Introduction: Sensors and RCS
(Chapter 1)

EC4630 Radar and Laser Cross Section

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Sensors and Signatures

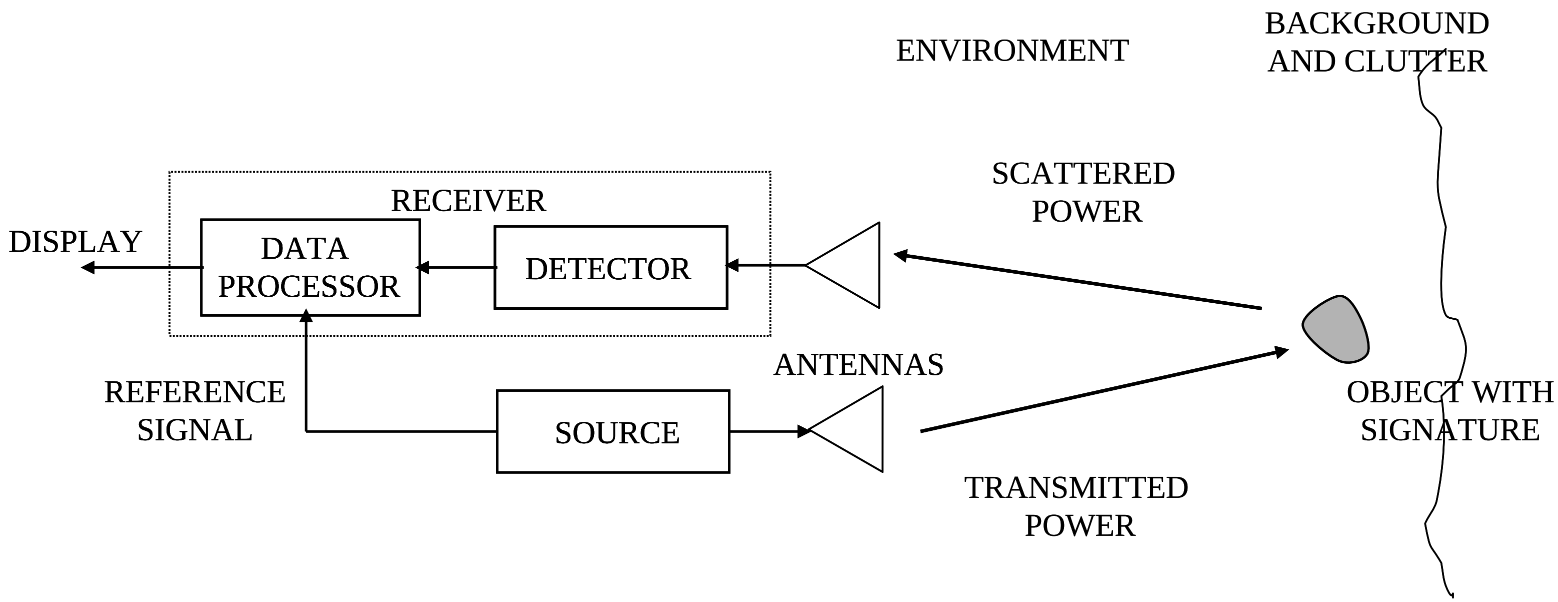
- Sensors collect data
- Sensor categories:
  1. Active (has a transmitter) vs. passive (only listens)
  2. Imaging (determines target features) vs. nonimaging (measures presence and movement)

- Sensor performance measures and attributes
  1. Detection range
  2. Resolution
  3. Field of view (FOV)
  4. Data collection rate
  5. Operating frequency band or regime: electromagnetic (IR, UV, MM, or radar), visual (as it relates to human vision), or acoustic

- Related issues
  1. Target signatures
  2. Probability of intercept ("quietness" of a system, covertness)
  3. Jamming and countermeasures to jamming
  4. Data fusing (network centric approaches)
  5. Background and clutter

Sensor System Components

- Antennas serve as transitions from guided wave structures to free space
- A generic block diagram of an active system is shown



Non-imaging vs. Imaging Sensors

Non-imaging Sensors

- Unresolved signatures are easiest to describe
- Very coarse background separation
- Long detection ranges – all signal concentrated on a single detector
- Spatially integrated total target emission important
- Spatial distribution of target emission not important

Suppression:

- Replace radiation occluded by target to blend in with background

Examples:

- Most radars
- Reticle missile seekers
- Infrared search & track (IRST)

Imaging Sensors

- Difficult to describe sensor due to spatial properties and numerous performance variables
- Ability to recognize pattern-related features (shape & edges; texture within shape)

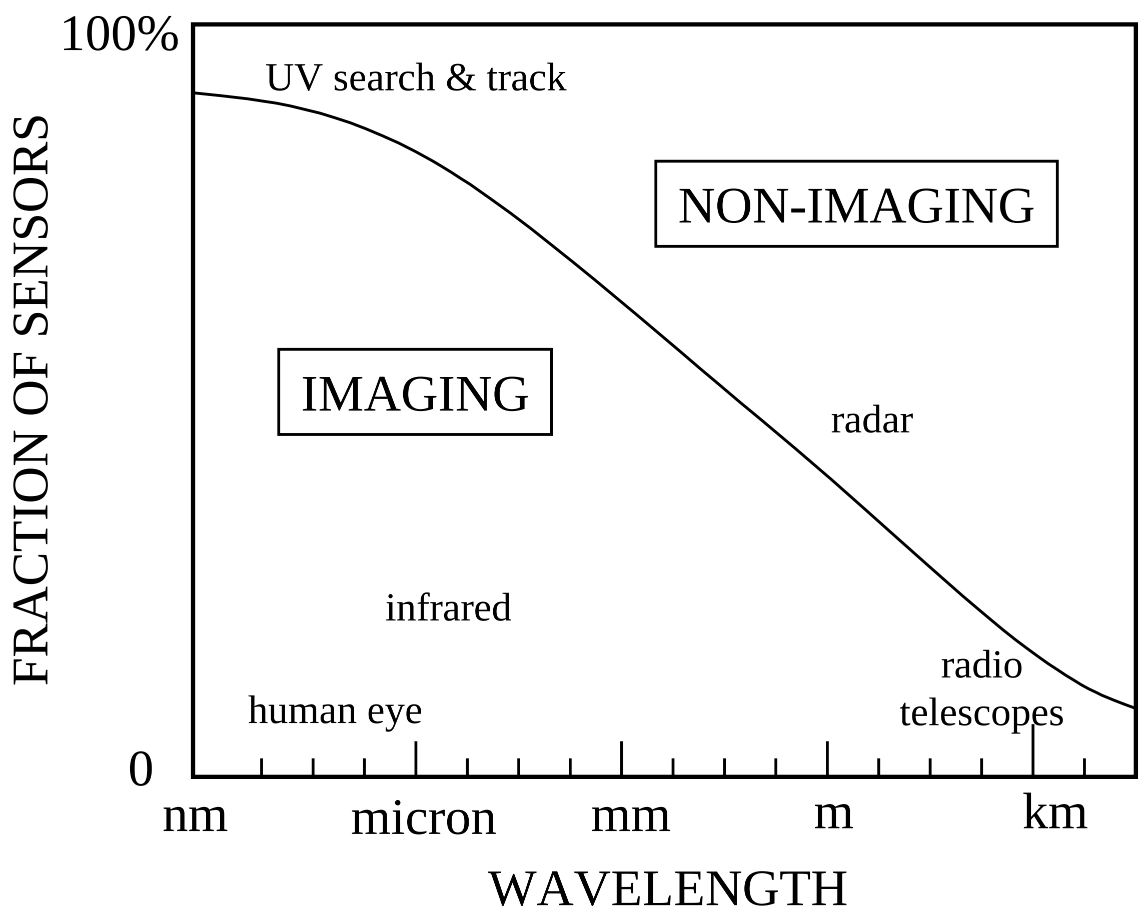
Suppression:

- Make target invisible to threat sensor (stealth)
- If signature reduction not practical then use camouflage (disguise shapes, blend shape into background, pattern deception)

Examples:

- Forward-looking infrared (FLIR)
- Television
- Human eye

Fraction of Imaging Sensors



From Prof. A. E. Fuhs

Performance Tradeoffs for Sensors

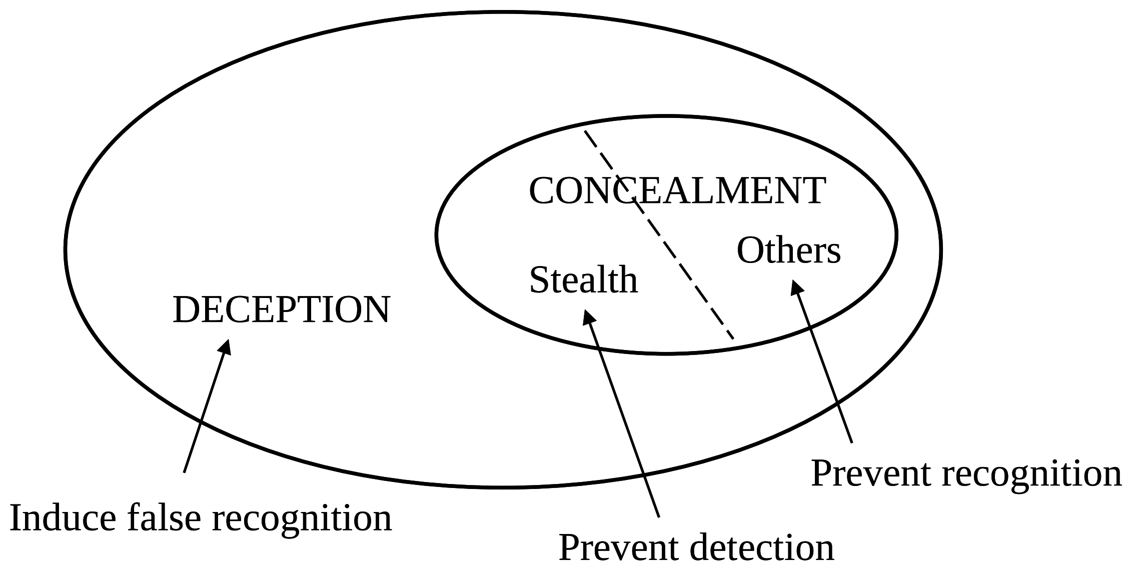
Sensor	Advantages	Issues
Forward looking Infrared (FLIR)	• High target to background contrast • Day or night operation • Penetrates fog, haze and dust	• False alarms from background clutter • Range uncertainty • Occlusions from terrain and vegetation • Aspect angle dependence
Millimeter wave (MMW) radar	• All weather operation • Day and night operation	• False alarms from background clutter, rocks, building, etc. • Terrain occlusions • Signature varies with aspect angle
Synthetic aperture radar (SAR)	• All weather operation • Day and night operation • Large target to background contrast	• False alarms from background clutter, rocks, building, etc. • Terrain occlusions • Signature varies with aspect angle
Laser radar	• High range resolution • Doppler measurement • Imaging capability • Penetrates fog, haze and dust	• Signature varies with aspect angle • Complex system and technology • Requires long dwell time • Requires precise tracking and stabilization
Passive electro-optic (EO)	• Lightweight • Inexpensive • High resolution • Reliable	• Relatively low target to background contrast • No night or all weather capability

Camouflage and Stealth

Camouflage is the military art of concealment and deception.

Stealth does not reveal itself by impairing enemy sensor operation.

Countermeasures impair the ability of sensors to "sense." We may be unconcerned with the enemy's awareness of the use of a countermeasure so long as the ability to sense is denied (e.g., chaff).



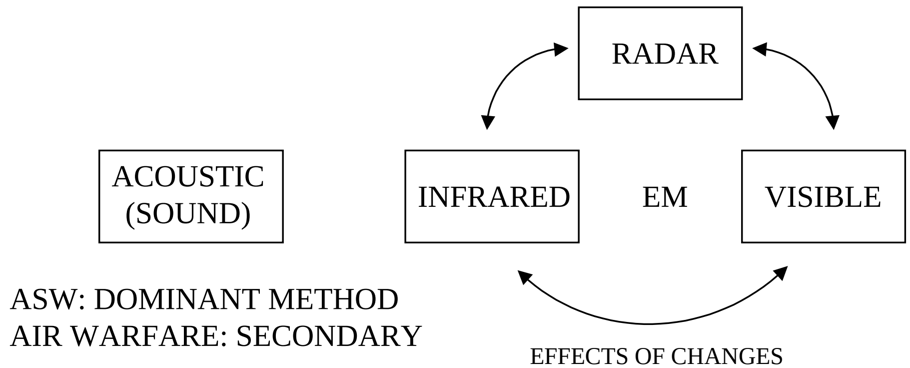
From Prof. A. E. Fuhs

Stealth, or low observability (LO) = undetectable to all sensors, both active and passive

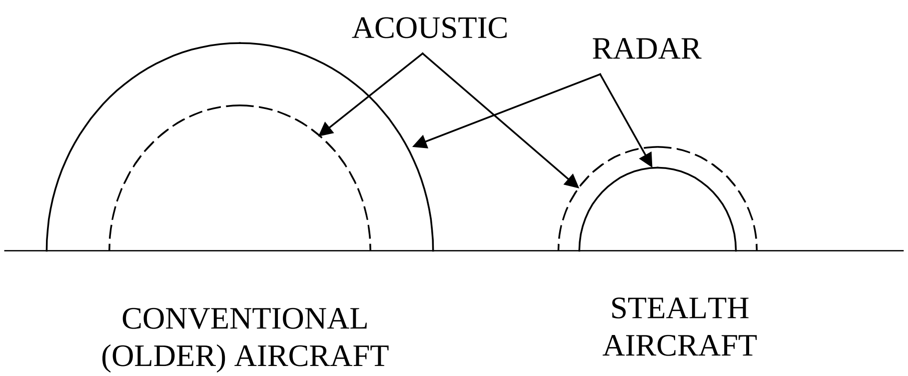
- optical/visible
- electromagnetic
    - laser
    - microwave/RF
    - passive infrared
- acoustic
- low emissions in all bands

Sensors and Low Observables (Stealth)

- LO = low observable, ULO = ultra-low observable
- Radar, visible and infrared signatures are coupled to a degree.

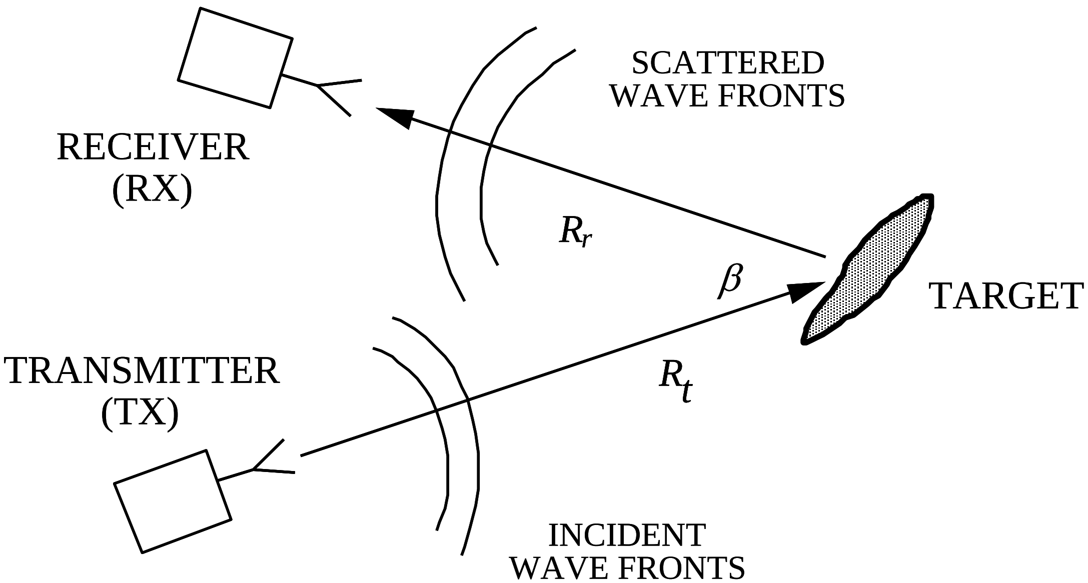


- Acoustics: why bother? The radar detection ranges of stealth aircraft have been reduced to the point where they are comparable to, or less than, those of acoustic sensors. (Circles represent detection envelopes.)



WWII vintage acoustic direction finder

Bistatic vs. Monostatic Radar



β is the bistatic angle

$\beta = 0^\circ$ direction is backscattering

$\beta = 180^\circ$ direction is forward scattering

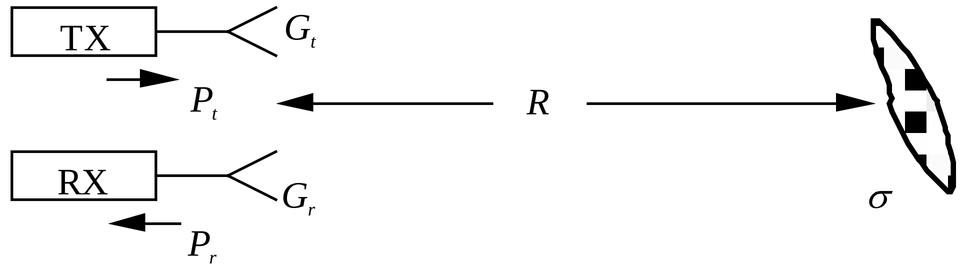
Bistatic: the transmit and receive antennas are at different locations as viewed from the target (e.g., ground transmitter and airborne receiver, $\beta \neq 0$)

Monostatic: the transmitter and receiver are co-located as viewed from the target (i.e., the same antenna is used to transmit and receive, $\beta = 0$)

Quasi-monostatic: the transmit and receive antennas are slightly separated but still appear to be at the same location as viewed from the target (e.g., separate transmit and receive antennas on the same aircraft, $\beta \approx 0$)

Basic Radar Range Equation (1)

"Quasi-monostatic" geometry



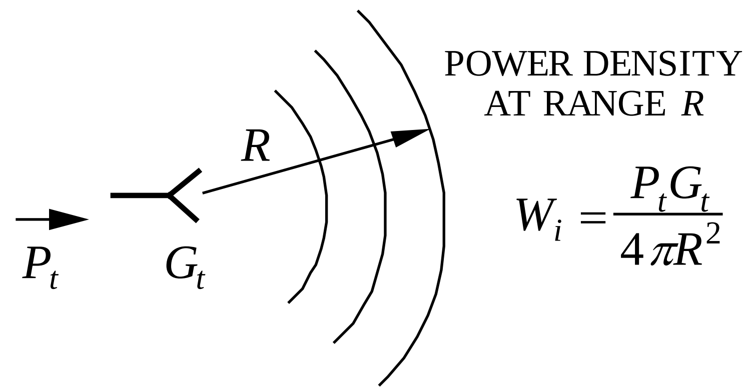
σ = radar cross section (RCS) in square meters
P_t = transmitter power, watts
P_r = received power, watts
G_t = transmit antenna gain in the direction of the target (assumed to be the maximum)
G_r = receive antenna gain in the direction of the target (assumed to be the maximum)
$P_t G_t$ = effective radiated power (ERP)

From antenna theory: $G_r = \dfrac{4\pi A_{er}}{\lambda^2}$ (for an aperture type antenna)

$A_{er} = eA$ = effective area of the receive antenna
A = physical aperture area of the antenna
λ = wavelength ($= c / f$)
e = antenna efficiency

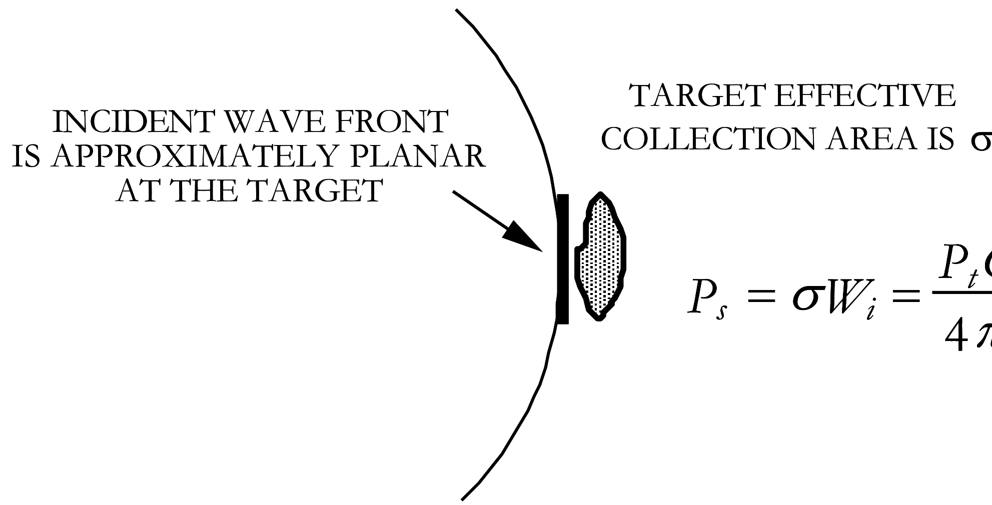
Basic Radar Range Equation (2)

Power density incident on the target



$$W_i = \frac{P_t G_t}{4\pi R^2} \quad (\mathrm{W/m^2})$$

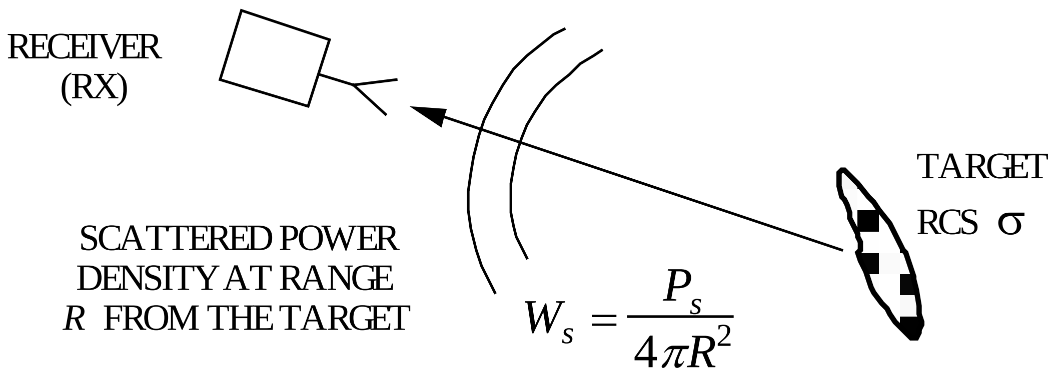
Power collected by the target and scattered back towards the radar is P_s



$$P_s = \sigma W_i = \frac{P_t G_t \sigma}{4\pi R^2}$$

Basic Radar Range Equation (3)

The RCS gives the fraction of incident power that is scattered back toward the radar. Therefore, the scattered power density at the radar is obtained by dividing P_s by $4\pi R^2$.



The target scattered power collected by the receiving antenna is $W_s A_{er}$. Thus the maximum target scattered power that is available to the radar is

$$P_r = \frac{P_t G_t \sigma A_{er}}{(4\pi R^2)^2} = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4}$$

This is the classic form of the radar range equation (RRE).

Characteristics of the Radar Range Equation

$$P_r = \frac{P_t G_t \sigma A_{er}}{(4\pi R^2)^2} = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4}$$

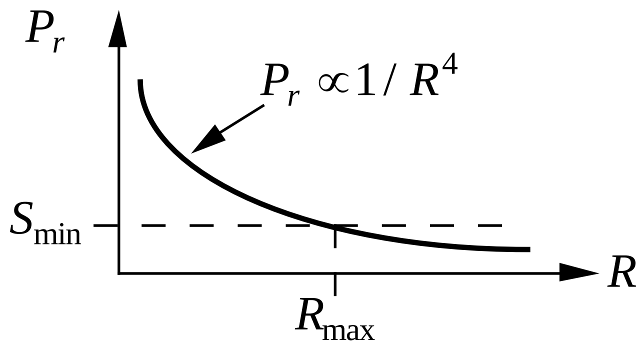
For monostatic systems a single antenna is generally used to transmit and receive so that $G_t = G_r \equiv G$. This form of the RRE is too crude to use as a design tool. Factors have been neglected that have a significant impact on radar performance:

- noise
- system losses
- propagation behavior
- clutter
- waveform limitations,
- receiver characteristics,
- etc.

The above form of the RRE does give some insight into the tradeoffs involved in radar design. The dominant feature of the RRE is the $1/R^4$ factor. Even for targets with relatively large RCS, high transmit powers must be used to overcome the $1/R^4$ when the range becomes large.

Maximum Detection Range

The minimum received power that the radar receiver can "sense" is referred to as the minimum detectable signal (MDS) and is denoted $S_{\min}$.



Given the MDS, the maximum detection range can be obtained:

$$P_r = S_{\min} = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4} \Rightarrow R_{\max} = \left(\frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 S_{\min}} \right)^{1/4}$$

The maximum detection range can be reduced by reducing the target RCS.

Other Radar System Factors

Radars typically operate in a "noise limited" condition. The target's received power must be greater than the receiver noise power. The noise power is generally modeled as Gaussian white noise

$$N_o = k_B \mathrm{T}_s B$$

The <u>signal-to-noise ratio</u> (SNR) is

$$SNR = \frac{P_r}{N_o} = \frac{P_r G_p L}{k_B \mathrm{T}_s B}$$

where: $k_B = 1.38 \times 10^{-23}$ = Boltzman's constant (J/K)

B = bandwidth (Hz)

$\mathrm{T}_s = \mathrm{T}_A + \mathrm{T}_e$ = system noise temperature (K)

T_A = antenna temperature (K)

T_e = effective noise temperature of the receiver (K)

G_p = processing gain (resulting from integration, correlation, or signal processing, etc.)

L = loss factor (≤ 1) for system hardware and processing losses

Typically SNRs of 10 to 20 dB are required for acceptable detection and tracking.

Search Radar Example (1)

Example 1.1: AN/SPS-10 radar has the following parameters:

peak transmitter power = 500 kW
antenna gain = 33.0 dB
frequency = 5.6 GHz
pulse width = 1.4 μs
PRF = 625 Hz
antenna scan rate = 16 rpm
azimuth half-power beamwidth = 1.5 degrees
antenna noise temperature = 75 K

receiver noise bandwidth = 1 MHz
receiver effective temperature = 2900 K
system losses ahead of the receiver = 5 dB
false alarm time (FAT) – average time between false alarms = 2 days
plan position indicator (PPI) display and operator
minimum SNR for the specified FAT = 16 dB
processing gain (integration of 10 pulses) = 9 dB

1. What is the thermal noise power in the receiver?

$$N_o = k_B \mathrm{T}_s B = (1.38\times10^{-23})(75+2900)(10^6) = 4.1\times10^{-14}\ \mathrm{W} = -133.9\ \mathrm{dBW}$$

2. Calculate the MDS in dBW. Note: dBW is decibels relative to 1 W, so that P_r in dBW = $10\log\left(P_r \text{ in watts}\right)$. Convert the SNR from dB: $\mathrm{SNR}_{\min} = P_{r_{\min}} / N_o = 10^{16/10} = 39.8$. The MDS is where the signal equals the noise, $\mathrm{SNR}_{\min} = 0$ dB

$$P_{r_{\min}} = (39.8)(4.1\times10^{-14}) = 1.6\times10^{-12}\ \mathrm{W} = -118\ \mathrm{dBW}$$

Search Radar Example (2)

3. Calculate the peak effective radiated power in dBW.

$$\text{ERP} = P_t G_t = (500\times 10^3)(10^{33/10}) = 997.6 \text{ MW}$$

4. Calculate the effective area of the antenna in square meters.

$$A_{er} = \frac{G\lambda^2}{4\pi} = \frac{(1995.3)(0.054)^2}{4\pi} = 0.46 \text{ m}^2$$

5. Calculate the maximum free space detection range on a 0 dBsm target.
   Convert the loss from dB (note that "loss" implies a negative sign in the exponent): $L = 10^{-5/10} = 0.316$. Convert the processing gain from dB: $G_p = 10^{9/10} = 7.9$. The dB unit of RCS is dBsm (decibels relative to a square meter), which is defined as: $\sigma \text{ in dBsm} = 10\log\left(\sigma \text{ in m}^2\right)$. Therefore, the target RCS is $\sigma = 10^{0/10} = 1\,\text{m}^2$. Thus,

$$R_{\max} = \left[\frac{P_t G^2 \sigma \lambda^2 L G_p}{(4\pi)^3 N_o \text{SNR}_{\min}}\right]^{1/4} = \left[\frac{500\times 10^3 (1995.3)^2 (1)(0.054)^2 (0.316)(7.9)}{(4\pi)^3 (4.1\times 10^{-14})(39.8)}\right]^{1/4} = 46 \text{ km}$$

Antenna as a Radar Target

Example 1.4: Antenna with gain G_a, effective area, A_{ea} and RCS σ has its terminals shorted (i.e., all of the received radar signal is sent back out the antenna). The scattered signal back at the radar receiver (monostatic) is

$$P_r = \underbrace{\frac{P_t G}{4\pi R^2} A_{ea}}_{P_{ra}} G_a \frac{1}{4\pi R^2} \underbrace{\frac{\lambda^2 G}{4\pi}}_{=A_{er}} = \frac{P_t G^2 \lambda^2}{(4\pi)^3 R^4} A_{ea} G_a$$

Based on the antenna's RCS, the received power is

$$P_r = \frac{P_t G^2 \lambda^2 \sigma_a}{(4\pi)^3 R^4}$$

Comparing gives: $$\sigma_a = A_{ea} G_a = A_{ea} \frac{4\pi A_{ea}}{\lambda^2} = \frac{4\pi A_{ea}^2}{\lambda^2} = \frac{4\pi e^2 A^2}{\lambda^2}$$

where $A_{ea} = eA$ (A is the physical area; e is the aperture efficiency). For an electrically large uniformly illuminated aperture we can use $A_{ea} \approx A$ and get the well-known formula

$$\sigma = \frac{4\pi A^2}{\lambda^2} \quad \text{(flat surface backscatter)}$$

Flat Surface Backscatter Formula

The "flat surface backscatter formula" applies to any arbitrary contoured flat surface of area A. Normal incidence on a plane surface gives $\sigma = \frac{4\pi A^2}{\lambda^2}$ m^2. The decibel unit is <u>decibels relative to a square meter</u> (dBsm) defined as: $\sigma,\ \text{dBsm} = 10\log_{10}(\sigma,\ \text{m}^2)$

Example 1.5: Normal incidence on a L by L square plate with L = 1 m

At 300 MHz, $\lambda = 1\text{m}$: $\sigma = \frac{4\pi\left[(1)^2\right]^2}{1^2} = 12.56\ \text{m}^2 = 11\ \text{dBsm}$

At 3 GHz, $\lambda = 0.1\text{m}$: $\sigma = \frac{4\pi\left[(1)^2\right]^2}{0.1^2} = 1256\ \text{m}^2 = 31\ \text{dBsm}$

Note the "gain" effect with increasing frequency (similar to an aperture antenna). A 1 m^2 area plate has a much higher RCS than 1 m^2 at these frequencies. This is because the plate scattering has some directivity (i.e., it does not scatter isotropically).

Wavelength Dimensions

A wavelength dimension is often used for targets. At a frequency with wavelength λ the plate edge length L can be expressed as a constant ℓ times wavelength, $L = \ell\lambda$.

For example, at 300 MHz a 1 m by 1 m plate is 1λ is 1λ ($\ell = 1$).

The RCS is often expressed in the dimensionless quantity σ / λ^2

$$\sigma = \frac{4\pi\left[(\ell\lambda)^2\right]^2}{\lambda^2} = 4\pi\ell^4\lambda^2 \quad \rightarrow \quad \sigma/\lambda^2 = 4\pi\ell^4$$

and in decibels, σ/λ^2, dB = $10\log_{10}(\sigma/\lambda^2)$.

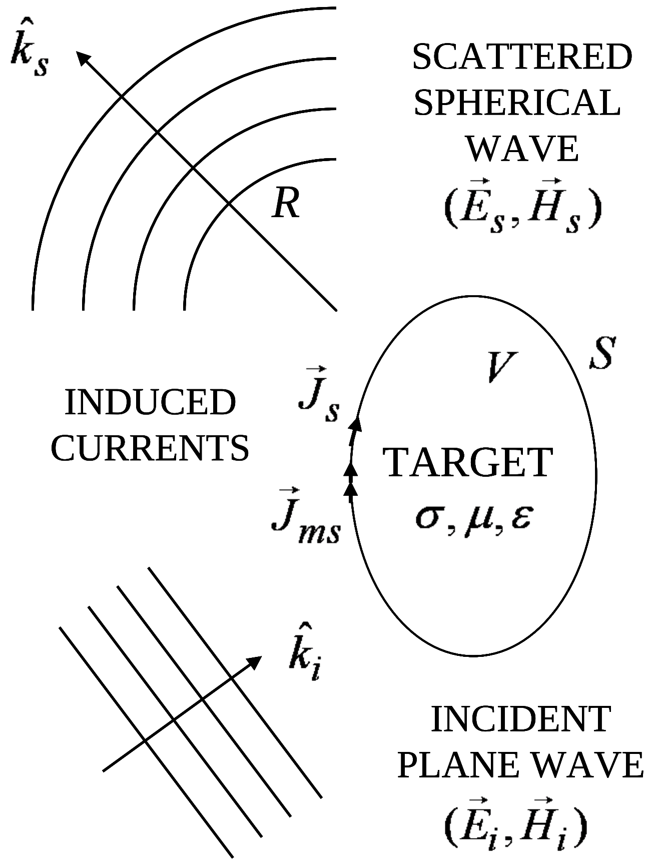
Example: The RCS of a 5λ square plate is

$$\sigma/\lambda^2 = 4\pi 5^4 = 7850 = 39 \text{ dB}$$

To obtain the RCS in dBsm at 600 MHz ($\lambda = 0.5$ m)

$$\sigma = \left(\sigma/\lambda^2\right)\lambda^2 = (7850)(0.5)^2 = 1962.5 \text{ m}^2 = 32.9 \text{ dBsm}$$

Scattering Nomenclature



Note: surface currents are shown, but there may also be volume currents in *V*.

Incident fields $(\vec{E}_i, \vec{H}_i)$

- Fields that exist in the absence of the object
- For RCS they are assumed to be plane wave.

Total fields $(\vec{E}, \vec{H})$

- Fields that exist with the object present

Scattered fields $(\vec{E}_s, \vec{H}_s)$

- The difference between the incident and total field due to the radiation of the induced currents

$$(\vec{E}_s, \vec{H}_s) = (\vec{E} - \vec{E}_i, \vec{H} - \vec{H}_i)$$

Typical integral for the far-scattered field for electric currents $(k = 2\pi / \lambda)$:

$$\vec{E}_s(R) \sim \frac{e^{-jkR}}{R} \iint_S \vec{J}_s \cdots ds'$$

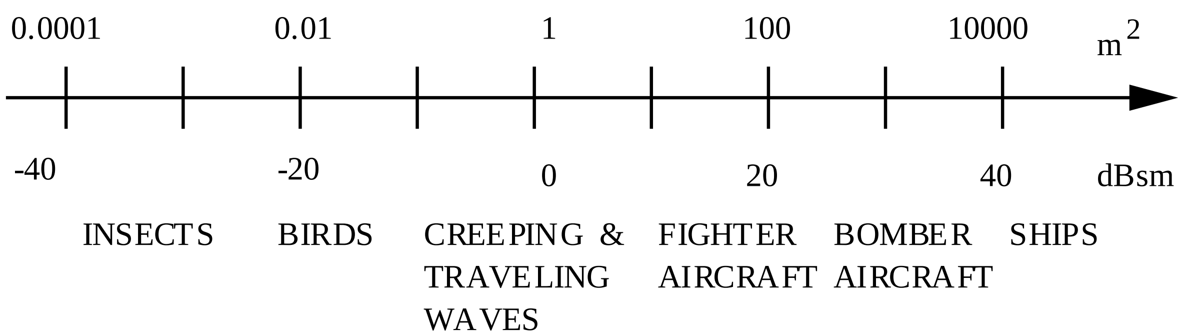
Definition of RCS

Formal definition of RCS:

$$\sigma = \frac{\text{power reflected to receiver per unit solid angle}}{\text{incident power density}/4\pi} = \lim_{R\to\infty} 4\pi R^2 \frac{|\vec{E}_s|^2}{|\vec{E}_i|^2}$$

- In the definition we could also use the ratio of scattered and incident magnetic field intensities squared or power densities.
- In the far scattered field of the target $\vec{E}_s \sim 1/R^2$ so RCS is range independent.
- Functional dependencies: $\sigma_{pq}(f, \theta_i, \phi_i, \theta_s, \phi_s)$
  - (p,q) denote polarizations of the scattered and incident fields, respectively
  - f is frequency
  - (θ_i, ϕ_i) incident wave (source) direction
  - (θ_s, ϕ_s) scattered wave (observation) direction

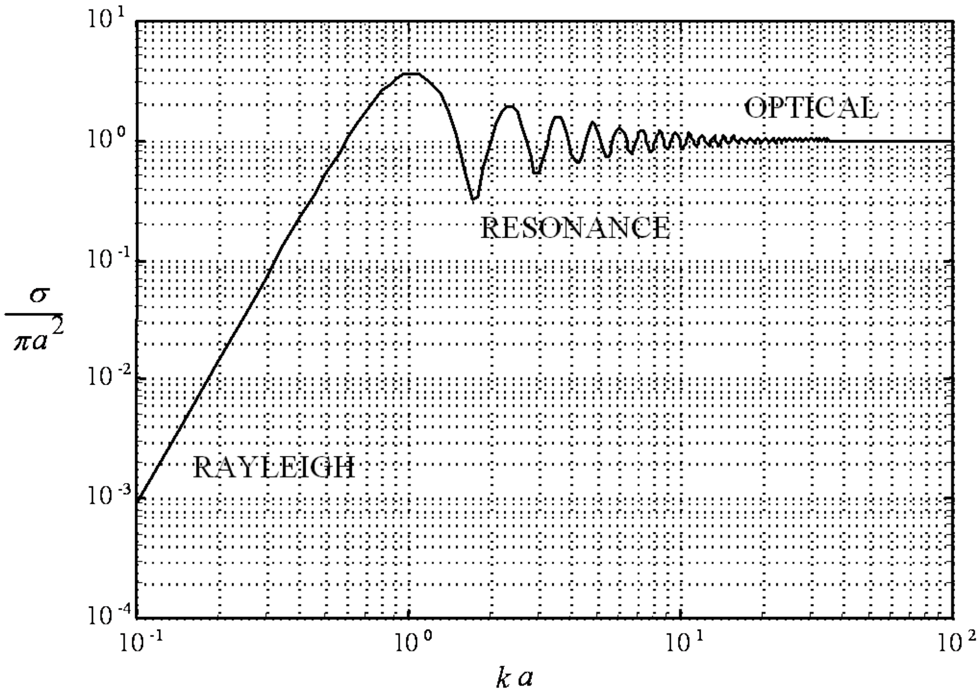
Typical values:



Frequency Regions

RCS of a sphere vs. frequency illustrates the common behavior in the three frequency regions.

$a =$ radius, $k = 2\pi / \lambda$



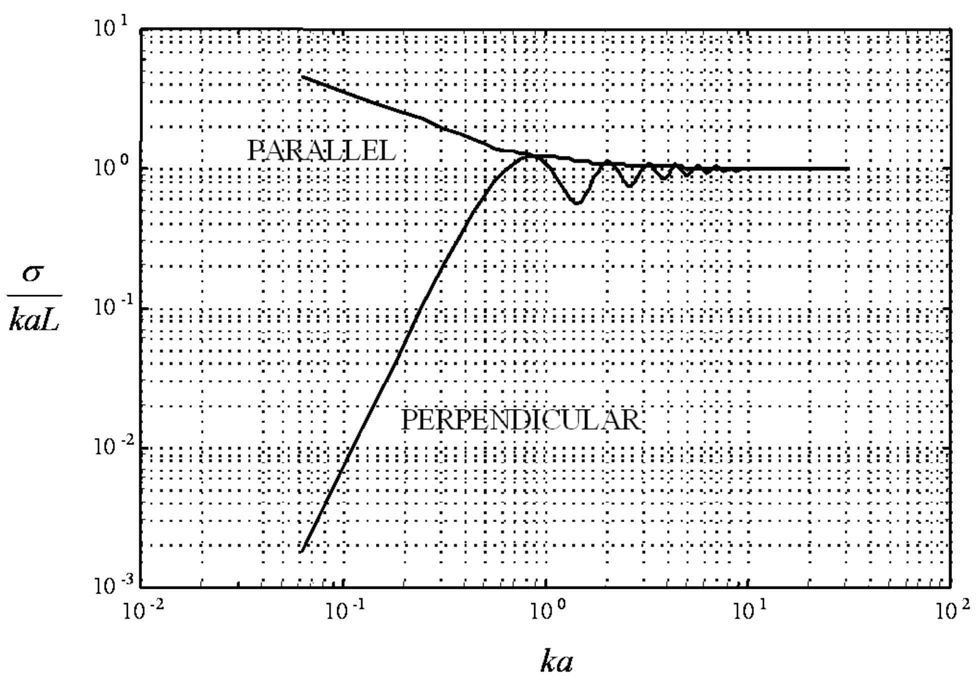
- Rayleigh region (low frequency): $ka << 1, \sigma \sim 1/\lambda^4$
- Mie region (resonance region): $ka \approx 1, \sigma$ vs. frequency oscillates
- Optical region (high frequency): $ka >> 1, \sigma$ vs. frequency smooth and may become independent of frequency

For other target shapes a can be replaced by a "characteristic length, L."

Polarization Dependence

A circular cylinder illustrates RCS polarization dependence with frequency.

a = cylinder radius, L = length



Polarization reference:

- Parallel: $\vec{E}_i$ parallel to cylinder axis
- Perpendicular: $\vec{E}_i$ perpendicular to cylinder axis

"Thin wire" behavior when $a/\lambda << 1$

- Wire scatters for parallel polarization
- Wire invisible for perpendicular polarization

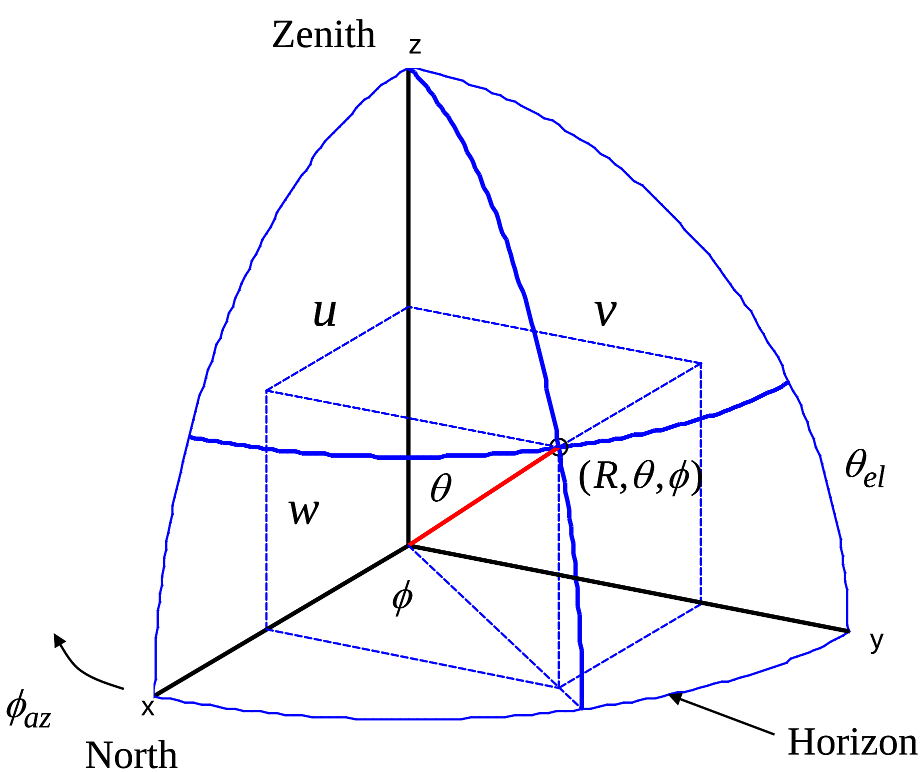
Three frequency regions are observed for perpendicular polarization.

Coordinate Systems

Radar coordinate systems: spherical polar: (r,θ,ϕ)

azimuth/elevation: (Az,El) or (α,γ) or (ϕ_{az},θ_{el})

The radar is located at the origin of the coordinate system; the Earth's surface lies in the x-y plane. Azimuth is generally measured clockwise from a reference (e.g., from North like a compass) but ϕ is measured counterclockwise from the x axis



Azimuth: $\alpha = 360^\circ - \phi$

Elevation: $\gamma = 90^\circ - \theta$

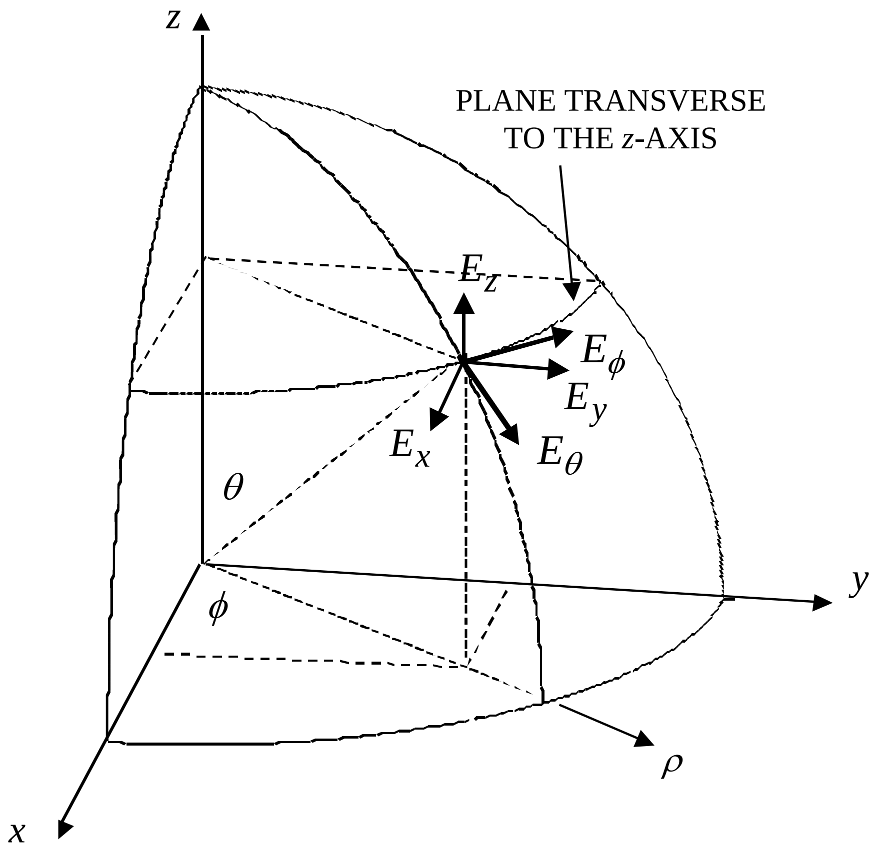
R is the distance between the source and observer. If one of them is at the origin then $R = r$.

Direction cosines (when $R = 1$)

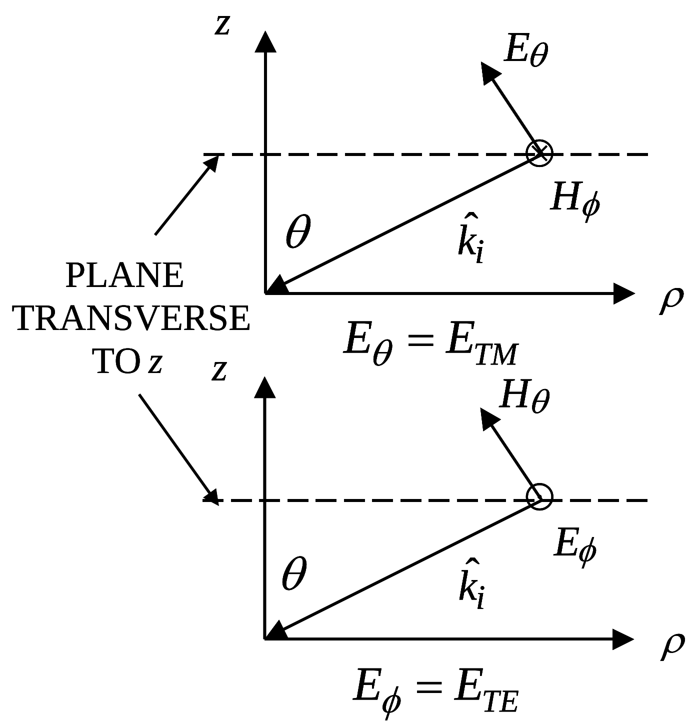
$$\cos\alpha_x = u = \sin\theta\cos\phi$$
$$\cos\alpha_y = v = \sin\theta\sin\phi$$
$$\cos\alpha_z = w = \cos\theta$$

Polarization Definitions

- Spherical, cylindrical and Cartesian components are shown.
- The *x*-*y* plane is the horizontal plane (parallel to the ground).

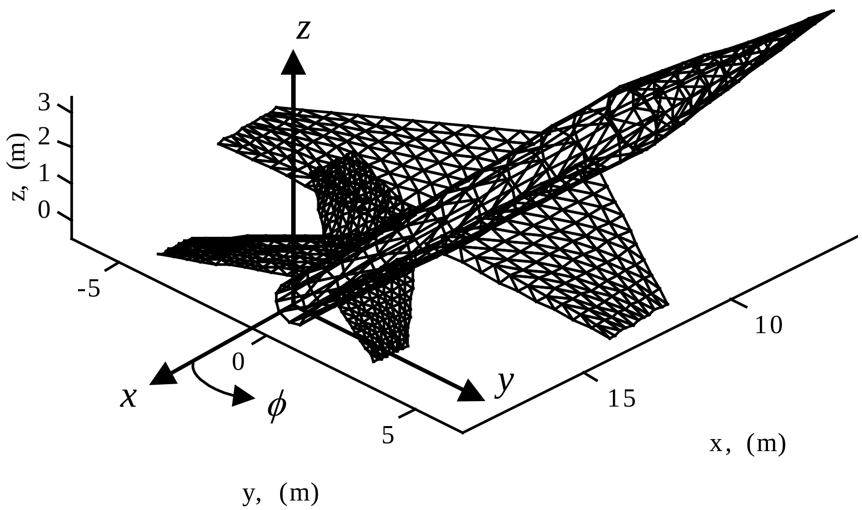


- Spherical: $\vec{E} = E_\theta \hat{\theta} + E_\phi \hat{\phi}$
- Horizontal (H) and vertical (V):
  $\vec{E} = E_H \hat{\phi} + E_V \hat{\theta}$ (when $\theta \approx 90^\circ$)
- Transverse electric (TE_z) and transverse magnetic (TM_z): $\vec{E} = \vec{E}_{TM} + \vec{E}_{TE}$



Scattering Mechanisms

The scattered fields $(\vec{E}_s, \vec{H}_s)$ that arise due to the induced currents determine the RCS. A bistatic scattering pattern of a generic aircraft is shown. Often the pattern peaks or shape can be identified with specific <u>scattering mechanisms</u> (or modes) associated with scattering sources on the target. Examples include: reflections from large (in terms of wavelength) surfaces, diffraction from edges, and surface waves.



$\theta_i = \phi_i = 90^\circ, \theta = 90^\circ, f = 1$ GHz, TM pol

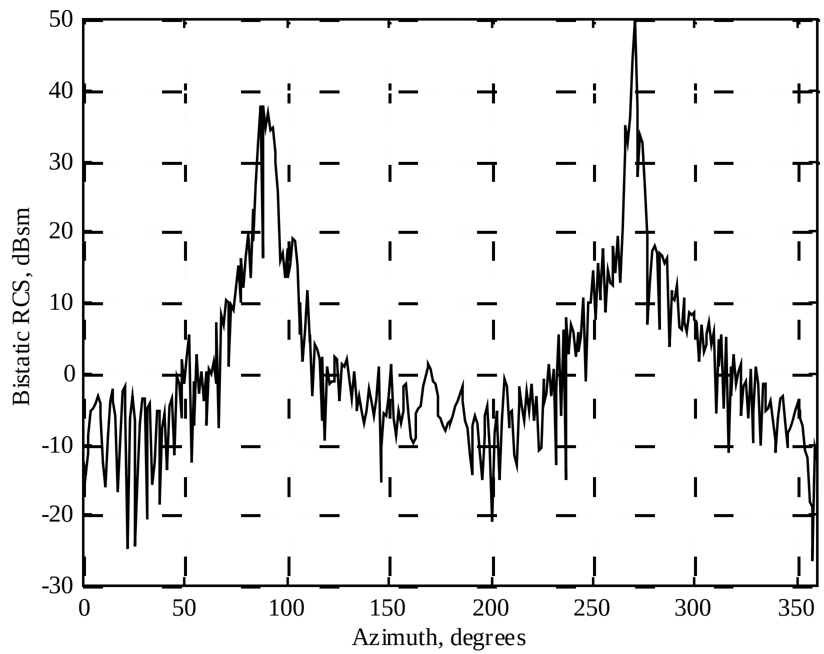
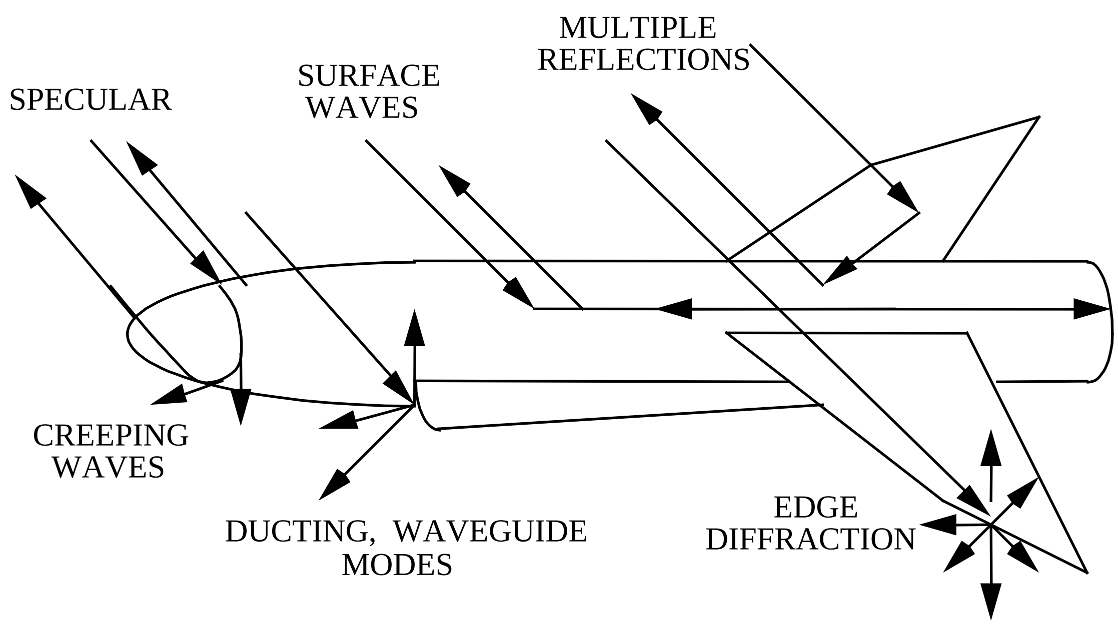


Illustration of Scattering Mechanisms

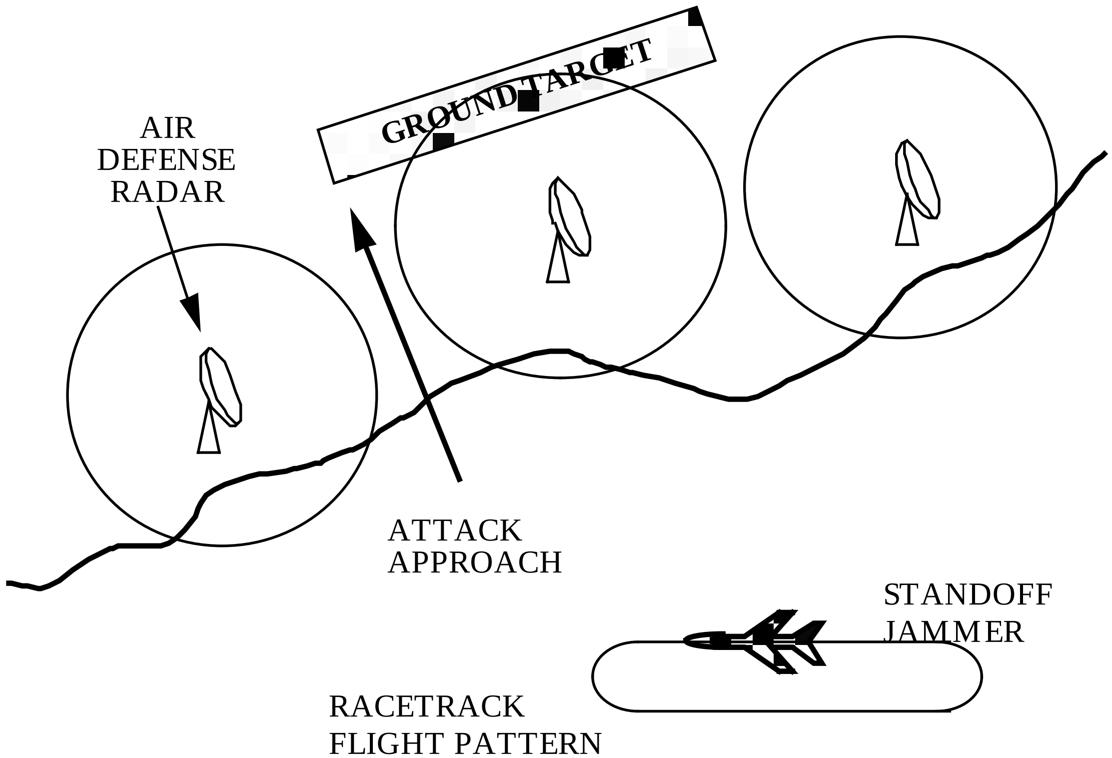


Important scattering mechanisms:

- Reflections, multiple reflections (multipath)
- Diffraction from edges
- Surface waves
    - Travelling waves
    - Creeping waves
    - Leaky waves
- Ducting (waveguide or cavity modes)
- Hybrid or "mixed" modes

<u>Reduction techniques</u> are dependent on the scattering source and mechanism.

Defeating Radar by Jamming

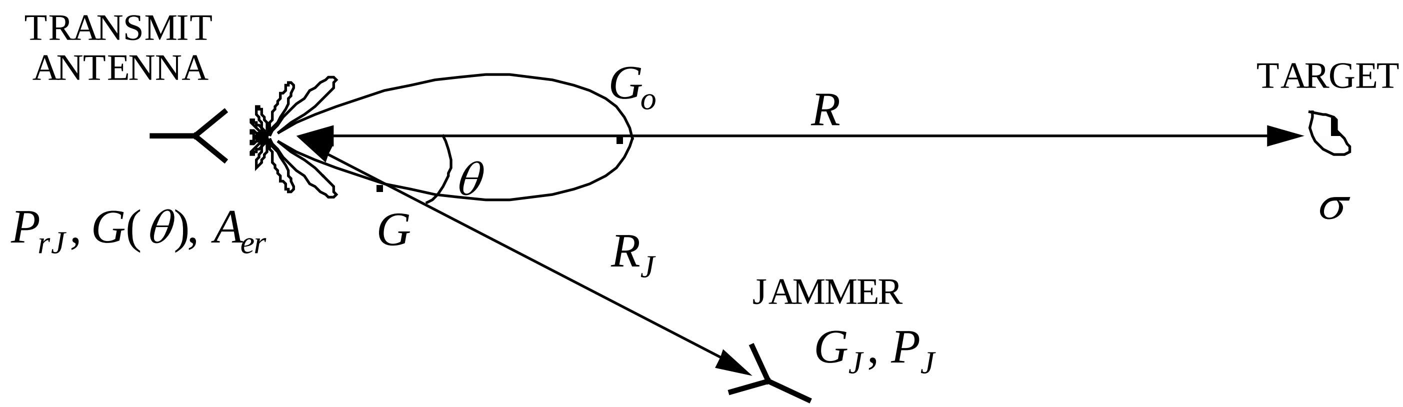


The barrage jammer floods the radar with noise and therefore decreases the SNR.

The radar knows it's being jammed.

Jammer Burnthrough Range (1)

Consider a standoff jammer operating against a radar that is tracking a target



The jammer power received by the radar is

$$P_{rJ} = W_i A_{er} = \left(\frac{P_J G_J}{4\pi R_J^2}\right)\left(\frac{\lambda^2 G(\theta)}{4\pi}\right) = \frac{P_J G_J \lambda^2 G(\theta)}{\left(4\pi R_J\right)^2}$$

Defining $G_o \equiv G(\theta = 0)$, the target return is

$$P_r = \frac{P_t G_o^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

Jammer Burnthrough Range (2)

The signal-to-jam ratio is

$$\mathrm{SJR} = \frac{S}{J} = \frac{P_r}{P_{rJ}} = \left(\frac{P_t G_o}{P_J G_J}\right)\left(\frac{R_J^2}{R^4}\right)\left(\frac{\sigma}{4\pi}\right)\left(\frac{G_o}{G(\theta)}\right)$$

The burnthrough range for the jammer is the range at which its signal is equal to the target return (SJR=1).

Important points:

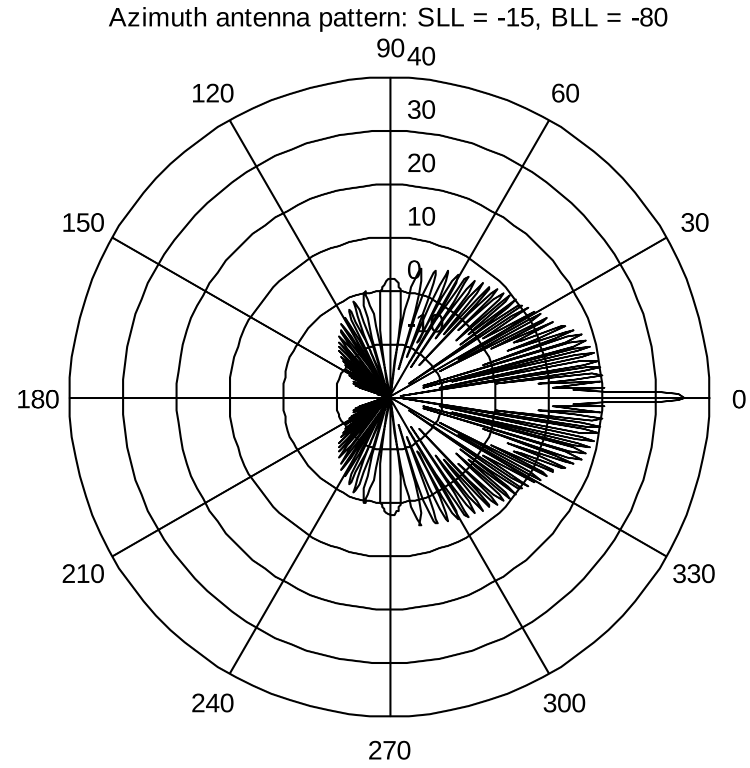
- R_J^2 vs R^4 is a big advantage for the jammer.
- G vs $G(\theta)$ is usually a big disadvantage for the jammer. Low sidelobe radar antennas reduce jammer effectiveness.
- Given the geometry, the only parameter that the jammer has control of is the ERP ($P_J G_J$).
- The radar knows it is being jammed. The jammer can be countered using waveform selection and signal processing techniques.

Jammer Example

Example 1.3: Radar detection range with and without jamming.

AN/SPS-10 from Example 1.1.
Reflector antenna has the pattern shown.
Detection range without jamming is 61 km

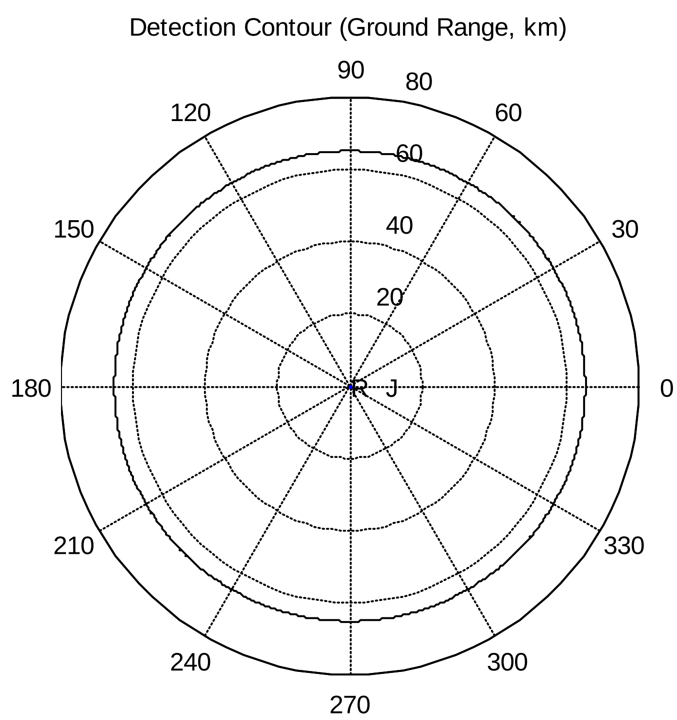
- Using software RADJAM
- "J" is jammer location (fixed)
- "R" is radar location (center)
- Radar beam is "on target" as it moves from 0 to 360 degrees in azimuth
- 0 dBsm target
- no multipath



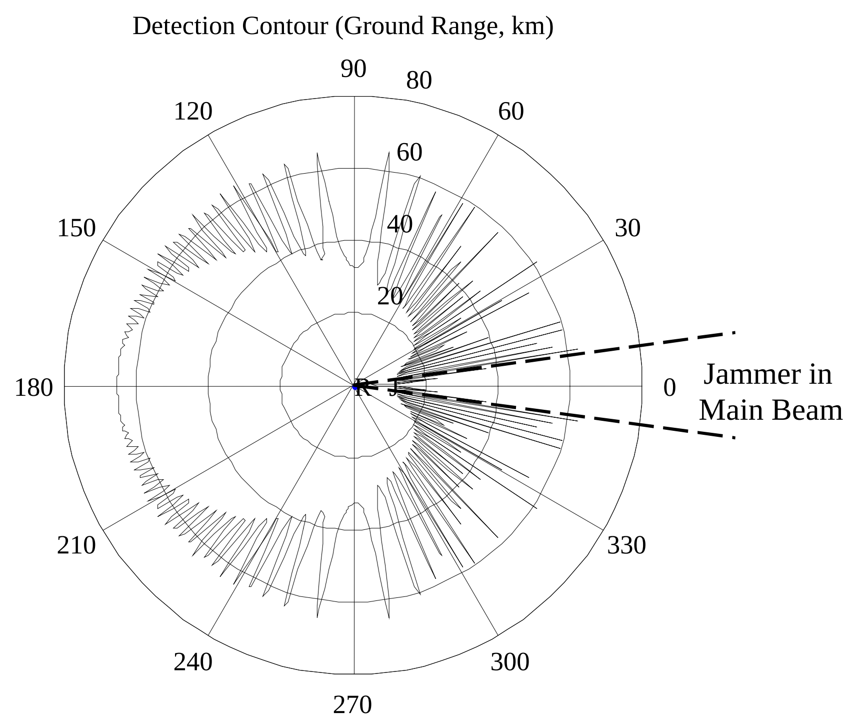
Note: RADJAM plots in Figures 1.19 and 1.20 used e_r = 0.85 not 0.55 as stated in Example 1.3.

Jammer Example

Without jamming



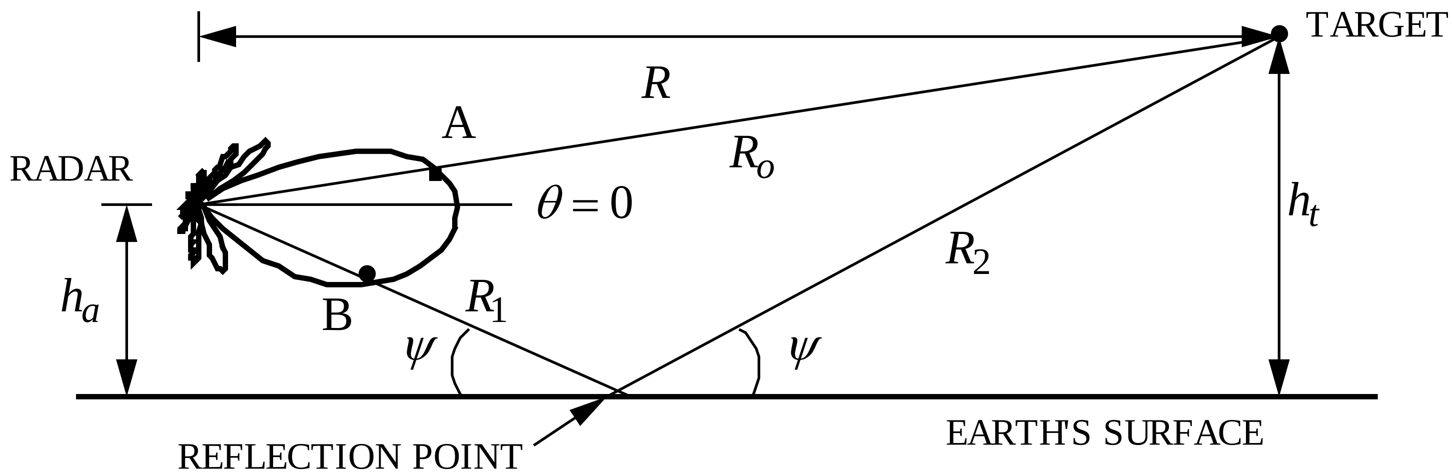
10 W jammer



- The jammer is ineffective when it is in a pattern null.
- The jammer effectiveness is reduced by low sidelobes.
- A jammer in the main beam can incapacitate the radar.

Ground Bounce (1)

When a radar and target are both operating near the surface of the earth, multipath (multiple reflections) can cause extremely large angle errors. Assume a flat earth:



At low altitudes the reflection coefficient is approximately constant ($\Gamma \approx -1$) and $G_D(\theta_A) \approx G_D(\theta_B)$. The difference between the direct and reflected paths is:

$$\Delta R = \underbrace{(R_1 + R_2)}_{\text{REFLECTED}} - \underbrace{R_o}_{\text{DIRECT}}$$

Ground Bounce (2)

The total signal at the target is:

$$E_{\text{tot}} = \underbrace{E_{\text{ref}}}_{\text{REFLECTED}} + \underbrace{E_{\text{dir}}}_{\text{DIRECT}} = E(\theta_A) + \Gamma\, E(\theta_B) e^{-jk\Delta R}$$

From the low altitude approximation, $E_{\text{dir}} = E(\theta_A) \approx E(\theta_B)$ so that

$$E_{\text{tot}} \approx E_{\text{dir}} + \Gamma\, E_{\text{dir}} e^{-jk\Delta R} = E_{\text{dir}} \left| \underbrace{\left[1 + \Gamma\, e^{-jk\Delta R}\right]}_{\equiv F,\ \text{PATH GAIN FACTOR}} \right|$$

The path gain factor takes on the values $0 \le F \le 2$. If $F = 0$ the direct and reflected rays cancel (destructive interference); if $F = 2$ the two waves add (constructive interference).

An approximate expression for the path difference can be obtained as follows:

$$R_o = \sqrt{R^2 + (h_t - h_a)^2} \approx R + \frac{1}{2}\frac{(h_t - h_a)^2}{R}$$

$$R_1 + R_2 = \sqrt{R^2 + (h_t + h_a)^2} \approx R + \frac{1}{2}\frac{(h_t + h_a)^2}{R}$$

Ground Bounce (3)

Therefore,

$$\Delta R \approx \frac{2 h_a h_t}{R}$$

and

$$|F| = \left|1 - e^{-jk2h_a h_t / R}\right| = \left|e^{jkh_a h_t / R}\left(e^{-jkh_a h_t / R} - e^{jkh_a h_t / R}\right)\right| = 2\left|\sin\left(kh_a h_t / R\right)\right|$$

Incorporate the path gain factor into the RRE:

$$P_r \propto |F|^4 = 16 \sin^4\left(\frac{kh_t h_a}{R}\right) \approx 16\left(\frac{kh_t h_a}{R}\right)^4$$

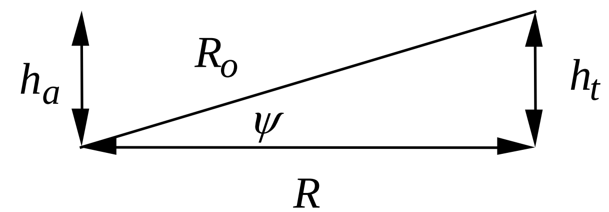
The last form is based on the small angle approximation

$$\frac{kh_t h_a}{R} \to 0.$$

Finally, the RRE can be written as

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4} |F|^4 \approx \frac{4\pi P_t G_t G_r \sigma (h_t h_a)^4}{\lambda^2 R^8}$$

Ground Bounce (4)



Define ψ as the elevation angle from the ground,. Therefore $\tan\psi = h_t / R$ and

$$|F|^4 = 16\sin^4\left(kh_a \tan\psi\right).$$

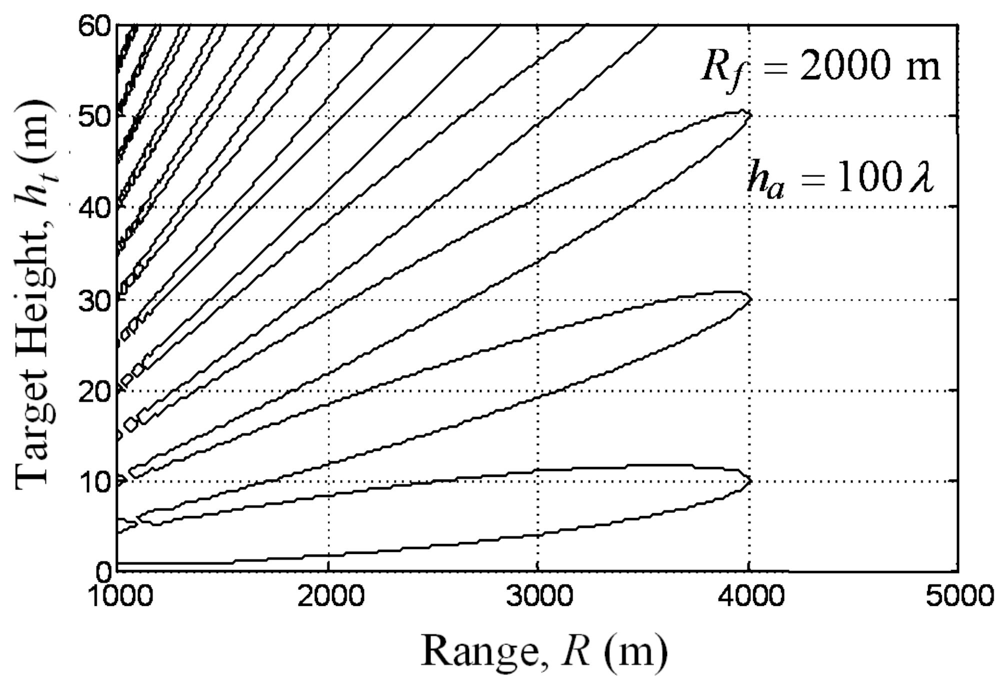
A coverage diagram consists of a contour plot of signal in dB for combinations of h_t and R, where R is normalized to a reference range R_f.

Curves are contours of power equal to that of the free space (direct path) at the reference range. From the RRE:

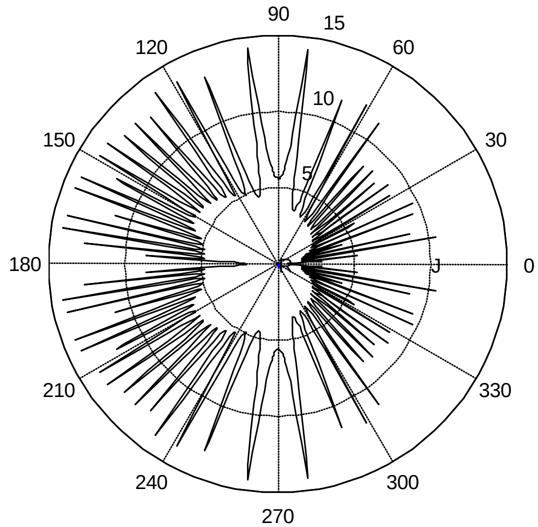
$$SNR = \frac{\text{constant}}{R_f^4} \times \underbrace{\left| 2\left(\frac{R_f}{R}\right)\sin\left(kh_a \tan\psi\right)\right|}_{|F|R_f/R}^4$$

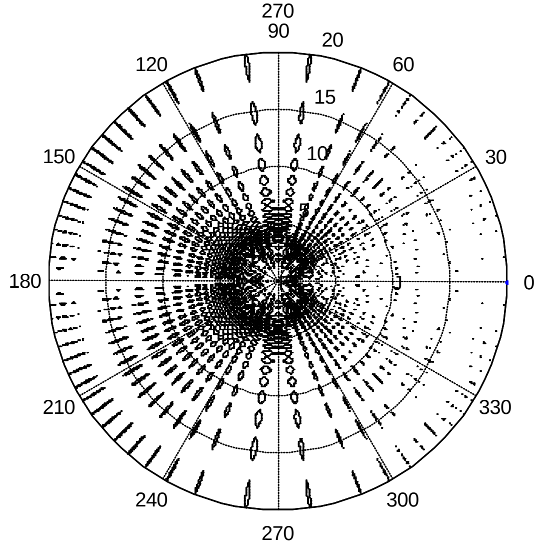
Coverage diagrams plot contours of

$$\left| 2\left(\frac{R_f}{R}\right)\sin\left(kh_a \tan\psi\right)\right|$$

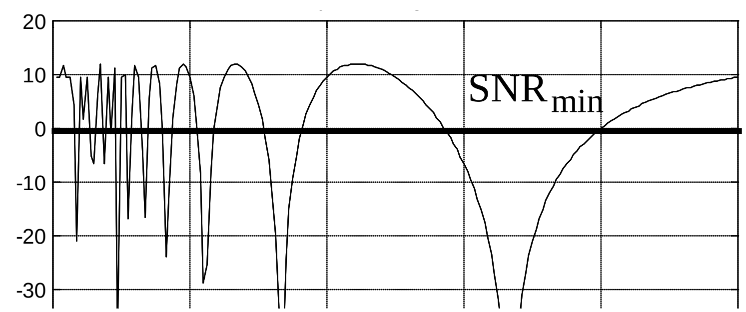


Jamming With Multipath

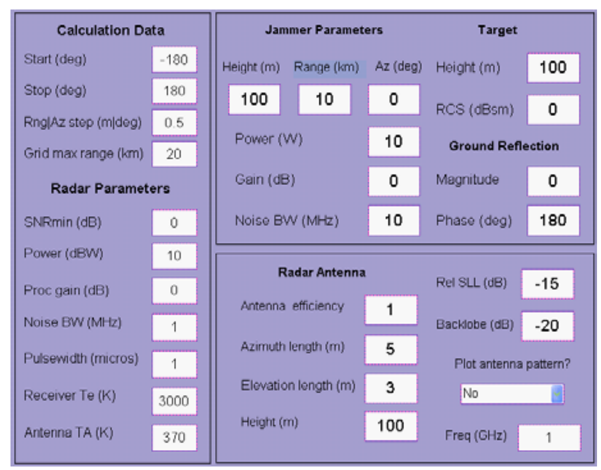




- Sample plot of SNR vs. range with multipath. There are detection gaps between lobes.



- Note that multipath can increase detection range in some cases.



Radjam Data