Microwave Optics
(Chapter 5)

EC4630 Radar and Laser Cross Section

Fall 2011
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Geometrical Optics (1)

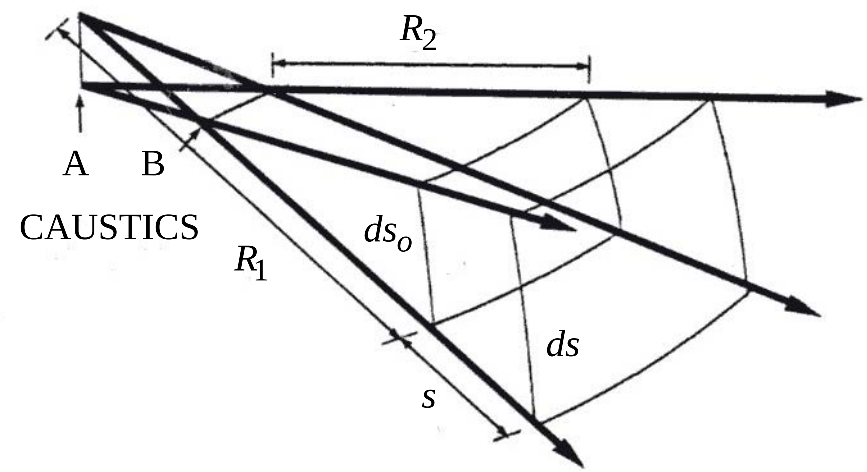
Geometrical optics (GO) refers to the simple ray tracing techniques that have been used for centuries at optical frequencies. The basic postulates of GO are:

1. Wavefronts are locally plane and waves are TEM
2. The wave direction is specified by the normal to the equiphase planes ("rays")
3. Rays travel in straight lines in a homogeneous medium
4. Polarization is constant along a ray in an isotropic medium
5. Power in a flux tube ("bundle of rays") is conserved

$$\iint_{\text{Area 1}} \vec{W} \cdot d\vec{s}_o = \iint_{\text{Area 2}} \vec{W} \cdot d\vec{s}$$

6. Reflection and refraction obey Snell's law

The figure shows an <u>astigmatic</u> flux tube.



7. The reflected field is linearly related to the incident field at the reflection point by a reflection coefficient

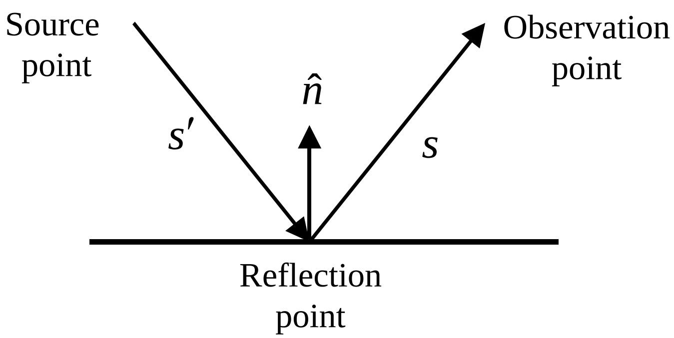
Geometrical Optics (2)

We have already used GO for the simple case of a plane wave reflected from an infinite flat boundary between two dielectrics

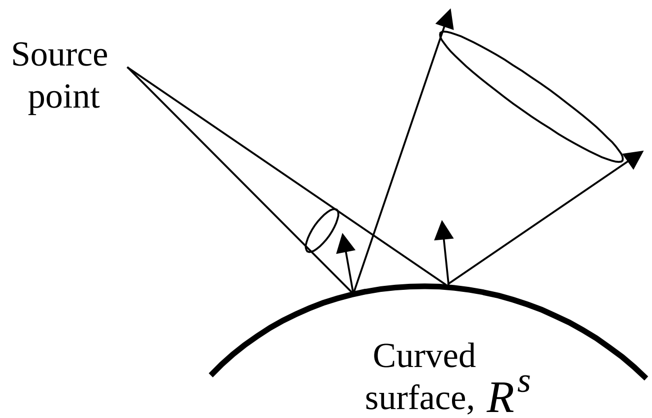
For example, for perpendicular polarization ($\vec{E} \perp$ to the plane defined by s' and $\hat{n}$, which is TE if $\hat{n} = \hat{z}$):

$$E_{r\perp}(s) = \Gamma_{\perp} E_{i\perp}(s') e^{-jks}$$

where s' is the distance from the source to the reflection point and s the distance from the reflection point to the observation point. This has the general form of postulate 7.



The curvature of the reflected wavefront determines how the power spreads as a function of distance and direction. It depends on the curvature of both the incident wavefront, R^i, and reflecting surface, R^s.



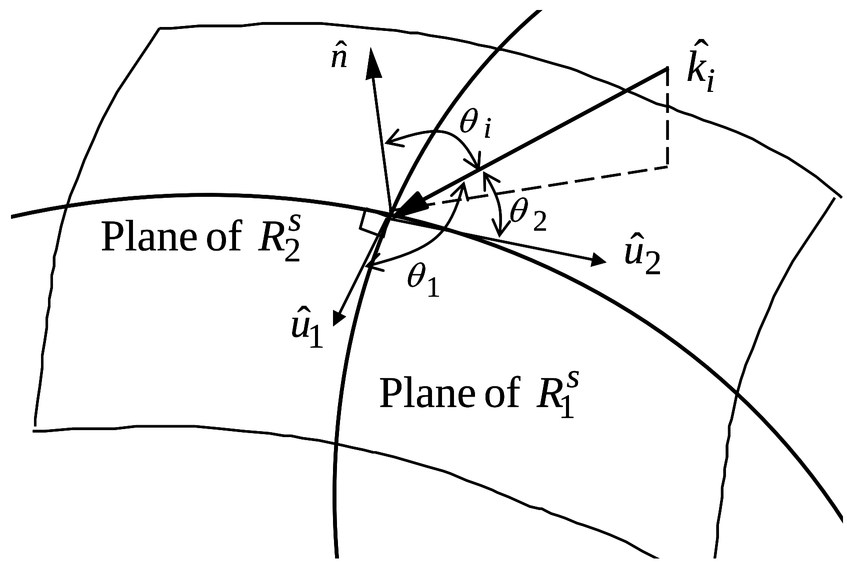
Geometrical Optics (3)

A doubly curved surface (or wavefront) is defined by two principal radii of curvature in two orthogonal planes: R_1^s, R_2^s for a surface or R_1^i, R_2^i for the incident wavefront. The reflected wavefront curvature (R_1^r, R_2^r) can be computed by first finding the focal lengths in the principal planes. When the principal planes of the incident wavefront and surface can be aligned, then

$$\frac{1}{f_{1,2}} = \frac{1}{\cos\theta_i}\left[\frac{\sin^2\theta_2}{R_1^s} + \frac{\sin^2\theta_1}{R_2^s}\right] \pm \left[\frac{1}{\cos^2\theta_i}\left(\frac{\sin^2\theta_2}{R_1^s} + \frac{\sin^2\theta_1}{R_2^s}\right)^2 - \frac{4}{R_1^s R_2^s}\right]^{1/2}$$

where

$$\frac{1}{R_{1,2}^r} = \frac{1}{2}\left[\frac{1}{R_1^i} + \frac{1}{R_2^i}\right] + \frac{1}{f_{1,2}}$$



$\hat{u}_1, \hat{u}_2$ = unit vectors tangent to the surface in the two principal planes

$\hat{n}$ = surface normal at the reflection point

Geometrical Optics (4)

For an arbitrary angle of incidence and polarization, the field is decomposed into parallel and perpendicular components. The reflected field can be cast in matrix form as:

$$\begin{bmatrix} E_{r\perp}(s) \\ E_{r\parallel}(s) \end{bmatrix} = \begin{bmatrix} \Gamma_{\perp\perp} & \Gamma_{\perp\parallel} \\ \Gamma_{\parallel\perp} & \Gamma_{\parallel\parallel} \end{bmatrix} \begin{bmatrix} E_{i\perp}(s') \\ E_{i\parallel}(s') \end{bmatrix} \underbrace{\sqrt{\frac{R_1^r R_2^r}{(R_1^r + s)(R_2^r + s)}}}_{A(s),\ \text{Spreading Factor}} e^{-jks} \, e^{j\phi_c}$$

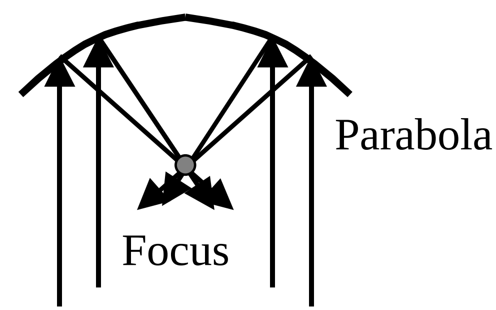
where:

ϕ_c = phase change when the path traverses a caustic (a point at which the cross section of the flux tube is zero)

Γ_{pq} = reflection coefficient for p polarized reflected wave, q polarized incident wave

Disadvantages of GO:

1. Does not predict the field in shadows
2. Cannot handle backscatter from flat or singly curved surfaces (R_1^s or $R_2^s = \infty$)



Example of a caustic: the focus of a parabola. All reflected rays pass through the focus. The cross section of a tube of reflected rays is zero

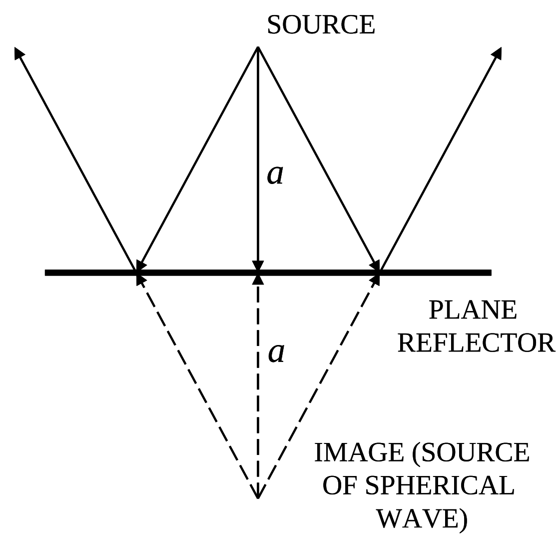
GO Examples

Example: Plane wave $(R_1^i = R_2^i = \infty)$ reflected from a plane surface $(R_1^s = R_2^s = \infty)$. The reflected wave is plane.

$$f_1 = f_2 = \infty \quad \rightarrow \quad R_1^r = R_2^r = 2\left[\frac{1}{R_1^i} + \frac{1}{R_2^i}\right]^{-1} = \infty$$

Example: Spherical wave $(R_1^i = R_2^i = s' = a)$ reflected by a plane surface $(R_1^s = R_2^s = \infty)$.

$$f_1 = f_2 = \infty \quad \rightarrow \quad R_1^r = R_2^r = 2\left[\frac{1}{a} + \frac{1}{a}\right]^{-1} = a$$



The reflected wave is spherical, which is also predicted by using image theory.

RCS of a Sphere

Example: A plane wave incident on a sphere of radius *a*.

At the reflection point, the wave is normally incident $\theta_i = 0^\circ \rightarrow \cos\theta_i = 1$ and $\theta_1 = \theta_2 = 90^\circ \rightarrow \sin\theta_1 = \sin\theta_2 = 1$

$$\frac{1}{f_{1,2}} = \frac{1}{1}\left[\frac{1}{R_1^s} + \frac{1}{R_2^s}\right] \pm \left[\left(\frac{1}{R_1^s} + \frac{1}{R_2^s}\right)^2 - \frac{4}{R_1^s R_2^s}\right]^{1/2}$$

For the sphere $R_1^s = R_2^s = a$ and $f_1 = f_2 = a/2$. For a plane wave $R_1^i = R_2^i = \infty$ so that

$$\frac{1}{R_{1,2}^r} = \frac{1}{2}\left[\frac{1}{R_1^i} + \frac{1}{R_2^i}\right] + \frac{1}{f_{1,2}} = \frac{2}{a} \rightarrow \quad A(s) = \sqrt{\frac{(a/2)^2}{s^2}} = \frac{a}{2s}$$

where in the denominator it was assumed that $s >> a$. For a PEC, $\Gamma = -1$, and the reflected field is

$$E_r = -\frac{E_i a}{2}\left(\frac{e^{-jks}}{s}\right)$$

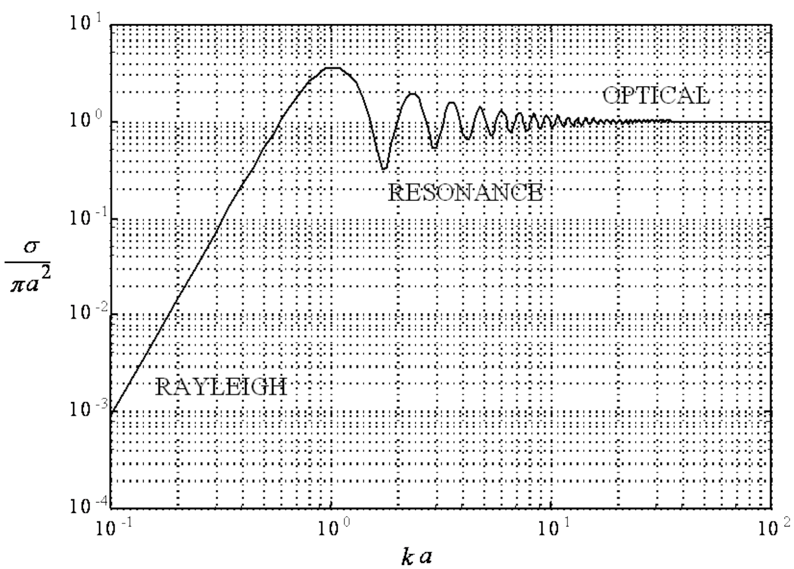
which is, a spherical wave.

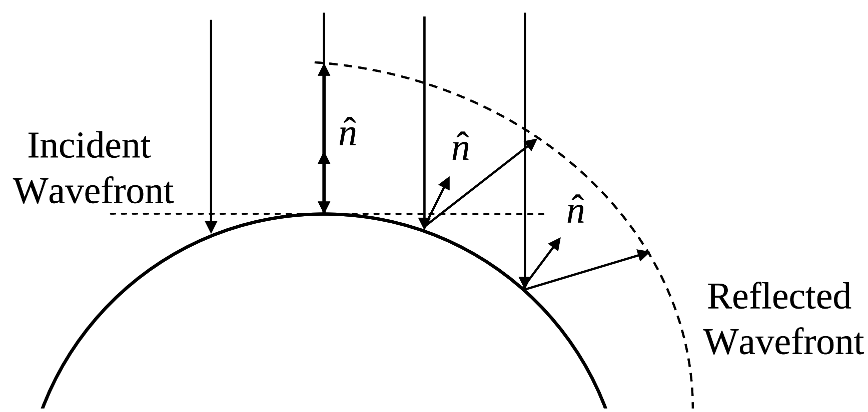
RCS of a Sphere

From the definition of RCS, when the observer is at a distance *s* from the sphere

$$\sigma = 4\pi s^2 \frac{\left| -\frac{E_i a}{2} \frac{e^{-jks}}{s} \right|^2}{|E_i|^2} = \pi a^2$$

The is the asymptotic value obtained with the Mie series.

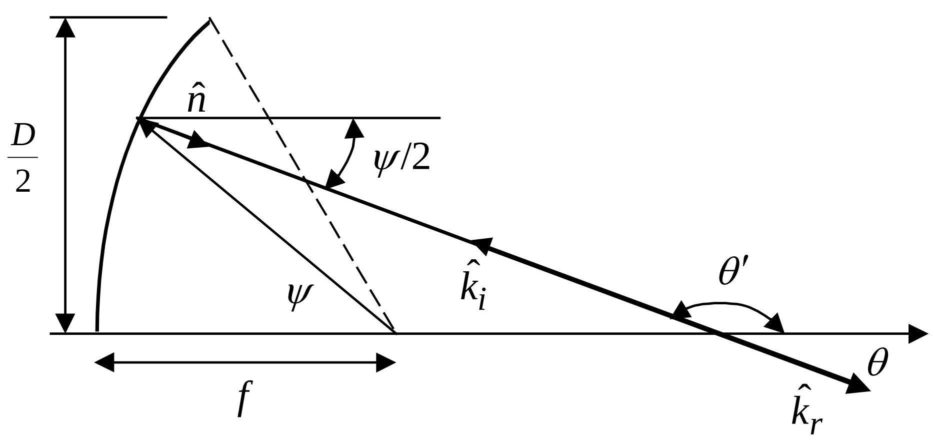


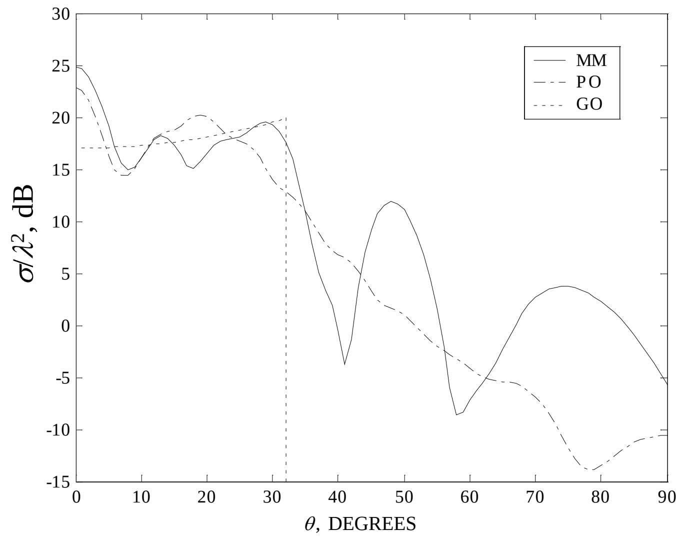


RCS of a Parabolic Reflector

A plane wave is incident on a parabolic reflector of diameter D and focal length f. The reflector edge angle is $\psi_e = 2\tan^{-1}\left(\frac{f}{4D}\right)$ and the normal vector (for $\theta < \psi_e/2$) is $\hat{n} = -\hat{r}\cos(\psi/2) - \hat{\theta}\sin(\psi/2)$. For plane wave incidence $R_1^r R_2^r = f_1 f_2 = \frac{R_1^s R_2^s}{4}$ and for a paraboloid $R_1^s = R_2^s = \frac{2f}{\cos^2(\psi/2)}$. The RCS is $\sigma = 4\pi f^2\left[\frac{2}{1+\cos(2\theta)}\right]^2 \quad \theta < \psi_e/2$.

The plot is for D=5λ, f/D=0.4.





Geometrical Theory of Diffraction (1)

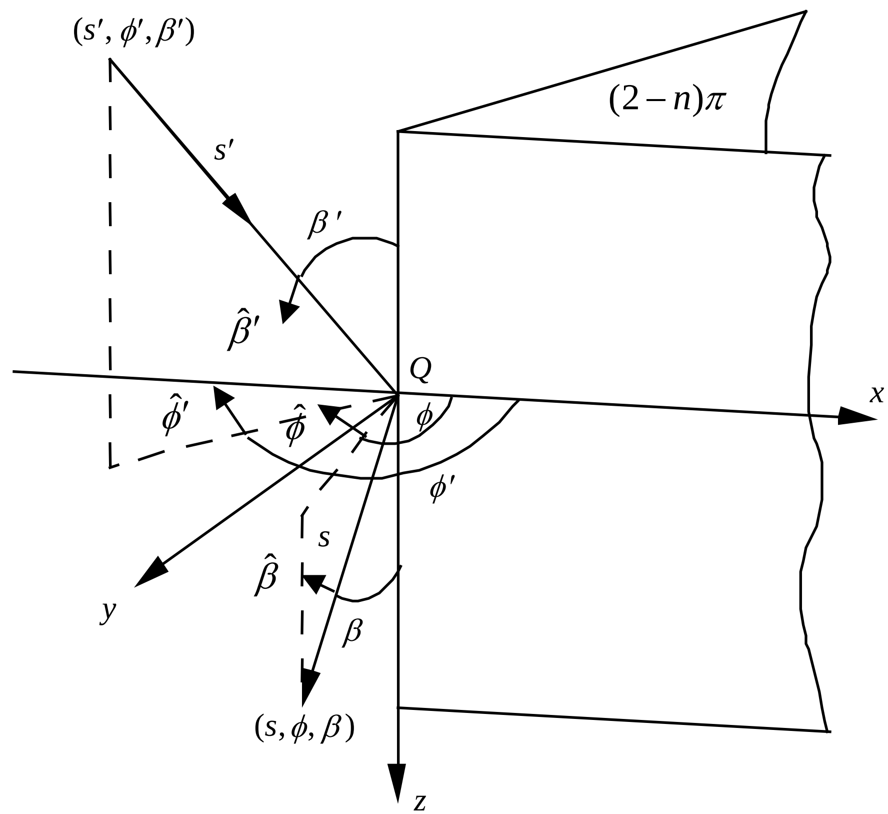
The geometrical theory of diffraction (GTD) was devised to eliminate many of the problems associated with GO. The strongest diffracted fields arise from edges, but ones of lesser strength originate from point discontinuities (tips and corners). The total field at an observation point *P* is decomposed into GO and diffracted components

$$\vec{E}_r(P) = \vec{E}_{GO}(P) + \vec{E}_{GTD}(P)$$

The behavior of the diffracted field is based on the following postulates of GTD:

1. Wavefronts are locally plane and waves are TEM.
2. Diffracted rays emerge radially from an edge.
3. Rays travel in straight lines in a homogeneous medium
4. Polarization is constant along a ray in an isotropic medium
5. The diffracted field strength is inversely proportional to the cross sectional area of the flux tube
6. The diffracted field is linearly related to the incident field at the diffraction point by a diffraction coefficient

Geometrical Theory of Diffraction (2)



Define a local ray fixed coordinate system: the *z* axis is along the edge; the *x* axis lies on the face and points inward.

The internal wedge angle is $(2-n)\pi$, where *n* is not necessarily an integer. A knife edge is the case of $n = 2$.

Primed quantities are associated with the source point; unprimed quantities with the observation point. Variable unit vectors are tangent in the direction of increase (like spherical unit vectors)

Diffracted rays lie on a cone of half angle $\beta = \beta'$ (the *Keller cone*)

The matrix form of the diffracted field is

$$\begin{bmatrix} E_{d\beta}(s) \\ E_{d\phi}(s) \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & -D_h \end{bmatrix} \begin{bmatrix} E_{i\beta'}(s') \\ E_{i\phi'}(s') \end{bmatrix} A(s, s')\, e^{-jks}$$

Geometrical Theory of Diffraction (3)

The <u>diffraction coefficients</u>, D_s and D_h[1], are determined by an appropriate "canonical" problem (i.e., a fundamental related problem whose solution is known).

The scattered field from an infinite wedge was solved by Sommerfeld. The infinitely long edge can represent a finite length edge if the diffraction point is not near the end.

The diffraction coefficient can be "backed out" of Sommerfeld's exact solution because we know the GO field and the form of the GTD field from the postulates

$$\underbrace{\vec{E}_r(P)}_{\substack{\text{known}\\ \text{from}\\ \text{Sommerfeld}}} = \underbrace{\vec{E}_{GO}(P)}_{\substack{\text{can be}\\ \text{found}}} + D\, \underbrace{\vec{E}_i(Q)A(s,s')e^{-jks}}_{\text{known}}$$

The basic expressions for the diffraction coefficients of a knife edge are simple (they contain only trig functions), however they have singularities at the shadow and reflection boundaries.

More complicated expressions have been derived that are well behaved everywhere. The most common is the uniform theory of diffraction (UTD). (See Stutzman and Thiele.)

[1] This notation is borrowed from optics: s = soft or parallel polarization; h = hard or perpendicular polarization. The reference plane is the plane defined by the edge and the incident ray (for incident polarization) or diffracted ray (for diffracted polarization).

Diffraction Coefficients

These are the uniform theory of diffraction (UTD) coefficients which have be modified by the transition function *F* to cancel singularities that occur in the original Keller formulas at the shadow and reflection boundaries:

$$D_{s,h}(\ell,\phi,\phi',\beta,\beta') = \frac{\mathrm{e}^{-j\pi/4}}{2\sqrt{2\pi k}\sin\beta}\left\{\frac{F[k\ell a(\phi^-)]}{\cos(\phi^-/2)} \pm \frac{F[k\ell a(\phi^+)]}{\cos(\phi^+/2)}\right\}$$

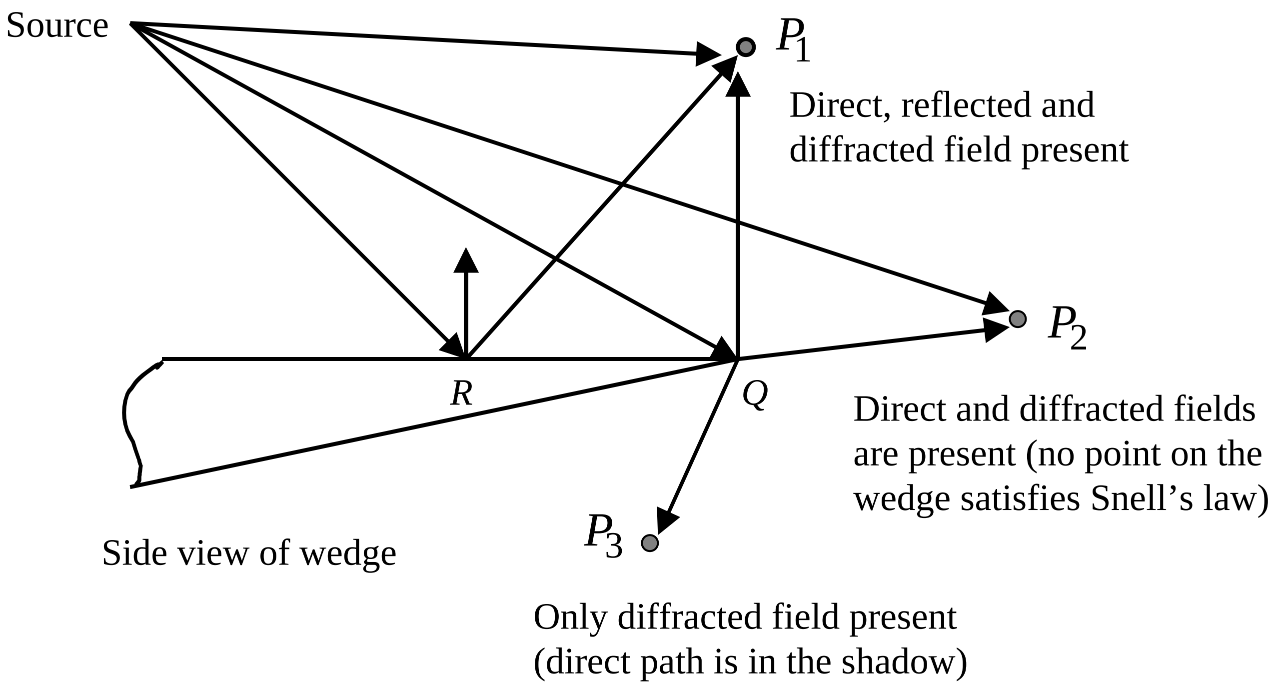
where $\phi^{\pm} = \phi \pm \phi'$, $a(\phi^{\pm}) = 2\cos^2(\phi^{\pm}/2)$, and $F[x] = 2j\,|\sqrt{x}|\,e^{jx}\int_{|\sqrt{x}|}^{\infty} e^{-jt^2}\,dt$

$$A(s,s') = \begin{cases} \sqrt{\dfrac{s'}{s(s'+s)}}, & \text{spherical wave incident} \\ \sqrt{\dfrac{1}{s}}\,, & \text{cylindrical and plane waves} \end{cases}$$

$$\ell = \begin{cases} \dfrac{s's}{s'+s}\sin^2\beta, & \text{spherical} \\ \dfrac{\rho\rho'}{\rho+\rho'}, & \text{cylindrical} \\ s\sin^2\beta', & \text{plane} \end{cases}$$

GO and GTD for a Wedge

Example: scattering from a wedge for three observation points



GO and GTD rays can "mix" (reflected rays can subsequently be diffracted, etc.) to obtain:

- reflected – reflected (multiple reflection)
- reflected – diffracted and diffracted – reflected
- diffracted – diffracted (multiple diffraction)

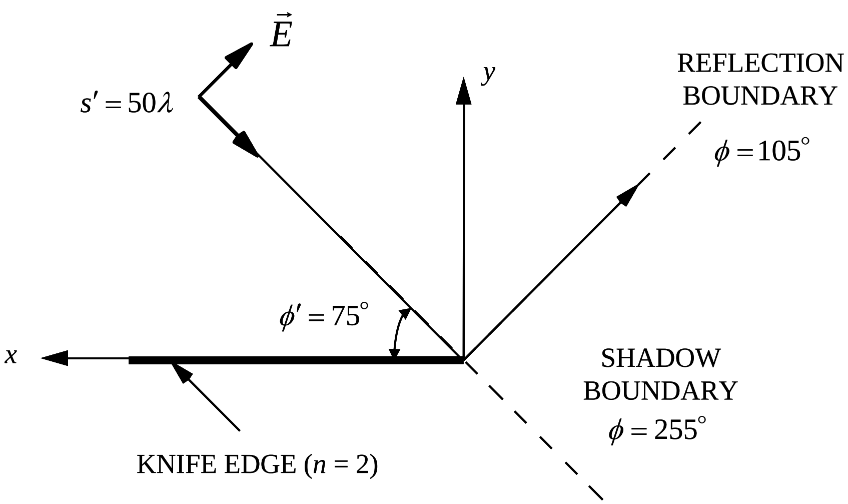
An accurate (converged) solution must include all significant contributions.

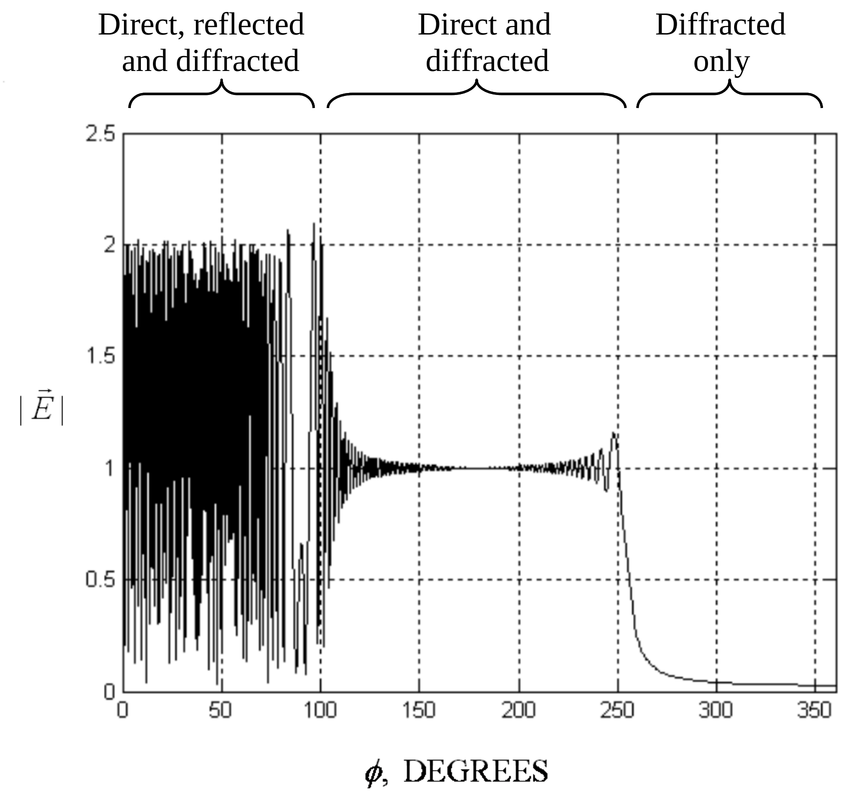
Application of GO and GTD

Example 5.4: Scattering from a knife edge (n=2). The field components are:

$$\vec{E}_{i_\perp} = e^{-jk\rho\cos(\phi-\phi')}$$
$$\vec{E}_{r_\perp} = e^{-jk\rho\cos(\phi+\phi')}$$
$$\vec{E}_{d_\perp} = \vec{E}_i(Q)D_\perp e^{-jk\rho} / \sqrt{\rho}$$

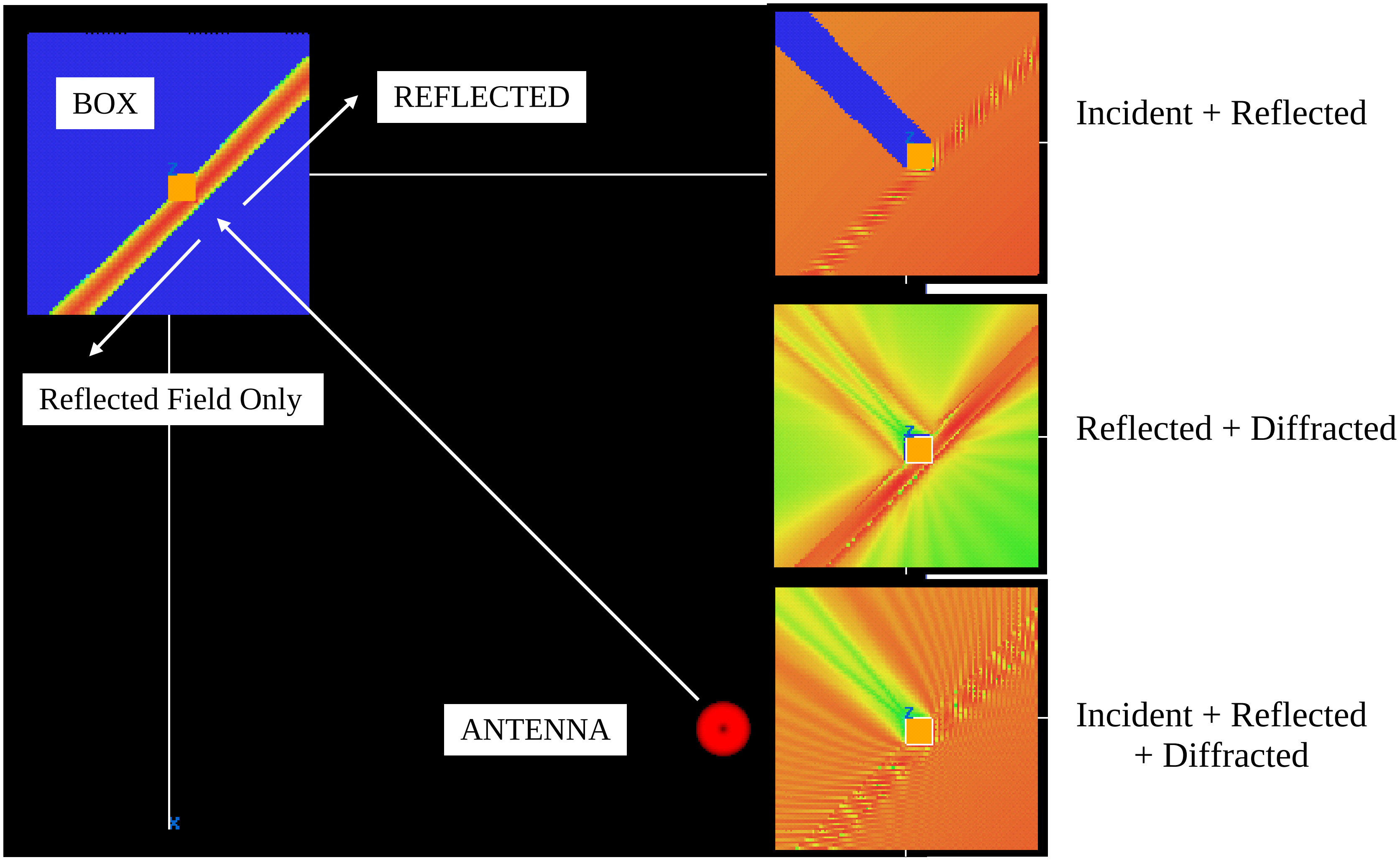
where ρ is the distance from the edge to the observation point.





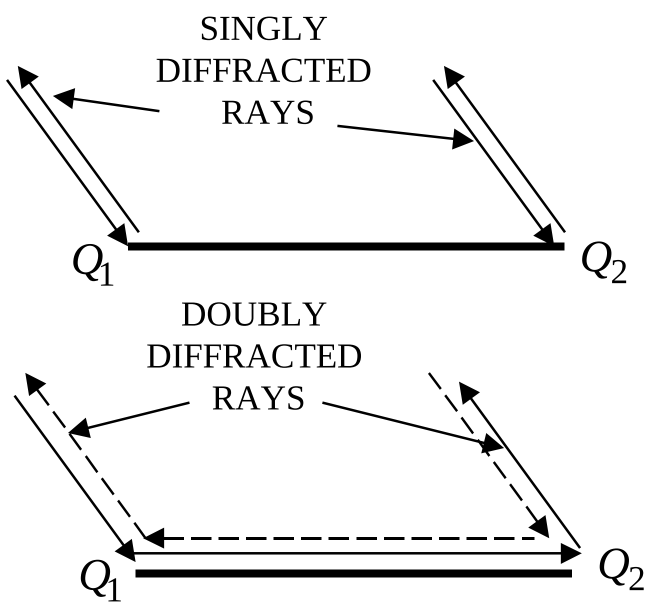
Example: Fields Scattered From a Box

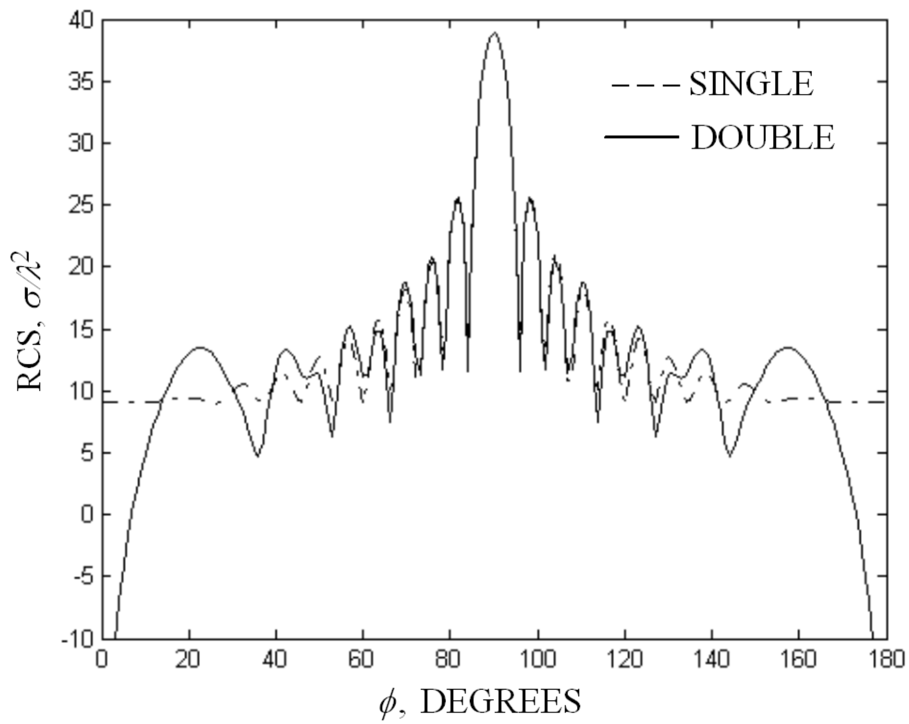
f =1 GHz, –100 dBm (blue) to –35 dBm (red), 0 dBm Tx power , 1 m PEC box computed using *Urbana*



Surface Waves

GTD accounts for surface waves as combinations of rays that propagate along the surface. See example 5.5 for a strip of width $w=5\lambda$. When double diffractions are included, the traveling wave is present.





Physical Theory of Diffraction (PTD)

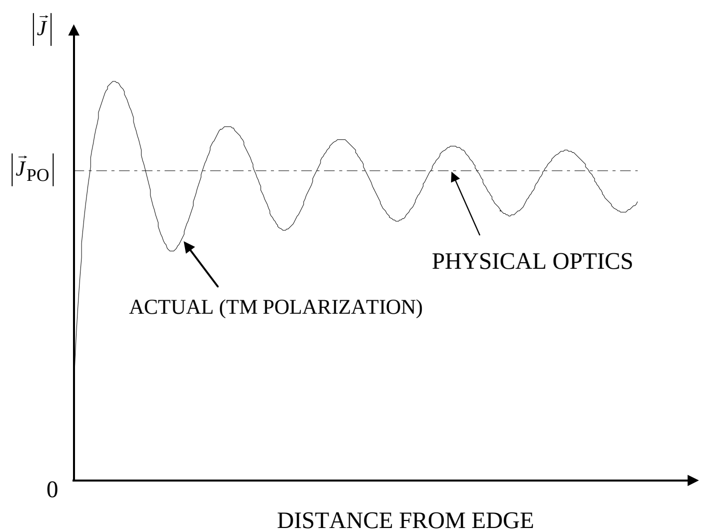
The PO current approximation is not accurate near edges. The ripples in the current decay with distance from the edge, as shown below. They are significant out to about 1λ from the edge. The edge (fringe) current can be found from the exact knife edge currents:

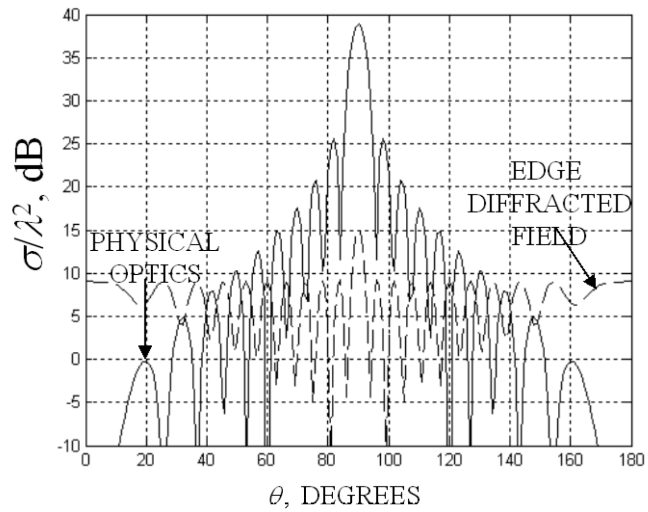
$$\vec{J}_{total} = \vec{J}_{PO} + \vec{J}_{edge} \quad \rightarrow \quad \vec{J}_{edge} = \vec{J}_{total} - \vec{J}_{PO}$$

$\vec{J}_{edge}$ is used to obtain an edge scattered field that can be added to PO:

$$\vec{E}_s(P) = \vec{E}_{PO}(P) - \frac{jk\eta_o}{4\pi r} e^{-jkr} \iint_{edge} \vec{J}(\vec{r}')_{edge} e^{jkg} ds'$$

Adding the edge scattered fields give the same results as singly diffracted GTD rays.



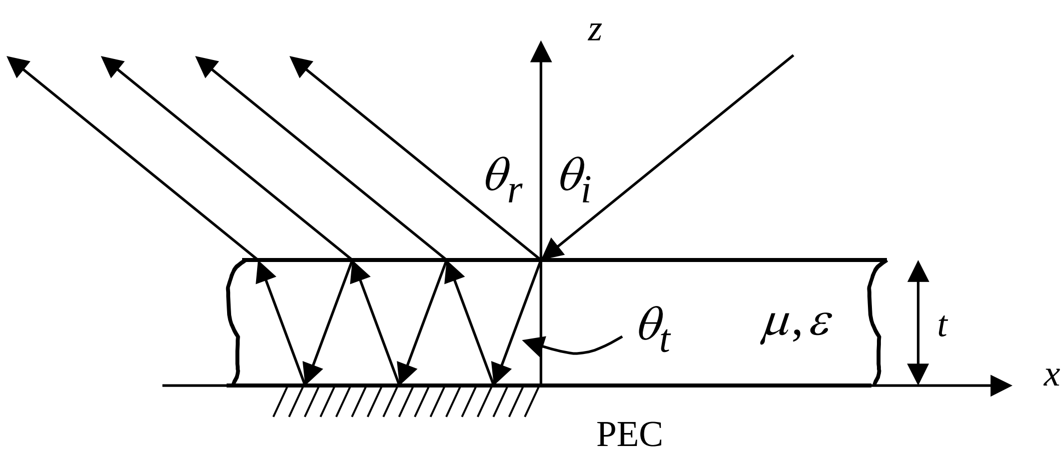


Multiple Reflections in Layers

The figure shows a layer of material (μ, ε) on a PEC conductor. This structure is generally referred to as a Dallenbach layer. If the material is lossless there is an infinite number of reflections that converge to give an input reflection of (see Problem 5.8):

$$\Gamma_{in} = \frac{\Gamma_d - P_d P_o}{1 - P_d^2 \Gamma_d P_o}, \quad \Gamma_d = \frac{Z_L - Z_o}{Z_L + Z_o}, \quad Z_L = \begin{cases} \eta \cos\theta_t, & \text{for TM} \\ \eta / \cos\theta_t, & \text{for TE} \end{cases}$$

where $P_d = e^{-\gamma\ell}$, $\ell = t/\cos\theta_t$, $P_o = e^{-jk\Delta}$, $\Delta = 2kt\tan\theta_t \sin\theta$, $Z_o = \eta_o \cos\theta$. A thickness of $\lambda/4$ gives cancellation of the reflections from the two interfaces (similar to the quarter wave transformer effect in transmission lines)



Reflections in layers are conveniently handled using wave matrices (see p. 354 in the book.)

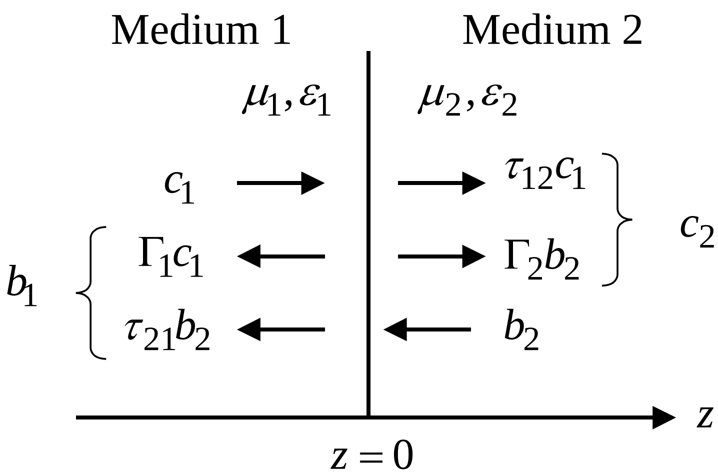
Wave Matrices For Layered Media (1)

For multilayered (stratified) media, a matrix formulation can be used to determine the net transmitted and reflected fields. The figure below shows incident and reflected waves at the boundary between two media. The positive *z* traveling waves are denoted *c* and the negative *z* traveling waves *b*. We allow for waves incident from both sides simultaneously.

Thus,

$$b_1 = \Gamma_1 c_1 + \tau_{21} b_2$$
$$c_2 = \Gamma_2 b_2 + \tau_{12} c_1$$

where Γ and τ are the appropriate Fresnel reflection and transmission coefficients.



Rearranging the two equations:

$$b_1 = \left(\tau_{21} - \frac{\Gamma_1 \Gamma_2}{\tau_{12}} \right) b_2 + \frac{\Gamma_1}{\tau_{12}} c_2$$

$$c_1 = \frac{c_2}{\tau_{12}} - \frac{\Gamma_2 b_2}{\tau_{12}}$$

which can be written in matrix form:

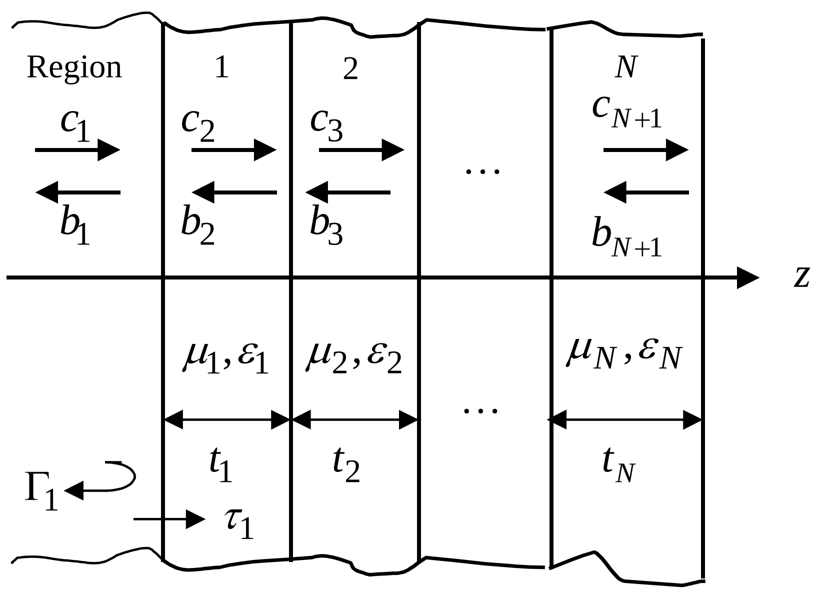
$$\begin{bmatrix} c_1 \\ b_1 \end{bmatrix} = \frac{1}{\tau_{12}} \begin{bmatrix} 1 & -\Gamma_2 \\ \Gamma_1 & \tau_{12}\tau_{21} - \Gamma_1 \Gamma_2 \end{bmatrix} \begin{bmatrix} c_2 \\ b_2 \end{bmatrix}$$

This is called the <u>wave transmission matrix</u>. It relates the forward and backward propagating waves on the two sides of the boundary.

Wave Matrices For Layered Media (2)

As defined, c and b are the waves incident on the boundary, $z = 0$. At some other location, $z = z_1$ the forward traveling wave becomes $c_1 e^{-j\beta z_1}$ and the backward wave becomes $b_1 e^{j\beta z_1}$. For a plane wave incident from free space onto N layers of different material (μ_n, ε_n) and thickness (t_n), the wave matrices can be cascaded

$$\begin{bmatrix} c_1 \\ b_1 \end{bmatrix} = \prod_{n=1}^{N} \frac{1}{\tau_n} \begin{bmatrix} e^{j\Phi_n} & \Gamma_n e^{-j\Phi_n} \\ \Gamma_n e^{j\Phi_n} & e^{-j\Phi_n} \end{bmatrix} \begin{bmatrix} c_{N+1} \\ b_{N+1} \end{bmatrix} \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} c_{N+1} \\ b_{N+1} \end{bmatrix}$$



where, for normal incidence $\Phi_n = \beta_n t_n$ is the electrical length of layer n. If the last layer extends to $z \to \infty$ then $b_{N+1} = 0$. We can arbitrarily set $\Phi_N = 0$ if the transmission phase is not of interest.

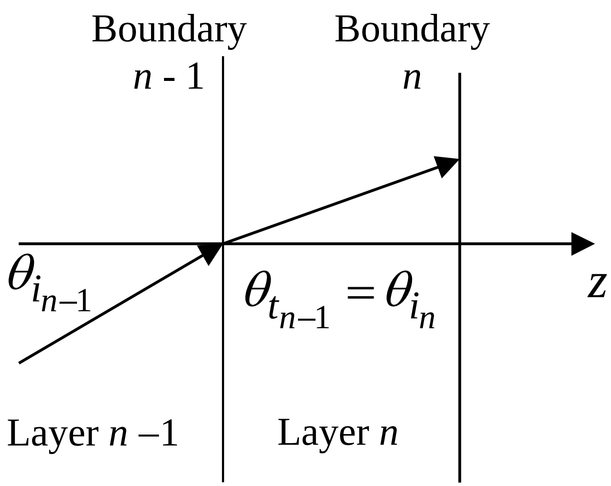
The overall transmission coefficient of the layers (i.e., the transmission into layer N when $b_{N+1} = 0$) is $c_{N+1} / c_1 = 1/A_{11}$. The overall reflection coefficient is $b_1 / c_1 = A_{21} / A_{11}$.

Wave Matrices For Layered Media (3)

If the incidence angle in region 1 is not normal, then Φ_n must be determined by taking into account the refraction in all of the previous $n-1$ layers. The transmission angle for layer n becomes the incidence angle for layer $n+1$ and they are related by Snell's law:

$$\beta_o \sin\theta_i = \beta_1 \sin\theta_{i_1} = \beta_2 \sin\theta_{i_2} = \cdots = \beta_{N-1} \sin\theta_{i_{N-1}}.$$

Oblique incidence and loss can be handled by modifying the transmission and reflection formulas as listed below, where θ_i is the incidence angle at the first boundary:



$$\Phi_n = \beta_o t_n \left(\varepsilon_{r_n}\mu_{r_n} - \sin^2\theta_i\right)^{1/2}$$

$$\Gamma_n = \frac{Z_n - Z_{n-1}}{Z_n + Z_{n-1}}, \quad \tau_n = 1 + \Gamma_n$$

$$\frac{Z_n}{Z_o} = \frac{\sqrt{\varepsilon_{r_n}\mu_{r_n} - \sin^2\theta_i}}{\varepsilon_{r_n}\cos\theta_i}, \text{ (parallel polarization)}$$

$$\frac{Z_n}{Z_o} = \frac{\mu_{r_n}\cos\theta_i}{\sqrt{\varepsilon_{r_n}\mu_{r_n} - \sin^2\theta_i}}, \text{ (perpendicular polarization)}$$

For lossy materials: $\varepsilon_{r_n} \rightarrow \varepsilon_{r_n} - j\dfrac{\sigma_n}{\omega\varepsilon_o}$