Dynamic Scheduling and Maintenance of a Deteriorating Server

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Problem: In what order should the jobs be worked on?

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Waiting class k jobs incur holding costs at the (constant) rate c_k.

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- Reward rate of $c_k\mu_k$ when a class k job is being served.
- Allows one to use an interchange argument on the sample paths of the process.

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Is the "$c\mu$-rule" optimal?

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Q: Does the "$c\mu$-rule" minimize the expected long-run average cost?

A: No!

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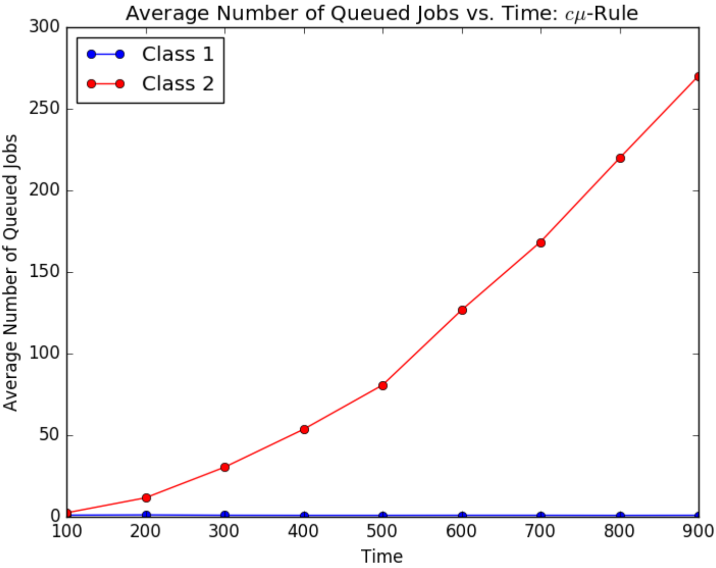

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- When it's not performed, which job class should be served?

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- The successive lengths of time that the server is down for maintenance are independent and identically distributed.

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For the joint scheduling and preventive maintenance problem, suppose Assumptions CR and QO hold. Then it is without loss of optimality to always schedule according to the $c\mu$-rule.

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Idea: Use the theory of SMDPs to study the structure of optimal policies.

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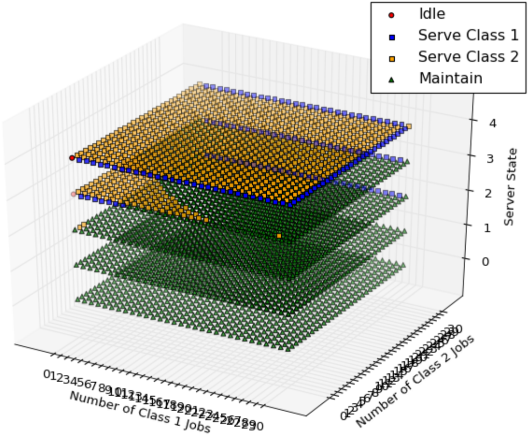

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