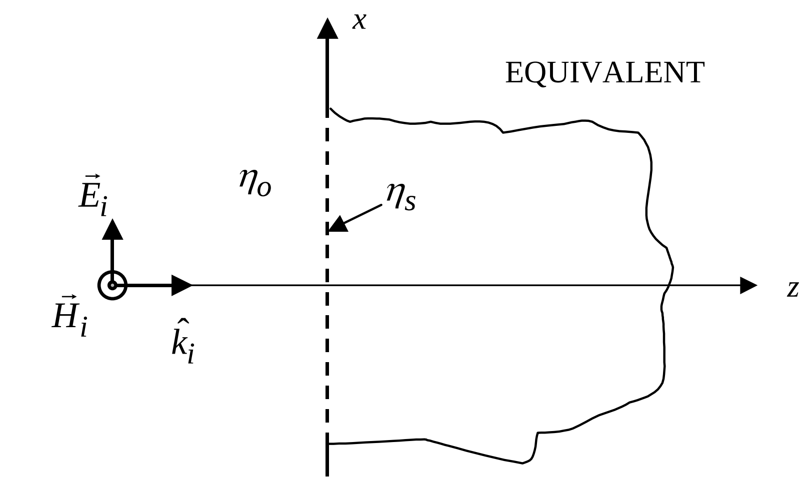
$$\left.\begin{aligned} \vec{E}_{\tan} &= \vec{E} - \hat{n}\left(\hat{n}\bullet\vec{E}\right) = \eta_s \hat{n}\times\vec{H} = \eta_s \vec{J}_s \\ \vec{H}_{\tan} &= \vec{H} - \hat{n}\left(\hat{n}\bullet\vec{H}\right) = -\frac{1}{\eta_s}\hat{n}\times\vec{E} = -\frac{1}{\eta_s}\vec{J}_{ms} \end{aligned}\right\} \rightarrow -\vec{J}_{ms} = \eta_s \hat{n}\times\vec{J}_s$$

Comments:

- For RCS calculations, we replace the complex scatterer with an impedance surface.
- η_s is known or can be estimated from the physical properties of the target shape and materials.
- η_s can be complex.
- η_s can be a function of angle since $E_{\tan}$ and $H_{\tan}$ are changing with incidence. Thus it does not always correspond to a "real" physical impedance.
- Generally it is an approximation, but may be exact in some cases.
- Most accurate near normal incidence and for high impedance materials $(Z_s << 1)$.

Surface Impedance Example





Infinite plane boundary between free space and medium (μ, ε) where $\eta = \sqrt{\mu / \varepsilon}$. A plane wave is normally incident

$$E_{\tan} = E_i + E_s = E_t \quad \rightarrow \quad 1 + \Gamma = \tau$$
$$H_{\tan} = H_i + H_s = H_t \quad \rightarrow \quad (1 - \Gamma)/\eta_o = \tau / \eta$$

$$\eta_s = \frac{E_{\tan}}{H_{\tan}} = \eta$$

Thus, we replace the half space with an infinite plane sheet of surface impedance. Note this gives the same reflection coefficient as the original problem:

$$\Gamma = \frac{\eta_s - \eta_o}{\eta_s + \eta_o} = \frac{\eta - \eta_o}{\eta + \eta_o}$$

Impedance Loaded Plate

Example 2.12: Plate of Example 2.4 with surface impedance η_s, TM polarization, using the physical optics approximation. The electric current is given by Equation (2.40) with $E_{0\phi} = 0$:

$$\vec{J}_s = 2\hat{n} \times \vec{H}_i = -2E_{0\theta} \underbrace{\hat{z} \times \hat{\phi}}_{=-\hat{\rho}} \frac{e^{jkh}}{\eta_o} = 2E_{0\theta}\hat{\rho}\frac{e^{jkh}}{\eta_o}$$

There is also a magnetic current on the surface:

$$\vec{J}_{ms} = -\eta_s \hat{n} \times \vec{J}_s = -2\eta_s E_{0\theta} \hat{z} \times \hat{\rho} \frac{e^{jkh}}{\eta_o} = -2\eta_s E_{0\theta} \hat{\phi} \frac{e^{jkh}}{\eta_o}$$

The far-field radiation integral now has two terms. The electric current term is the same as for the PEC plate. The second term is due to the magnetic current

$$E_\theta(P) = -\frac{jk\eta_o}{4\pi}\left(\frac{e^{-jkr}}{r}\right)\iint_S \left[\vec{J}_s \bullet \hat{\theta} + \frac{\vec{J}_{ms} \bullet \hat{\phi}}{\eta_o}\right] e^{jkg} dx'dy'$$

$$= -\frac{jkE_{0\theta}}{2\pi}\left(\frac{e^{-jkr}}{r}\right)\iint_S \left[\hat{\rho} \bullet \hat{\theta} + \frac{\eta_s \hat{\phi} \bullet \hat{\phi}}{\eta_o}\right] e^{j2kg} dx'dy'$$

Impedance Loaded Plate

Separable integrals result

$$E_\theta(P) = -\frac{jkE_{0\theta}}{2\pi r} e^{-jkr} \left(\cos\theta - Z_s\right) \underbrace{\int_{-a/2}^{a/2} e^{j2x'u} dx'}_{a\,\mathrm{sinc}(kau)} \underbrace{\int_{-b/2}^{b/2} e^{j2y'v} dy'}_{b\,\mathrm{sinc}(kbv)}$$

$$= -\frac{jkE_{0\theta}A}{2\pi r} e^{-jkr} \left(\cos\theta - Z_s\right) \mathrm{sinc}(kau)\,\mathrm{sinc}(kbv)$$

$$\sigma_{\theta\theta} = \frac{4\pi A^2}{\lambda^2} \left| \cos\theta \left(1 - Z_s / \cos\theta\right) \mathrm{sinc}(kau)\,\mathrm{sinc}(kbv) \right|^2$$

Comments:

- The difference from the TM PEC case is the factor $\left(1 - Z_s / \cos\theta\right)$.
- Starting with the Fresnel reflection coefficient for TM polarization

$$\Gamma_{\mathrm{TM}} = \frac{\eta_s \cos\theta_t - \eta_o \cos\theta_i}{\eta_s \cos\theta_t + \eta_o \cos\theta_i} \approx \frac{Z_s - \cos\theta}{Z_s + \cos\theta} \approx Z_s - \cos\theta$$

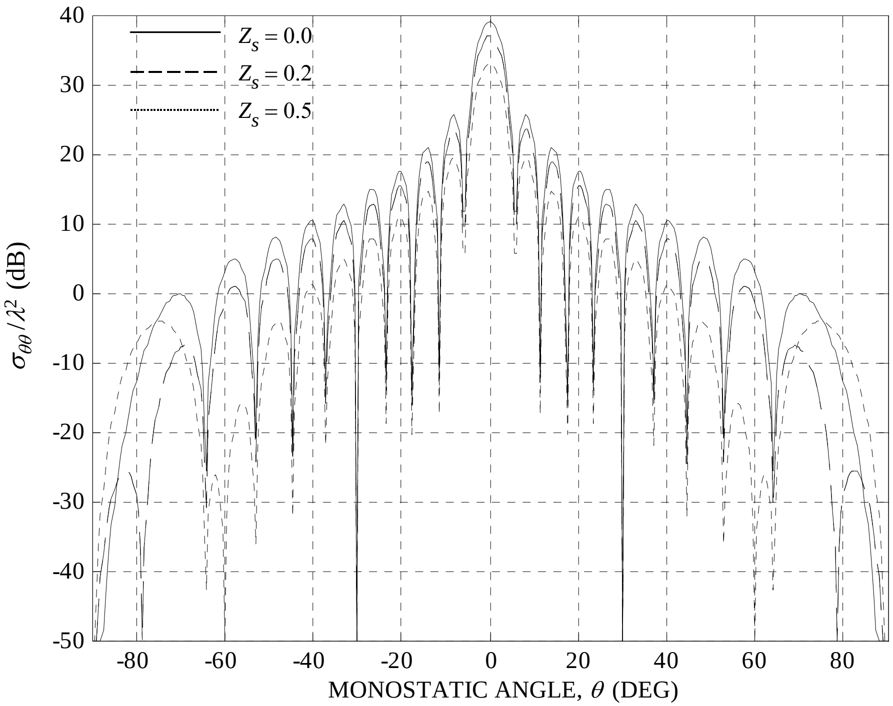
under the conditions $\theta_t = \theta, \cos\theta_t \approx 1, Z_s << \cos\theta$.

- Thus $\sigma_{\theta\theta} \approx \sigma_{\mathrm{PEC}} \left|\Gamma_{\mathrm{TM}}\right|^2$ where $\sigma_{\mathrm{PEC}} = \dfrac{4\pi A^2}{\lambda^2} \mathrm{sinc}^2(kau)\,\mathrm{sinc}^2(kbv)$

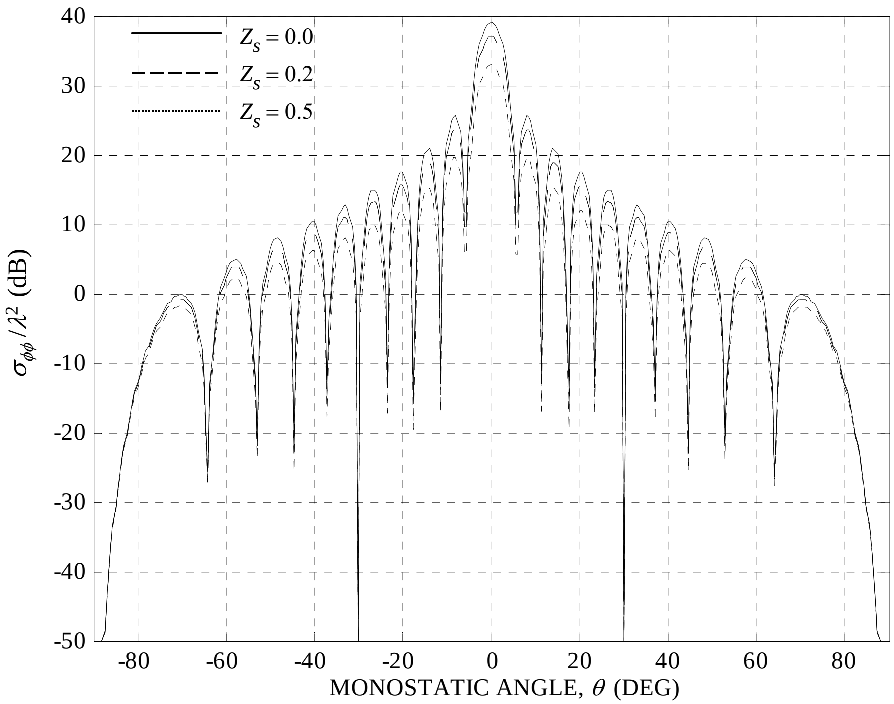
RCS Patterns of Impedance Loaded Plates

- $a = b = 5\lambda, \quad \phi = 0^{\circ}$

TM polarization



TE polarization



Discontinuity Boundary Conditions

"Sheets" are infinitesimally thin penetrable surfaces.

- A resistive sheet supports only electric currents $(\vec{J}_{ms} = 0)$. The boundary condition at the sheet is

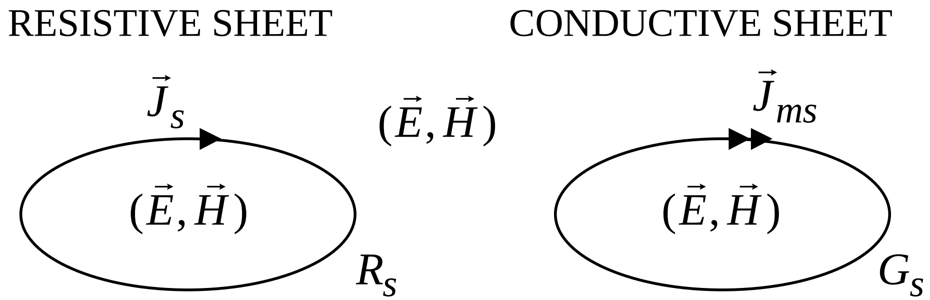
$$\vec{E}_{\tan} = \hat{n} \times \hat{n} \times \vec{E} = -R_s \vec{J}_s$$

where R_s is the surface resistivity in ohms/square $(\Omega/\square)$.

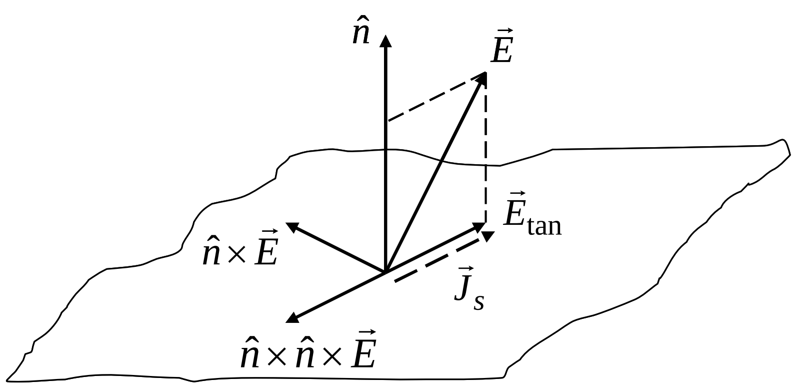
- A conductive sheet (as in "conductance") supports only magnetic currents $(\vec{J}_s = 0)$. The boundary condition at the sheet

$$\vec{J}_{ms} = -\frac{1}{G_s} \hat{n} \times \hat{n} \times \vec{H}$$

where G_s is the surface conductivity.



Vector relationships



Resistive Sheets

The resistance of a block of metal with conductivity σ is

$$R = \frac{\ell}{\sigma t w} \xrightarrow[t \to 0]{\ell = w} R_s = \frac{1}{\sigma t} \qquad (\ell = w \text{ is a square of surface})$$

The sheet reflection coefficient (see Example 2.13)

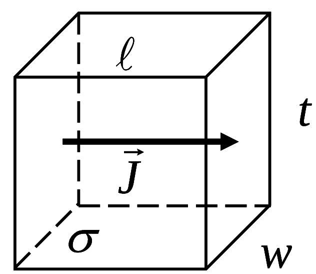
$$\Gamma = \frac{-\eta_o}{2R_s + \eta_o} = \frac{-1}{2R_s' + 1}$$

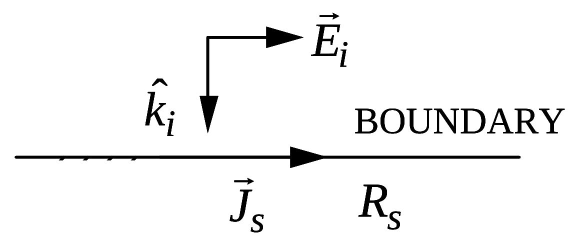
where $R_s' = R_s / \eta_o$ is the normalized surface resistance.

Limiting cases:

- PEC, $\sigma \to \infty, R_s = 0, \Gamma = -1$
- No sheet, $t \to 0, R_s = \infty, \Gamma = 0$
- Example of a resistive film is "window tinting" applied to automobiles (silvered mylar)

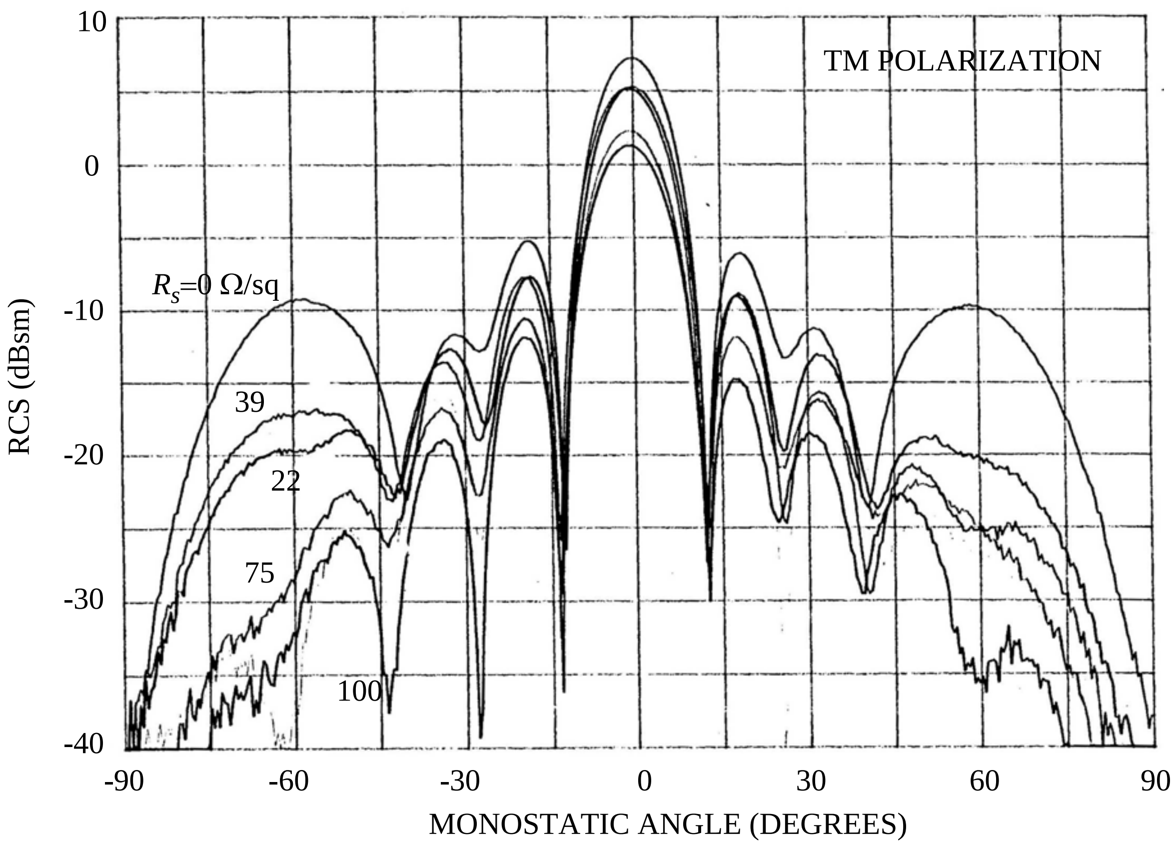
Take the surface current to be the current that flows in a skin depth, $J_s \approx J\delta_s \quad (t = \delta_s)$





RCS of Resistive Sheets

TM polarization, 2.2λ by 2.2λ sheets, measured RCS



- $R_s = 0$ is the same as a PEC plate
- Traveling wave lobes occur at $\approx \pm 58^\circ$
- As $R_s \to \infty$ there is more transmission and less reflection
- As $R_s \to \infty$ the traveling wave diminishes
- No traveling wave for TE polarization

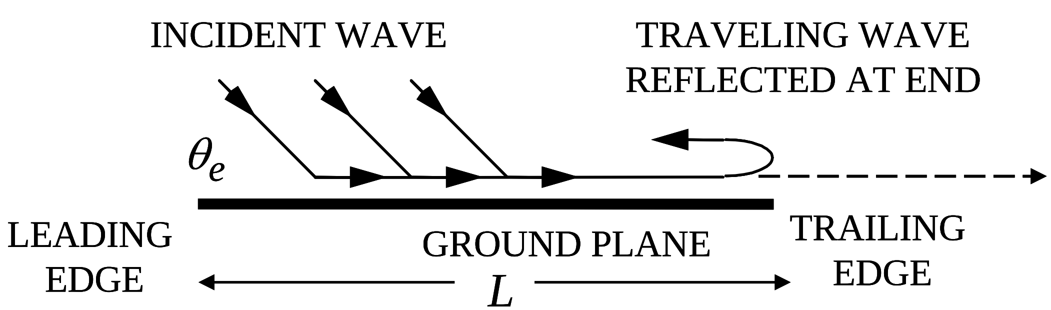
Traveling Waves

A traveling wave is one type of surface wave. The surface impedance supports a transmission line mode. The incident wave is captured by the surface and transformed to a wave guided along the interface.

If the surface is finite (i.e., has an end) then some of the traveling wave will be radiated off the edge in the forward direction and some reflected. The reflected wave radiates as it travels in the reverse direction. This effect is a maximum at an edge incidence angle of approximately

$$\theta_e = 90^\circ - \theta \approx 49.35^\circ \sqrt{\lambda / L}$$

Example: For the 2.2λ sheet, $\theta_e \approx 49.35^\circ \sqrt{\lambda / L} = 33.3^\circ \quad \to \quad \theta \approx 56.7^\circ$



Surface Waves

Surface wave structures are best modeled by transmission lines. It is customary to use *z* as the direction of propagation. The scalar wave equation in region 1 (a source free region) is

$$(\nabla^2 + k^2)\,\psi(x,z) = 0$$

where $\psi(x,z)$ is a wave function that represents either *E* or *H*. The scattered wave function is separable with the general plane wave solution (see Appendix A.7)

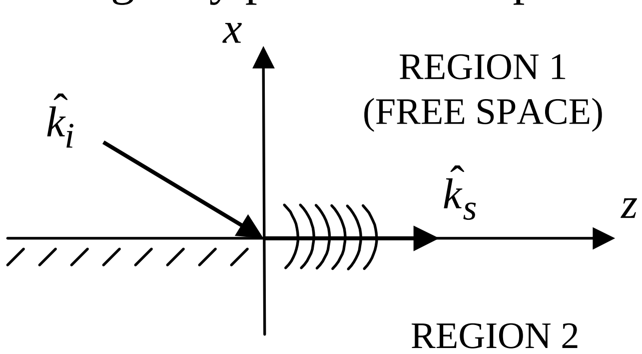
$$\psi(x,z) = \psi_x(x)\,\psi_z(z) = e^{-\gamma_x x}\, e^{-\gamma_z z}$$

Insert this back into the wave equation, and the result is the <u>separation equation</u>

$$\gamma_x^2 + \gamma_z^2 + k^2 = 0$$

Let $\gamma_x = \alpha_x + j\beta_x$ and $\gamma_z = \alpha_z + j\beta_z$. Equate real and imaginary parts of the separation equation to get two equations

$$\alpha_x^2 + \alpha_z^2 - \beta_x^2 - \beta_z^2 + k^2 = 0$$
$$\alpha_x\beta_x + \alpha_z\beta_z = 0$$



The second equation implies equiphase planes are perpendicular to constant amplitude planes.

Surface Waves

Equiphase planes are perpendicular to z. Equal amplitude planes are perpendicular to x. The field in region 1 decays as $|E_1| \sim \left|e^{-\gamma_x x}\right| = e^{-\alpha_x x}$

$\alpha_x > 0$ are proper waves. They decay with distance from the interface.
$\alpha_x < 0$ are improper waves. They grow with distance from the interface.

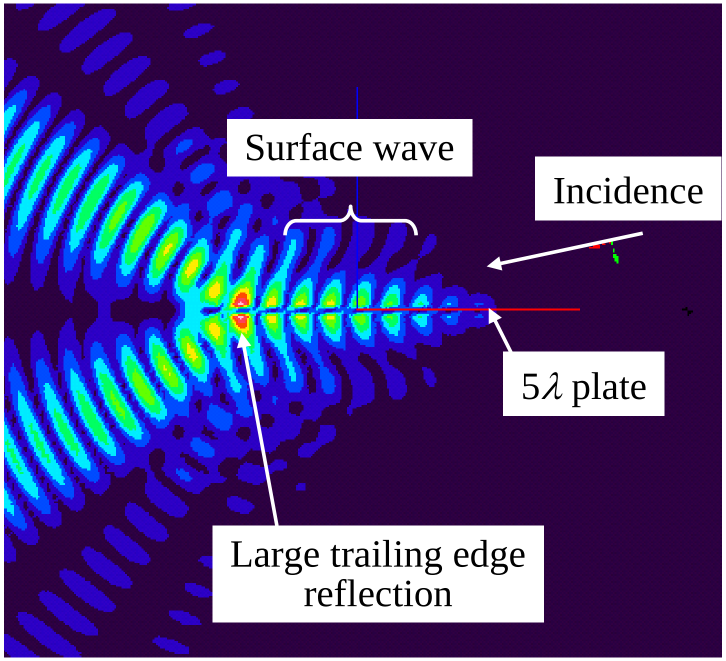
Trapped or tightly bound surface waves decay rapidly.

Loosely bound surface waves decay slowly.

Slow waves have phase velocities less than that of the unbounded medium (free space in this case)

Fast waves have phase velocities greater than that of the unbounded medium (free space in this case)

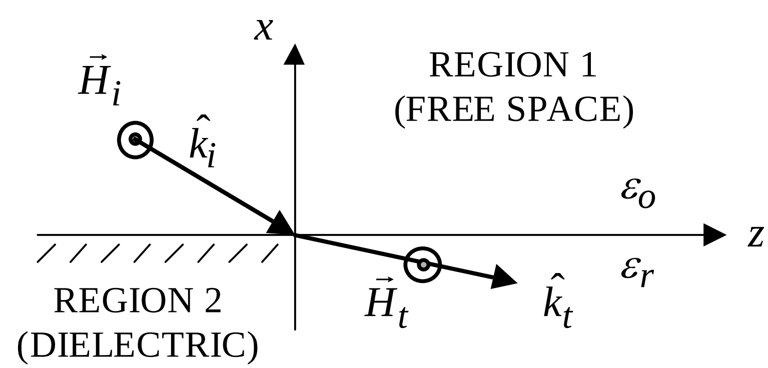
Scattered field plot using EMVIZ



Surface Waves and Surface Impedance

Examine a dielectric/air boundary shown

- the incident wave propagates in the $-x$ direction
- the angle of incidence is the Brewster's angle so as to excite a surface wave



The incident magnetic field intensity is

$$H_{y1} = H_{yi} = H_0 \exp(\gamma_{x1} x - \gamma_{z1} z) = H_0 \exp\left\{ -jk_o \frac{z\sqrt{\varepsilon_r} - x}{\sqrt{\varepsilon_r - 1}} \right\}, \quad x > 0$$

The impedance on the surface is (see book for details):

$$\eta_s = \frac{E_{z1}}{H_{y1}} = \frac{\gamma_{x1}}{j\omega\varepsilon_o} = \underbrace{-\frac{j\gamma_{x1}}{k_o}}_{=Z_s} \eta_o \quad \rightarrow \quad Z_s \equiv R_s' + jX_s' = \frac{\beta_{x1} - j\alpha_{x1}}{k_o}$$

Propagation constant of the traveling wave

$$\gamma_z = -k_o^2 - \gamma_{x1}^2 = k_o \sqrt{(R_s')^2 - [1 + \left(X_s'\right)^2 + 2jR_s' X_s'}$$

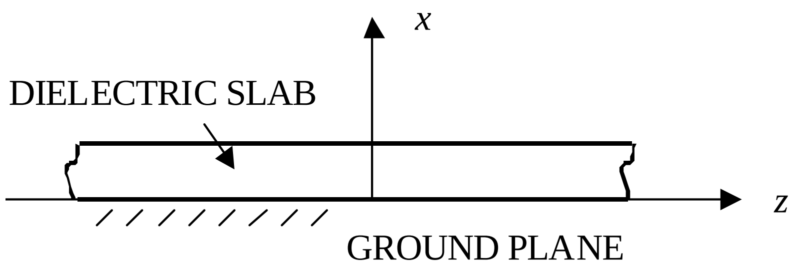
Surface Waves and Surface Impedance

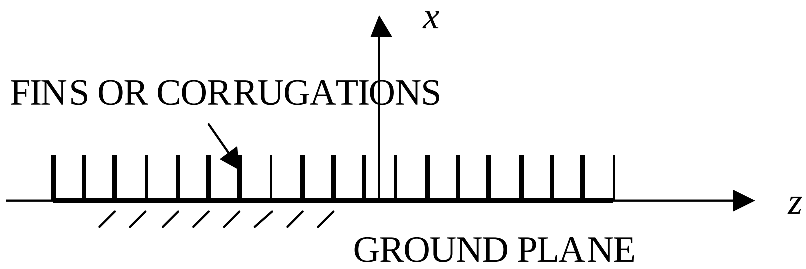
Conditions for the surface impedance to support a traveling wave:

$$R_s' = \frac{\beta_{x1}}{k_o} \text{ and } X_s' = -\frac{\alpha_{x1}}{k_o}$$

A proper wave in region 1 has $\alpha_{x1} < 0$, therefore $X_s' > 0$ and the surface impedance is inductive (positive). To support a surface wave for TM polarization an inductive surface impedance is required.

Examples of surfaces with inductive impedance:



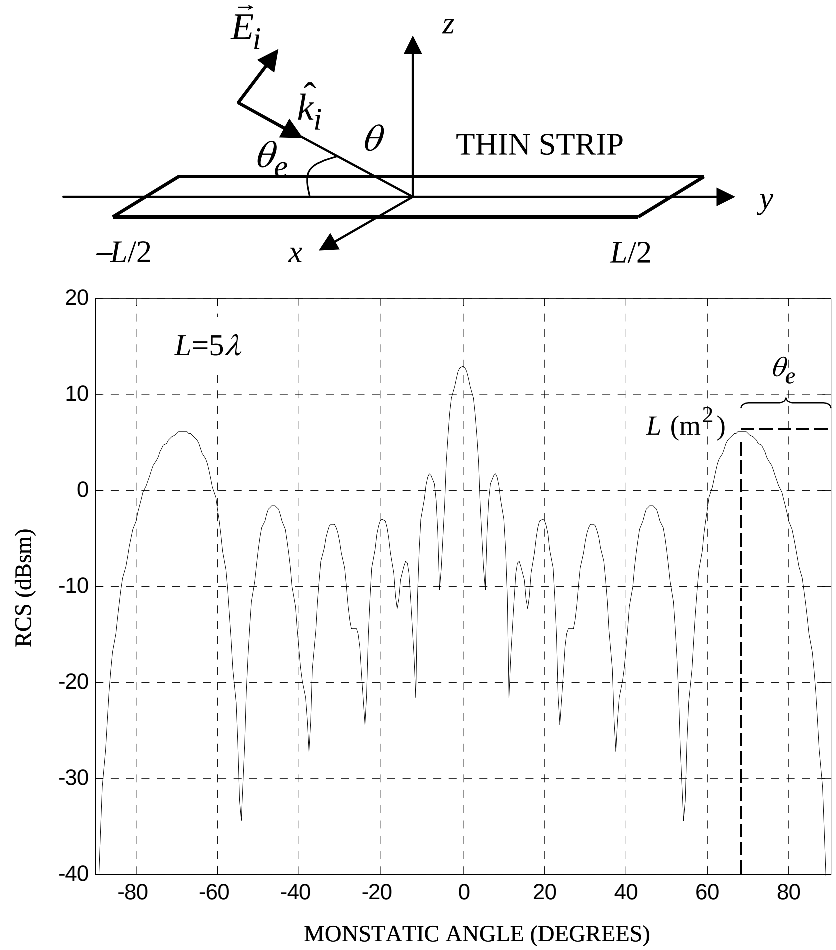


Traveling Wave Lobes

(Example 2.16) A conducting strip has a surface impedance of

$$Z_s = (1+j)\sqrt{\frac{k_o}{2\sigma_c \eta_o}} = R_s' + jX_s'$$

$$\gamma_z = jk_o\sqrt{1 - j\frac{k_o}{2\sigma_c \eta_o}} \approx jk_o + \frac{k_o^2}{2\sigma_c \eta_o}$$



Typical values of $\sigma_c = 10^7$ S/m and $k_o = 200$ rad/m ($\lambda = 3.14$ cm) give

$$\alpha_z = 5.3 \times 10^{-6} \text{ Np/m}$$
$$\alpha_{x1} = 3.2 \times 10^{-6} \text{ Np/m}$$

Significant levels occur in the far field. The lobe edge angle is approximately $\theta_e \approx 49.35\sqrt{\lambda / L}$ degrees.