IONOSPHERIC WAVE PROPAGATION

by Professor David Jenn

(version 1.6.5)
Ionospheric Radiowave Propagation (1)

The ionosphere refers to the upper regions of the atmosphere (90 to 1000 km). This region is highly ionized, that is, it has a high density of free electrons (negative charges) and positively charged ions. The charges have several important effects on EM propagation:

1. Variations in the electron density \( N_e \) cause waves to bend back towards Earth, but only if specific frequency and angle criteria are satisfied. Some examples are shown below. Multiple skips are common thereby making global communication possible.
2. The Earth’s magnetic field causes the ionosphere to behave like an anisotropic medium. Wave propagation is characterized by two polarizations (denoted as “ordinary” and “extraordinary” waves). The propagation constants of the two waves are different. An arbitrarily polarized wave can be decomposed into these two polarizations upon entering the ionosphere and recombined on exiting. The recombined wave polarization angle will be different that the incident wave polarization angle. This effect is called Faraday rotation.

The electron density distribution has the general characteristics shown on the next page. The detailed features vary with

- location on Earth,
- time of day,
- time of year, and
- sunspot activity.

The regions around peaks in the density are referred to as layers. The F layer often splits into the $F_1$ and $F_2$ layers.
Electron Density of the Ionosphere

(From Rohan. Note unit is per cubic centimeter.)
The Earth’s Magnetosphere

Interaction of the sun and Earth magnetic fields

(From George Parks, *Physics of Space Plasmas*, Addison-Wesley)
Barometric Law (1)

Ionized layers are generated by solar radiation, mainly ultraviolet, which interacts with neutral molecules to produce electron-ion pairs. As a first step in predicting the ionized layers it is necessary to know the distribution of molecules with height.

Consider a differential volume of air, as shown in the figure. The pressure $p$ decreases with altitude, so $dp$ is negative. There is a net buoyant force on the volume of cross sectional area $A$

$$F_b = Ap - A(p + dp) = -Adp.$$ 

This force must be balanced by the gravitational force of the weight of the gas in the volume

$$F_g = \rho gdV = \rho gAdh$$

where $\rho$ is the mass density of the gas (mass per unit volume) and $g$ is the gravitational constant. If $m$ is the mean molecular mass

$$F_g = \rho gdV = N\bar{m}gAdh$$

Equating the last two equations gives $-dp = N\bar{m}gdh$. $N$ is the number of molecules per unit volume.
Barometric Law (2)

Pressure and temperature are related by the ideal gas law: \( p = Nk_B T \) (\( k_B \) is Boltzman’s constant and \( T \) is temperature). Use the ideal gas law to substitute for \( N \) in \( dp \):

\[
-\frac{dp}{p} = -\frac{\bar{m}g}{k_B T} dh
\]

then integrate from starting height \( h_o \) to a final height \( h \)

\[
\ln \frac{p(h)}{p(h_o)} = \int_{h_o}^{h} \frac{\bar{m}g}{k_B T} dh'
\]

Finally, assuming that all of the quantities are approximately constant over the heights of concern, and defining a new constant \( \frac{1}{H} = \frac{\bar{m}g}{k_B T} \), gives the barometric law for pressure vs. height:

\[
p(h) = p(h_o) \exp \left( h - h_o \right) = p(h_o) \exp \left( \int_{h_o}^{h} \frac{dh'}{H} \right) = p(h_o) \exp \left( -\frac{h - h_o}{H} \right)
\]

\( H \) is called the local scale height and thus the exponent, when normalized by \( H \), is in “\( H \) units.” The reference height is arbitrary. The pressure, mass density, and number density are seen to vary exponentially with height difference measured in \( H \) units.
Chapman’s Theory (1)

Chapman was the first to quantify the formation of ionized regions due to flux from the sun. In general we expect ion production to vary as follows:

1. Small at high altitudes – solar flux is high, but the number of molecules available for ionization is small
2. Small at low altitudes – the number of molecules available is large, but solar flux is low due to attenuation at higher altitudes
3. High at intermediate altitudes – sufficient number of molecules and flux

Chapman’s derivation proceeds along the following steps:

1. Find the amount of flux penetrating to an altitude \( h \).
2. By differentiation, find the decrease in flux with incremental height \( dh \).
3. The decrease in flux represents absorbed energy. From the result of step 2 an expression for ion production per unit volume at height \( h \) is derived.
4. The barometric law is used to find the optical depth at height \( h \) (optical depth is a measure of the opacity of the atmosphere above \( h \)).
5. The maximum height of electron-ion production is set as the reference height.
6. Assuming equilibrium (no vertical wind) laws of the variation of the electron density are derived based on the rates of electron production and loss.
7. A scaling law is developed so that ionization can be represented by a single variable rather than two (altitude and solar zenith angle).
Chapman’s Theory (2)

Only the final results are presented here. $S$ denotes the solar flux and $\chi$ is the zenith angle. The electron density at height $h$ and zenith angle $\chi$ is given by

$$N_e(\chi, h) = N_o \exp\left[\frac{1}{2} \left(1 - z - \sec \chi \ e^{-z}\right)\right]$$

where $z = (h - h_o)/H$ and $N_o$ is the electron density at the reference height $h_o$. This equation specifies a recombination type of layer (as opposed to an attachment type layer\(^1\)). It can be rewritten (by a substitution of variables and rescaling) as

$$N_e(\chi, z) = \sqrt{\cos \chi} \ N_e(0, z - \ln \sec \chi)$$

The value for any $\chi$ can be obtained from the $\chi = 0^\circ$ curve.

\(^1\)There are several reactions which can remove electrons from the ionosphere. The two most important classes of recombination are (1) the electron attaches itself to a positive ion to form a neutral molecule, and (2) it attaches itself to a neutral molecule to form a negative ion.
Chapman’s Theory (3)

It is found that the E and F1 regions behave closely to what is predicted by Chapman’s equation.
Dielectric Constant of a Plasma (1)

When an electric field is applied, charges in the ionosphere are accelerated. The mass of the ions (positive charges) is much greater than the electrons, and therefore the motion of the ions can be neglected in comparison with that of the electrons. The polarization vector is

\[ \vec{P} = -N_e \vec{p} = -N_e e \vec{d} \]

where \( N_e = \) electron density \( / \ m^3 \), \( e = 1.59 \times 10^{-19} \ C \), electron charge, and \( \vec{d} \) is the average displacement vector between the positive and negative charges. The relative dielectric constant is

\[
\varepsilon_r = \frac{\vec{D}}{\varepsilon_0 \vec{E}} = \varepsilon_0 \vec{E} + \frac{\vec{P}}{\varepsilon_0 \vec{E}} = 1 + \frac{\vec{P}}{\varepsilon_0 \vec{E}}
\]

The propagation constant is

\[
\gamma = jk_c = j \frac{\omega}{c} \sqrt{\varepsilon_r} = jk_o n
\]

where \( n = \sqrt{\varepsilon_r} \) is the index of refraction. Both the dielectric constant and index of refraction can be complex (\( \varepsilon_r = \varepsilon'_r - j\varepsilon''_r \) and \( n = n' - jn'' \)).
Dielectric Constant of a Plasma (2)

Assume a $x$ polarized electric field. The electrons must follow the electric field, and therefore, $\ddot{d} = \dot{x}$. The equation of motion of an electron is

$$m \ddot{x} + v_m \dot{x} = -eE_x e^{j\omega t}$$

where $m = 9.0 \times 10^{-31}$ kg, electron mass, and $\nu = \text{collision frequency}$. The solution of the equation of motion is of the form $x = ae^{j\omega t}$. Substituting this back into the equation gives

$$m(-a \omega^2 e^{j\omega t}) + v_m(j \omega a e^{j\omega t}) = -eE_x e^{j\omega t}$$

with $a = \frac{eE_x}{m\omega^2(1 - j \nu / \omega)}$. The polarization vector is

$$\vec{P} = -\frac{N_e e^2 E_x e^{j\omega t} \dot{x}}{m\omega^2(1 - j \nu / \omega)}$$

Now define an angular plasma frequency, $\omega_p = \sqrt{\frac{N_e e^2}{m\varepsilon_0}}$ and constants $X = \left(\frac{\omega_p}{\omega}\right)^2$ and $Z = \frac{\nu}{\omega}$ so that

$$\vec{P} = -\varepsilon_0 E_x e^{j\omega t} \frac{X}{(1 - jZ)}.$$  

(Note that in the book $\omega_N$ is used.)
Dielectric Constant of a Plasma (3)

The ratio of $P$ to $E$ gives the complex dielectric constant, which is equal to the square of the index of refraction

$$\varepsilon_r = \varepsilon'_r - j\varepsilon''_r = n^2 = 1 - \frac{X}{(1 - jZ)} = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu)}$$

Separating into real and imaginary terms gives an equivalent conductivity

$$\varepsilon_r = \varepsilon'_r - j\frac{\sigma}{\omega\varepsilon_o}$$

where

$$\varepsilon'_r = 1 - \frac{N_e e^2}{\varepsilon_o m(\nu^2 + \omega^2)}$$

and

$$\sigma = \frac{N_e e^2 \nu}{m(\nu^2 + \omega^2)}$$

For the special case of no collisions, $\nu = 0$, and the corresponding propagation constant is real

$$k_c = \omega\sqrt{\mu_o \varepsilon_r \varepsilon_o} = k_o \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

with $k_o = \omega\sqrt{\mu_o \varepsilon_o}$. 
Dielectric Constant of a Plasma (4)

Consider three cases:

1. $\omega > \omega_p$: $k_c$ is real and $e^{-jk_cz} = e^{-j|k_c|z}$ is a propagating wave

2. $\omega < \omega_p$: $k_c$ is imaginary and $e^{-jk_cz} = e^{-|k_c|z}$ is an evanescent wave

3. $\omega = \omega_p$: $k_c = 0$ and this value of $\omega$ is called the critical frequency\(^1\), $\omega_c$

At the critical frequency the wave is reflected. Note that $\omega_c$ depends on altitude because the electron density is a function of altitude. For electrons, the highest frequency at which a reflection occurs is

$$f_c = \frac{\omega_c}{2\pi} \approx 9\sqrt{N_{e\text{max}}}$$

Reflection at normal incidence requires the greatest $N_e$.

---

\(^1\) The critical frequency is where the propagation constant is zero. Neglecting the Earth’s magnetic field, this occurs at the plasma frequency, and hence the two terms are often used interchangeably.
Ionospheric Radiowave Propagation (1)

If the wave’s frequency and angle of incidence on the ionosphere are chosen correctly, the wave will curve back to the surface, allowing for very long distance communication. For oblique incidence, at a point of the ionosphere where the critical frequency is $f_c$, the ionosphere can reflect waves of higher frequencies than the critical one. When the wave is incident from a non-normal direction, the reflection appears to occur at a virtual reflection point, $h'$, that depends on the frequency and angle of incidence.
To predict the bending of the ray we use a layered approximation to the ionosphere just as we did for the troposphere.

Snell’s law applies at each layer boundary

\[ \sin \psi_i = \sin(\psi_1) \sqrt{\varepsilon_r(z_1)} = \cdots \]

The ray is turned back when \( \psi(z) = \pi / 2 \), or \( \sin \psi_i = \sqrt{\varepsilon_r(z)} \)
Note that:

1. For constant $\psi_i$, a higher $N_e$ is required for higher frequencies if the ray is to return to Earth (because $\varepsilon_r$ decreases with $\omega$).

2. Similarly, for a given maximum $N_e$ ($N_{e_{\text{max}}}$), the maximum value of $\psi_i$ that results in the ray returning to Earth increases with increasing $\omega$.

There is an upper limit on frequency that will result in the wave being returned back to Earth. Given $N_{e_{\text{max}}}$ the required relationship between $\psi_i$ and $f$ can be obtained:

\[
\sin \psi_i = \sqrt{\varepsilon_r(z)}
\]

\[
\sin^2 \psi_i = 1 - \frac{\omega_p^2}{\omega^2}
\]

\[
1 - \cos^2 \psi_i = 1 - \frac{81N_{e_{\text{max}}}}{f^2}
\]

\[
N_{e_{\text{max}}} = \frac{f^2 \cos^2 \psi_i}{81} \quad \Rightarrow \quad f_{\text{max}} = \sqrt{\frac{81N_{e_{\text{max}}}}{\cos^2 \psi_i}}
\]
Ionospheric Radiowave Propagation (4)

Examples:

1. \( \psi_i = 45^\circ, \ N_{e_{\text{max}}} = 2 \times 10^{10} \text{ / m}^3 \): \( f_{\text{max}} = \sqrt{(81)(2 \times 10^{10}) / (0.707)^2} = 1.8 \text{ MHz} \)

2. \( \psi_i = 60^\circ, \ N_{e_{\text{max}}} = 2 \times 10^{10} \text{ / m}^3 \): \( f_{\text{max}} = \sqrt{(81)(2 \times 10^{10}) / (0.5)^2} = 2.5 \text{ MHz} \)

The value of \( f \) that makes \( \varepsilon_r = 0 \) for a given value of \( N_{e_{\text{max}}} \) is the critical frequency defined earlier:

\[ f_c = 9\sqrt{N_{e_{\text{max}}}} \]

Use the \( N_{e_{\text{max}}} \) expression from the previous page and solve for \( f \)

\[ f = 9\sqrt{N_{e_{\text{max}}} \sec \psi_i} = f_c \sec \psi_i \]

This is called the secant law or Martyn’s law. When \( \sec \psi_i \) has its maximum value (maximum angle of incidence on the ionosphere), the frequency is called the maximum usable frequency (MUF). A typical value is less than 40 MHz. It can drop as low as 25 MHz during periods of low solar activity. The optimum usable frequency\(^1\) (OUF) is 50% to 80% of the MUF. Data for the MUF is available. This is only correct for horizontally stratified media.

\(^1\)Sometimes called the optimum working frequency (OWF).
Maximum Usable Frequency

Shown below is the MUF in wintertime for different skip distances. The MUF is lower in the summertime.

Fig. 6.43 in R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985
Ionospheric Radiowave Propagation (5)

Multiple hops allow for very long range communication links (transcontinental). Using a simple flat Earth model, the virtual height \((h')\), incidence angle \((\psi_i)\), and skip distance \((d)\) are related by \(\tan\psi_i = \frac{d}{2h'}\). This implies that the wave is launched well above the horizon. However, if a spherical Earth model is used and the wave is launched on the horizon then \(d = 2\sqrt{2R'_{e}h'}\).\(^1\)

---

\(^1\)Whether to use \(R_e\) or \(R'_e\) depends on the amount of path through the troposphere where refraction is significant. In the remaining notes \(R'_e\) is used.
Ionospheric Radiowave Propagation (6)

Approximate virtual heights for layers of the ionosphere

<table>
<thead>
<tr>
<th>Layer</th>
<th>Range for $h'$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_2$</td>
<td>250 to 400 (day)</td>
</tr>
<tr>
<td>$F_1$</td>
<td>200 to 250 (day)</td>
</tr>
<tr>
<td>$F_1$</td>
<td>200 to 250 (day)</td>
</tr>
<tr>
<td>$F$</td>
<td>300 (night)</td>
</tr>
<tr>
<td>$E$</td>
<td>110</td>
</tr>
</tbody>
</table>

Example: Based on geometry, a rule of thumb for the maximum incidence angle on the ionosphere is about 74°. The MUF is

$$\text{MUF} = f_c \sec(74°) = 3.6 f_c$$

For $N_{e_{\text{max}}} = 10^{12} \text{ / m}^3$, $f_c \approx 9 \text{ MHz}$ and the MUF = 32.4 MHz. For reflection from the $F_2$ layer, $h' \approx 300 \text{ km}$. The maximum hop distance will be about

$$d_{\text{max}} \approx 2\sqrt{2R_e h'} = 2\sqrt{2(8500 \times 10^3)(300 \times 10^3)} = 4516 \text{ km}$$
For a curved Earth, using the law of sines for a triangle

\[ \frac{1 + h'/R'_e - \cos \theta}{\sin \theta} = \frac{1}{\tan \psi_i} \]

where

\[ \theta = \frac{d}{2R'_e} \]

and the launch angle (antenna pointing angle above the horizon) is

\[ \Delta = \phi - 90^\circ = 90^\circ - \theta - \psi_i \]

The great circle path via the reflection point is \( R \), which can be obtained from

\[ R = \frac{2R'_e \sin \theta}{\sin \psi_i} \]
Ionospheric Radiowave Propagation (8)

Example: Ohio to Europe skip (4200 miles = 6760 km). Can it be done in one hop?

To estimate the hop, assume that the antenna is pointed on the horizon. The virtual height required for the total distance is

\[
d / 2 = R_e^' \theta \quad \rightarrow \quad \theta = d / (2R_e^') = 0.3976 \text{ rad} = 22.8 \text{ degrees}
\]

\[
(R_e^' + h^') \cos \theta = R_e^' \quad \rightarrow \quad h^' = R_e^' / \cos \theta - R_e^' = 720 \text{ km}
\]

This is above the F layer and therefore two hops must be used. Each hop will be half of the total distance. Repeating the calculation for \( d / 2 = 1690 \text{ km} \) gives

\[
\theta = d / (2R_e^') = 0.1988 \text{ rad} = 11.39 \text{ degrees}
\]

\[
h^' = R_e^' / \cos \theta - R_e^' = 171 \text{ km}
\]

This value lies somewhere in the F layer. We will use 300 km (a more typical value) in computing the launch angle. That is, still keep \( d / 2 = 1690 \text{ km} \) and \( \theta = 11.39 \text{ degrees} \), but point the antenna above the horizon to the virtual reflection point at 300 km

\[
\tan \psi_i = \sin(11.39^\circ) \left[ 1 + \frac{300}{8500} - \cos(11.39^\circ) \right]^{-1} \quad \rightarrow \quad \psi_i = 74.4^\circ
\]
Ionospheric Radiowave Propagation (9)

The actual launch angle required (the angle that the antenna beam should be pointed above the horizon) is

\[ \Delta = 90^\circ - \theta - \psi_i = 90^\circ - 11.39^\circ - 74.4^\circ = 4.21^\circ \]

The electron density at this height (see chart, p.3) is \( N_{e_{\text{max}}} \approx 5 \times 10^{11} \text{ / m}^3 \) which corresponds to the critical frequency

\[ f_c \approx 9 \sqrt{N_{e_{\text{max}}}} = 6.36 \text{ MHz} \]

and a MUF of

\[ \text{MUF} \approx 6.36 \text{ sec } 74.4^\circ = 23.7 \text{ MHz} \]

Operation in the international short wave 16-m band would work. This example is oversimplified in that more detailed knowledge of the state of the ionosphere would be necessary: time of day, time of year, time within the solar cycle, etc. These data are available from published charts.
Ionospheric Radiowave Propagation (10)

Generally, to predict the received signal a modified Friis equation is used:

\[
P_r = \frac{P_t G_t G_r}{(4\pi R/\lambda)^2} L_x L_\alpha
\]

where the losses, in dB, are negative:

- \(L_x = L_{\text{pol}} + L_{\text{refl}} - G_{\text{iono}}\)
- \(L_{\text{refl}}\) = reflection loss if there are multiple hops
- \(L_{\text{pol}}\) = polarization loss due to Faraday rotation and earth reflections
- \(G_{\text{iono}}\) = gain due to focussing by the curvature of the ionosphere
- \(L_\alpha\) = absorption loss
- \(R\) = great circle path via the virtual reflection point

Example: For \(P_t = 30\) dBW, \(f = 10\) MHz, \(G_t = G_r = 10\) dB, \(d = 2000\) km, \(h' = 300\) km, \(L_x = 9.5\) dB and \(L_\alpha = 30\) dB (data obtained from charts).

From geometry compute: \(\psi_i = 70.3^\circ\), \(R = 2117.8\) km, and thus \(P_r = -108.5\) dBw
Charges in Magnetic Fields (1)

The motion of charged particles in a magnetic field can be derived from the Lorentz equation. For a charge $q$ of mass $m$ moving at velocity $u$

$$\vec{F}_m = q\vec{u} \times \vec{B} = m \frac{d\vec{u}}{dt}$$

Decomposing $\vec{u}$ into longitudinal and transverse components with respect to $\vec{B}$ gives

$$q(\vec{u}_L + \vec{u}_T) \times \vec{B} = m \frac{d}{dt}(\vec{u}_L + \vec{u}_T)$$

Let the magnetic field be uniform and in the $z$ direction, so that $\hat{\vec{B}} = \hat{z}$, $\vec{u}_L = u_z \hat{z}$, and $\vec{u}_T = u_x \hat{x} + u_y \hat{y}$

and therefore the vector equation can be written as two scalar equations

$$\frac{du_x}{dt} = qu_y B_o / m, \quad \frac{du_y}{dt} = -qu_x B_o / m$$

The classical gyrofrequency is $\omega_H = qB_o / (m = q\mu_0 H_o / m)$. To solve the two equations define a new variable $\zeta = u_x + ju_y$

$$\frac{d\zeta}{dt} + j\omega_H \zeta = 0$$
Charges in Magnetic Fields (2)

The solution is

\[ \zeta = |\vec{v}_T|e^{j\omega_H t} = u_T \left[ \cos(\omega_H t) + j \sin(\omega_H t) \right] \]

Therefore,

\[ u_x = \frac{dx}{dt} = u_T \cos(\omega_H t) \]
\[ u_y = \frac{dy}{dt} = u_T \sin(\omega_H t) \]
\[ u_z = \frac{dz}{dt} = u_L \]

Integrating with respect to time

\[ x - x_o = \frac{u_T}{\omega_H} \sin(\omega_H t + \phi) \]
\[ y - y_o = \frac{u_T}{\omega_H} \cos(\omega_H t + \phi) \]
\[ z - z_o = u_L t \]
Charges in Magnetic Fields (3)

The Larmor radius is defined as \( r_c = \frac{u_T}{\omega_H} \). The previous equations describe a circle of radius \( r_c \) with the constants \( x_o, y_o, z_o \) and \( \phi \) that depend on the initial conditions. Note that the particle motion has the following characteristics:

1. The particle rotates around the magnetic field line in a circle of radius \( r_c \) at frequency \( \omega_H \)
2. The center of the circle (the particle’s guiding center) travels along the field line with velocity \( \vec{u}_L \) so that the particle traces out a helix, as shown in the figure

\[
\vec{B} = B_o \hat{z}
\]
Charges in Magnetic Fields (4)

The Earth’s magnetic field is inhomogeneous. The magnetic field is strongest at the poles where the flux lines converge. The force on the charge varies as the gradient of the field

\[ \vec{F}_m = -|\vec{M}| \nabla \vec{B} = m \frac{d\vec{u}}{dt} \]

\[ |\vec{M}| = \frac{q^2 r_c B}{2m} \]

is the magnitude of the magnetic dipole moment. If momentum (energy) is conserved along the path, \( \frac{1}{2} m (u_L^2 + u_H^2) \) = constant. Conservation of momentum implies:

1. As \( |\vec{B}| \) increases \( \omega_H \) increases and \( u_T \) increases
2. As \( |\vec{B}| \) increases \( r_c \) decreases and \( u_T \) increases

The reverse is true when \( |\vec{B}| \) decreases. Thus, \( u_T \) increases as the flux lines converge, which requires that \( u_L \) decrease so that momentum is conserved. At some point all energy is in the transverse component, and longitudinal motion stops. This is a turning point where the particle reverses direction.
Earth Magnetic Field (1)

The Earth’s magnetic field is comprised of three main components:
1. The main field due to the outer core. The molten core consists of rotating liquid metal that give rise to a dynamo effect.
2. Fields that arise in the Earth’s crust and upper mantle.
3. Atmospheric magnetic field due to currents flowing in the atmosphere and in the oceans.

The first component dominates and is approximately a dipole field with the magnetic dipole pointing south.

Typical values:
At poles, $|\vec{B}| = 0.6 \text{ to } 0.7 \times 10^{-4} \text{ T}$
Near equator: $|\vec{B}| = 0.25 \times 10^{-4} \text{ T}$
(1 Wb/m$^2$ = 1 T = 10$^4$ G)

From: www.ngdc.noaa.gov/seg/WMM/back.shtml
Earth Magnetic Field (2)

The Earth’s main magnetic field reverses on the time scale of every several hundred thousand years. It is believed that the main field is currently in the process of reversal. The evidence includes the weakening of the magnetic field, relatively rapid migration of the poles (below left) and an increase in anomalies. Below is a simulation result showing the Earth’s magnetic field during the transition period.


(http://science.nasa.gov/headlines/y2003/29dec_magneticfield.htm?list770900)
Magneto-ionic Medium (1)

Thus far the Earth’s magnetic field has been ignored. An electron with velocity $\vec{u}$ moving in a medium with a magnetic field $\vec{B}_0 = \mu_0 \vec{H}_0$ experiences a force $\vec{F}_m = -e\vec{u} \times \vec{B}_0$.

Assume a plane wave that is propagating in the $z$ direction ($\vec{E}, \vec{H} \sim e^{-\gamma z}$). From Maxwell’s equations

$$\nabla \times \vec{H} = j \omega \vec{D} = j \omega \left[ \varepsilon \vec{E} + \vec{P} \right] \rightarrow \begin{cases} \gamma H_y = j \omega E_x \left( \varepsilon_0 + P_x / E_x \right) \quad (1) \\ \gamma H_x = j \omega E_y \left( \varepsilon_0 + P_y / E_y \right) \quad (2) \\ 0 = j \omega (\varepsilon_0 E_z + P_z) = j \omega D_z \quad (3) \end{cases}$$

$$\nabla \times \vec{E} = -j \omega \mu \vec{H} \rightarrow \begin{cases} \gamma E_y = -j \omega \mu_0 H_x \quad (4) \\ \gamma E_x = j \omega \mu_0 H_y \quad (5) \\ 0 = -j \omega \mu_0 H_z \quad (6) \end{cases}$$

These equations show that the plane wave will be transverse only with respect to the $\vec{B}$, $\vec{D}$ and $\vec{H}$ vectors, which is different from the isotropic case. The characteristic impedance of the medium is

$$\eta = E_x / H_y = -E_y / H_x = j \omega \mu_0 / \gamma.$$
Magneto-ionic Medium (2)

Solve (5) for $H_y$ and use the result in (1): $\gamma^2 = -\omega^2 \mu_0 \varepsilon_0 \left( 1 + \frac{P_x}{\varepsilon_o E_x} \right)$. A similar process starting with equation (4) would lead to $n^2 = 1 + \frac{P_y}{\varepsilon_o E_y}$. Equating this to the expression for $n^2$ above gives the polarization ratio of the wave, $R_{pol} \equiv \frac{P_x}{P_y} = \frac{E_x}{E_y}$.

Rewrite the equation of motion for an average electron when both electric and magnetic fields are present

$$m \ddot{r} + v m \dot{r} = -e \left( \vec{E} + \dot{r} \times \vec{B}_0 \right)$$

There is also a term in the parenthesis for the magnetic field of the wave, but it turns out to be negligible. We now define the longitudinal and transverse components, which are parallel and perpendicular to the direction of propagation, respectively.
Magneto-ionic Medium (3)

Definition of the coordinate system: \( y \) lies in the plane defined by \( B_o (= \mu_o H_o) \) and \( z \)

\[
H_L = H_z = H_o \cos \theta \quad \text{(longitudinal)}
\]
\[
H_T = H_y = H_o \sin \theta \quad \text{(transverse)}
\]

Note the direction of \( x \) is chosen to form a right-handed system (RHS). Any combination of propagation direction and magnetic field can be handled with this convention, as long as they are not parallel. If they are parallel any combination of \( x \) and \( y \) that forms a RHS is acceptable.

In this coordinate system the equation of motion can be written as three scalar equations, which for the \( x \) component is

\[
m\ddot{x} + v\dot{x} = -e\left( E_x + \dot{y}\mu_o H_L - \dot{z}\mu_o H_T \right)
\]
Magneto-ionic Medium (4)

For time-harmonic displacements, phasors can be used and the derivatives in time give a multiplicative factor \( j\omega \)

\[-\omega^2 m x + j \omega v m x = -e \left( E_x + j \omega y \mu_0 H_L - j \omega z \mu_0 H_T \right)\]

The polarization is caused by the electron displacements in \( x, y, \) and \( z \)

\[ x = \frac{P_x}{-eN_e}, \quad y = \frac{P_y}{-eN_e}, \quad z = \frac{P_z}{-eN_e} \]

Substituting:

\[ \frac{\omega^2 m P_x}{eN_e} - \frac{j \omega v m P_x}{eN_e} = -e E_x - \frac{j \omega \mu_0 P_y H_L}{eN_e} + \frac{j \omega \mu_0 P_z H_T}{eN_e} \]

Multiply by \( \varepsilon_0 / m \) and define the longitudinal and transverse gyro-frequencies\(^1\):

\[
\begin{align*}
\omega_L &\equiv \mu_0 e H_L / m = \omega_B \cos \theta \\
\omega_T &\equiv \mu_0 e H_T / m = \omega_B \sin \theta
\end{align*}
\]

\(^1\)In general, the gyro-frequency (also called the cyclotron frequency) of an electron in a magnetic field is defined as \( \omega_B = eB_o / m \) or \( \omega_H = e\mu_o H_o / m \).
Magneto-ionic Medium (5)

For convenience we define the frequency ratios

\[ X = \left( \frac{\omega_p}{\omega} \right)^2, \quad Y_L = \frac{\omega_L}{\omega}, \quad Y_T = \frac{\omega_T}{\omega}, \quad \text{and} \quad Z = \frac{\nu}{\omega}. \]

Recall that the plasma frequency is \( \omega_p = \sqrt{\frac{N e^2}{m \epsilon_o}} \). With this notation

\[ \epsilon_o X E_x = -P_x (1 - jZ) - jY_L P_y + jY_T P_z \]

Going back to the \( y \) and \( z \) components of the equation of motion, and treating them the same, gives two more equations

\[ \epsilon_o X E_y = -P_y (1 - jZ) + jY_L P_x \]

\[ \epsilon_o X E_z = -P_z (1 - jZ) - jY_T P_x \]

These are the equations needed to find \( n \) in terms of the components of \( \tilde{E} \), and \( \tilde{P} \). Arbitrarily we choose to solve for \( P_y \) and \( E_y \), eliminating \( P_x, E_x, P_z, E_z \).
Magneto-ionic Medium (6)

With several substitutions and a fair amount of algebra, two equations are obtained:

\[ \varepsilon_o X R_{\text{pol}} E_y = -P_y R_{\text{pol}} (1 - jZ) - j Y_L P_y + Y_T^2 R_{\text{pol}} P_y / (1 - X - jZ) \]
\[ \varepsilon_o X R_{\text{pol}} E_y = -P_y R_{\text{pol}} (1 - jZ) + j Y_L P_y R_{\text{pol}}^2 \]

Equate the right hand sides:
\[ -j Y_L + Y_T^2 R_{\text{pol}} / (1 - X - jZ) = j Y_L R_{\text{pol}}^2 \]

Solving this quadratic equation gives:
\[ R_{\text{pol}} = -\frac{j}{Y_L} \left\{ \frac{Y_T^2}{2(1 - X - jZ)} + \left[ \frac{Y_T^4}{4(1 - X - jZ)^2} + Y_L^2 \right]^{1/2} \right\} \]

Not all plane waves can pass through the medium. To remain plane waves they must have one of two polarizations that are characteristic of the medium. If this equation is satisfied, both equations at the top of the page give the same solution:
\[ P_y / E_y = -\frac{\varepsilon_o X}{1 - jZ - j Y_L R_{\text{pol}}} \]
Magneto-ionic Medium (7)

Use this ratio to find the index of refraction: 

\[ n^2 = 1 - X / \left[ 1 - jZ - jY_L R_{pol} \right] \]

and with \( R_{pol} \) from the quadratic equation yields the Appleton-Hartree formula

\[
\varepsilon_r = n^2 = (n' - jn'')^2 = 1 - \frac{X}{\left( 1 - jZ - \frac{Y_T^2}{2(1 - X - jZ)} \right) \pm \left\{ \frac{Y_T^4}{4(1 - X - jZ)^2} + Y_L^2 \right\}^{1/2}}
\]

Physical interpretation: The Earth’s magnetic field causes the ionosphere to be anisotropic. There are two modes of propagation, each with a particular polarization, that depend entirely on the properties of the medium. The phase velocities of the two modes, from the two values of \( n \) above, are different, and when they recombine they have different phase relationships. The solution with the positive sign is called the ordinary wave; that with the negative sign is the extraordinary wave. When the frequency is greater than about 1 MHz, which is always the case for systems that transmit through the ionosphere, the wave can be considered solely longitudinal (\( H_T, \omega_T \) and \( Y_T \) are zero) and therefore,

\[ n^2 \approx 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_L - j\nu)} \]

is sufficiently accurate for most calculations.
Anisotropic Media

In an isotropic medium the phase velocity at any given point is independent of the direction of propagation. For an isotropic medium, the direction of phase propagation (i.e., movement of the equiphase planes) is radial with velocity $u_p$. The energy propagation, which is generally taken as the group velocity, $u_g$ is also radial. In an anisotropic medium the directions of the equiphase planes and energy are different.

---

1 This is the velocity of propagation of a packet of frequencies centered about a carrier frequency. For simple amplitude modulated (AM) waveforms, it is the velocity of propagation of the envelope, whereas the phase velocity is the velocity of propagation of the carrier.
Phase and Group Velocities

The phase velocity is

$$u_p = \omega / \beta = c / n'$$

and the phase index of refraction (our normal index of refraction) is related to it by

$$n'u_p = c.$$  A group index of refraction, \(n_g\), can be defined analogous to the phase index of refraction:

$$n_g u_g = c$$

where \(u_g = d\omega / d\beta\). It is always true that \(u_p u_g = c^2\). The group index can be expressed as:

$$n_g = \frac{c}{u_g} = \frac{c d\beta}{d\omega} = \frac{d}{d\omega} \left( \frac{2\pi c}{\lambda} \right)$$

$$= \frac{d}{d\omega} \left( \frac{2\pi c n' u_p}{\lambda c 2\pi f} \right)$$

$$= \frac{d}{d\omega} (n' \omega)$$

$$n_g = n' + f \frac{dn'}{df}$$
Total Electron Content

If we neglect the magnetic field and consider the case where $Z = 0$ and $X << 1$, which is typical for links that penetrate the ionosphere, then

$$n' = \sqrt{1 - X} \approx 1 - \frac{1}{2}X = 1 - \frac{1}{2} \left( \frac{N_e e^2}{m \varepsilon_o \omega^2} \right) = 1 - \frac{40.3 N_e}{f^2}$$

For this approximation the group velocity is

$$n_g = n' + f \frac{dn'}{df} \approx 1 + 2 \left( \frac{40.3 N_e}{f^2} \right) = 1 + \frac{1}{2} X \approx \frac{1}{\sqrt{1 - X}} = \frac{1}{n'}$$

The angular (radian) phase path length between a transmitter at $P_1$ and receiver at $P_2$ is$^1$:

$$\Phi = -\frac{\omega}{c} \left[ \frac{P_2}{P_1} \right] \left[ 1 - \frac{1}{2} \left( \frac{N_e e^2}{m \varepsilon_o \omega^2} \right) \right] ds = -\frac{\omega}{c} \ell + \frac{1}{2} \left( \frac{e^2}{m \varepsilon_o \omega c} \right) \int_{P_1}^{P_2} N_e(s) ds$$

The integral $N_T (\text{}/\text{m}^2)$ represents the total electron content (TEC) along the path from $P_1$ to $P_2$. The second term is the phase error due to ionization. A value of $10^{16}$ is called a TEC unit (TECU).

$^1$The – sign means that traveling a distance $d$ in free space gives a phase delay of $\beta d$ radians. Sometimes the negative sign is omitted when the term delay is used.
Phase Delay and Time Delay (1)

Let the distance between point $P_1$ and $P_2$ be $\ell$. As a check, for a homogeneous medium,

$$\Phi = -\frac{\omega}{c} \ell \sqrt{\varepsilon_r} = -\frac{\omega}{c} n' \ell = -\beta_n n' \ell = -\beta \ell$$

Typical for a vertical path at 100 MHz is $3 \times 10^{17}$ electrons/m$^2$, which gives a 2$^{\text{nd}}$ term

$$\frac{1}{4\pi} \left(3 \times 10^{17}\right) \left(1.6 \times 10^{-19}\right)^2 \approx 2526 \text{ rad}$$

The time delay between a transmitter at $P_1$ and receiver at $P_2$ is:

$$t_d = \frac{1}{c} \int_{P_1}^{P_2} \frac{1}{n'(s)} ds$$

With the same assumptions on $Z$ and $X$, the (group) time delay is

$$t_d \approx \frac{1}{c} \int_{P_1}^{P_2} \left(1 + \frac{1}{2} X\right) ds = -\frac{\ell}{c} + \frac{40.3 N_T}{c f^2}$$
Phase Delay and Time Delay (2)

The phase rotation of the ordinary wave is found to be

\[ \Phi_o = -k_o \ell + \frac{N_T}{2 - Y_T^2 - \sqrt{Y_T^4 + 4Y_L^2}} \left( \frac{e^2}{m\varepsilon_o \omega c} \right) \]

A handy identity is \( \frac{e^2}{m\varepsilon_o c} = 1.06 \times 10^{-5} \). For the extraordinary wave

\[ \Phi_x = -k_o \ell + \frac{N_T}{2 - Y_T^2 + \sqrt{Y_T^4 + 4Y_L^2}} \left( \frac{e^2}{m\varepsilon_o \omega c} \right) \]

Faraday rotation is the rotation of the phase angle of a linearly polarized plane wave:

\[
\Omega = \frac{1}{2} (\Phi_o - \Phi_x) = \frac{1}{4} \left( \frac{e^2}{m\varepsilon_o \omega c} \right) N_T \frac{\sqrt{Y_T^4 + 4Y_L^2}}{1 - Y_T^2 - Y_L^2} \approx \frac{1}{4} \left( \frac{e^2}{m\varepsilon_o \omega c} \right) N_T \sqrt{Y_T^4 + 4Y_L^2}
\]

where the approximation assumes that the wave frequency is much larger than the gyro frequency.
Faraday Rotation Example

Find the Faraday rotation of a 135 MHz plane wave propagating a distance of 200 km through the ionosphere. The electron density is constant over the path and equal to $5 \times 10^{10} \text{ m}^{-3}$. The gyro-frequency is $8 \times 10^6 \text{ Hz}$. Assume collisions can be neglected and that the propagation is longitudinal.

The TEC is $N_T = \int_0^\ell N_e(s)ds = \ell N_e = 200 \times 10^3 \left(5 \times 10^{10}\right) = 10^{16} \text{ m}^2$. For longitudinal propagation and no collisions $Y_T = Z = 0$ and $Y_L = \omega_B / \omega$

$$\Omega \approx \frac{1}{4} \left(\frac{e^2}{m\varepsilon_0 \omega c}\right) N_T 2 \left(\frac{\omega_B}{\omega}\right) \left(1.06 \times 10^{-5}\right) \left(10^{16}\right) \left(2\pi\right) \left(8 \times 10^6\right) \approx 3.7 \text{ radians}$$

$$\approx 3.7 \text{ radians}$$

200 km

3.7 rad
GPS Error Due to Ionospheric Time Delay

An increase in phase velocity is accompanied by a decrease in group velocity:

\[
\begin{align*}
    u_p &= c \left( 1 - \frac{1}{2} X \right) \\
    u_g &= c \left( 1 + \frac{1}{2} X \right)
\end{align*}
\]

\[\rightarrow \Delta u = c \frac{1}{2} X\]

Differential relationships

\[
ds = c \left( 1 \pm \frac{1}{2} X \right) dt \quad \rightarrow \quad dt = \frac{1}{c} \left( 1 \mp \frac{1}{2} X \right) ds
\]

The path length error due to ionospheric time delay error is

\[
\Delta \ell = \int_{t_1}^{t_2} \Delta u \, dt = \int_{t_1}^{t_2} \frac{1}{2} c X \, dt = \int_{P_1}^{P_2} \frac{1}{2} X (1 + \frac{1}{2} X) \, ds = \frac{40.3}{f^2} N_T \ll 1
\]

Typical values (in meters):

<table>
<thead>
<tr>
<th>(N_T (/ \text{m}^3))</th>
<th>100</th>
<th>400</th>
<th>1227.6 (L2)</th>
<th>1575.42 (L1)</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^{16})</td>
<td>40</td>
<td>2.5</td>
<td>0.26</td>
<td>0.16</td>
<td>0.1</td>
</tr>
<tr>
<td>(10^{18})</td>
<td>4000</td>
<td>250</td>
<td>26</td>
<td>16</td>
<td>10</td>
</tr>
</tbody>
</table>
GPS Example

GPS uses two frequencies (L1 and L2 in the table on the previous page). If one can estimate the TEC then it is possible to correct the position errors due to the ionospheric time delay. Given that the one-way time delay for a path from the ground to the satellite differs by 15 ns for the two frequencies, estimate the total electron content of the path.

The change in time delay due to the ionosphere is given by the term

\[ \Delta t_d \approx \frac{40.3N_T}{cf^2} \]

For the two frequencies:

\[ \Delta t_{d2} - \Delta t_{d1} \approx \frac{40.3N_T}{c}\left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right) \equiv 15 \times 10^{-9} \]

\[ N_T = \frac{15 \times 10^{-9}}{40.3} c \left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right)^{-1} = \frac{15 \times 10^{-9}}{\left(1.34 \times 10^{-7}\right)\left(2.6 \times 10^{-19}\right)} = 4.2 \times 10^{17} \text{ / m}^3 \]

Applying this correction in practice is difficult because of other ambiguities that exist.
Absorption

Once the index of refraction is determined for the medium, the attenuation constant can be obtained from

\[ \gamma = \alpha + j\beta = j\frac{\omega}{c}n \]

For simplicity, the effect of the magnetic field is ignored in this discussion \((Y_T = Y_L = 0)\) so that

\[ n^2 = (n' - jn'')^2 = \left(1 - \frac{X}{1+Z^2} - j\frac{XZ}{1+Z^2}\right) \]

yielding \(\alpha = \frac{\omega}{c}n'' = \frac{\omega}{c} \frac{1}{2n'} \frac{XZ}{1+Z^2}\). In general, \(\alpha\) is not constant along a path through the ionosphere, because the electron density, and hence \(X\) and \(n'\) are changing along the path. The collision frequency, if not zero, can also change with location in the ionosphere. Thus, the loss due to absorption from \(P_1\) to \(P_2\) should be computed from

\[ L_{\text{attn}} = \exp\left(-\int_{P_1}^{P_2} \alpha(s) ds\right) \]

where \(s\) is the distance along the path.
Example

Find the attenuation (in dB/km) in the ionosphere for \( f = 30 \text{ MHz}, \nu = 10^7 \text{ /s and } N_e = 10^8 \text{ /m}^3 \).

Using the formula from the previous page with \( Z = \left( \frac{\nu}{\omega} \right) = \left( \frac{10^7}{2\pi 30 \times 10^6} \right) = 0.0531 \)

and \( X = \left( \frac{\omega_p}{\omega} \right)^2 = \left( \frac{56.34\sqrt{10^8}}{2\pi 30 \times 10^6} \right)^2 = 8.9 \times 10^{-6}, \varepsilon'_r = 1 - \frac{N_e e^2}{\varepsilon_o m (\nu^2 + \omega^2)} \approx 1 \). Substituting gives:

\[
\alpha = \frac{\omega}{c} \frac{1}{2 \sqrt{\varepsilon'_r}} \frac{XZ}{1 + Z^2} = 1.48 \times 10^{-7} \rightarrow -20 \log(e^{-1000\alpha}) = 0.0013 \text{ dB/km.}
\]

Alternatively, we could use the basic equation for attenuation

\[
\alpha = \omega \sqrt{\frac{\mu_o \varepsilon_o \varepsilon'_r}{2}} \left( \sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon_o \varepsilon'_r} \right)^2} - 1 \right) \text{ where } \sigma = \frac{N_e e^2 \nu}{m (\nu^2 + \omega^2)}.
\]

For \( N_e = 10^{11} \) the attenuation increases to 1.3 dB/km.
Ionospheric Soundings and Ionograms

An ionosonde is an instrument used to measure the variation of critical frequency with height. The measurements are used to determine $N_e(h')$. A train of short RF pulses is transmitted vertically and the time delay to reception is measured. The range to the reflection point can be computed. The frequency of the pulse train $f_v$ is swept to obtain a plot of $h'$ versus $f$, which is called an ionogram. The three segments of the curves between spikes represent reflections from the E, F₁, and F₂ layers.

From: www.ukssdc.ac.uk/ionosondes/ionogram_interpretation.html
Transmission Curves (1)

Two paths through the ionosphere are shown below, one for vertical incidence at frequency \( f_v \) and the other for oblique incidence at frequency \( f_{ob} \). We know that

\[ f < f_c, \text{ a reflection always occurs;} \]
\[ f > f_c, \text{ reflection only if } f \cos \psi_i < f_c. \]

From the Secant law, \( f_{ob} / f_v = \sec \psi_i \). For a flat earth, \( \tan \psi_i = d / (2h') \), and since \( \sec^2 \psi_i = 1 + \tan^2 \psi_i = 1 + [d / (2h')]^2 \), we can solve for \( h' \):

\[ \sec \psi_i = \sqrt{1 + [d / (2h')]^2} \]
\[ f_{ob} / f_v = \sqrt{1 + [d / (2h')]^2} \]
\[ h' = \frac{d}{2\sqrt{(f_{ob} / f_v)^2 - 1}} \]

Martyn’s theorem: \[ h'_v = h'_{ob} \]
Transmission Curves (2)

Plot a family of curves of $h'$ versus $f_v$ with $f_{ob}$ treated as a parameter. Then overlay them on an ionogram (heavy line in the figure). Together they determine what paths a signal radiated at frequency $f_{ob}$ may take over distance $d$ (neglecting loss and magnetic effects). They are called transmission curves.

Intersections of transmission curves with the ionogram represent valid reflection heights for the given frequency.

In this example, at frequencies above about 13 MHz there are no points common to the transmission curves and ionogram. This is the MUF.

(Curves shown are for $d = 800$ km, after K. Davies)
Time Delay Contours

Nightside transionospheric time delay at 30 MHz for receiver on a LEO platform at 1000 km.

Upper left: Receiver field-of-view showing time delay for ground-based transmitters whose signals range in elevation angle from 90° (zenith propagation) to 5° (defines boundary of circular area).
Upper right: GAIM vertical TEC, orbit path, and receiver location on nightside.
Lower left: Bisecting cut through image in upper left from E to W.
Lower right: Corresponding bisecting cut from S to N.

(http://www.cpi.com/splash/signalprop.html)
Ionospheric Ray Bending

The paths (red curves) taken by signals through two ionospheres under daytime conditions with different altitude distributions of the ionization. The ionospheres are displayed in color and the frequency and elevation angle considered for the illustration are 30 MHz and 5°, respectively. The blue curves show the corresponding straight-line paths that appear with curvature due to the use of a flat-Earth projection for the displays. The total electron content along the straight-line paths (TECSL) is the same for both cases (100 TEC units). The displays illustrate different degrees of ray bending due to different distributions of the ionization along the ray paths but constrained to the same TECSL values.

(http://www.cpi.com/splash/signalprop.html)
Scintillation (1)

*Scintillation* refers to rapid fluctuations in amplitude, phase, or angle of arrival of waves. It is produced by small-scale refractive index variations in the troposphere and ionosphere. An optical effect of scintillation is the “twinkling” of stars.

Tropospheric scintillation is produced in the first few kilometers of the surface and depends on season, local climate, frequency, and elevation angle. Typical peak-to-peak losses are:

<table>
<thead>
<tr>
<th>Loss (dB)</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Clear, summer</td>
</tr>
<tr>
<td>0.25 to 1</td>
<td>Winter</td>
</tr>
<tr>
<td>2 to 6</td>
<td>Some clouds</td>
</tr>
</tbody>
</table>

The fading rate ranges from 0.5 Hz to 10 Hz. Scintillation increases with frequency and is worst at low elevations (<10°). Gain reduction for 1 – 2 m apertures at 30 GHz ≈1 dB. It also causes depolarization.

Correlation length (distance) is the distance on average that the phase becomes uncorrelated (usually quantified by a $2\pi$ phase change). Correlation time is the time on average that phase becomes uncorrelated.
A simple model to explain scintillation is the dielectric layers or ellipsoids model.

- Ellipsoids with slightly different relative dielectric constants are changing shape as a function of time. The output wavefront is distorted due to “atmospheric multipath.”

- Wavefront distortion gives rise to phase errors and angle of arrival errors.

- Multipath causes signal fading.
Ionospheric scintillation is due to localized changes in the electron density $N_e$, and hence $\varepsilon_r$. The effect decreases as $1/f^2$ and usually can be neglected above 4 GHz, except when severe magnetic storms are occurring. Characteristics are:

1. At 4 GHz causes 0.5 to 10 dB fading 0.1 % of the time
2. Worst at the poles and in an equatorial belt $\pm 25^\circ$ from the equator
3. Worst 1 – 2 hours after sunset
4. Independent of elevation angle

Ionospheric irregularities can take on a wide range of sizes but are typically around 1 km in extent. They tend to align with the Earth’s magnetic field lines and form “striations” or “plumes.”
Scintillation Index

A measure of scintillation is the scintillation index

\[ S_4 = \sqrt{\frac{\langle P^2 \rangle - \langle P \rangle^2}{\langle P \rangle^2}} \]

\( P = \) signal power
\( \langle P \rangle = \) average power
(usually over 1 minute)

Significance:
- \( S_4 < 0.6 \): weak
- \( S_4 > 0.6 \): strong
- \( S_4 = 1.0 \): saturation

Current information available at National Weather Service Space Weather Prediction Center
http://www.swpc.noaa.gov/index.html
Ionospheric Disturbances

Sudden disturbances are usually due to solar flares, with the following effects:

1. **phase anomalies**: change in $h'$ over short paths of a few hundred km (D layer)
2. **frequency deviations**: change in frequency of a reflected signal (E and F regions)
3. **cosmic noise absorption**: sudden decrease in cosmic noise
4. **increase in TEC** (E and F regions)

The time extent of the effects is from several minutes to several hours.

**Magnetic storms** are disturbances of the geomagnetic field that last from a few hours to several days.

Factors that influence magnetic storm behavior include:

- polar heating
- thermal expansion
- electromagnetic drift
- neutral air drag
- global atmospheric circulation
- atmospheric composition

Consequences of magnetic storms:

- power outages
- increase in atmospheric drag on satellites
- affects the discharging of satellites
- induces large voltages in big conductors (e.g., Alaskan pipeline)
- adverse effects on radio communications