DYNAMICS OF FREE SURFACE AND PURE ELONGATIONAL FLOWS OF LIQUID CRYSTALLINE POLYMERS

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ABSTRACT

The performance properties of many polymers are dominated by the molecular alignment induced during processing by the elongational flow component, that is, the diagonal component of the velocity gradient tensor. Here we discuss two idealizations of this flow-microstructure interaction. We first summarize our recent studies of biaxial nematic patterns and phase transitions (Forest and Wang, 1998a) of lyotropic liquid crystalline polymers (LCPs) in response to imposed elongational flows. We show radially symmetric biaxial nematic patterns coexist with homogeneous biaxial patterns characterized by Rey (Rey, 1995). All biaxial patterns in unidirectional stretching flows are unstable, whereas planar stretching induces candidate stable biaxial nematic patterns. Non-homogeneous biaxial patterns have potential *core defects*, and two special director alignments are selected by imposing a finite-pressure condition along the axis of flow symmetry. The role of director dynamics in response to flow perturbations is described, and in particular *new, elongation-induced, director instabilities* are revealed, extending seminal stability and phase transition results (Khokhlov and Semenov, 1982; See et al., 1992; Bhave et al, 1993; Rey, 1995; Hu and Ryskin, 1992). We then examine two special solution families of the governing equations for free surface, radially symmetric, *cylindrical* LCP fiber flows, following analogous Newtonian results of Segur et al. (Segur et al., 1997). The cylindrical free surface only accommodates uniaxial orientation, and from these ideal transient solutions we analyze the linearized stability of the coupled flow and microstructure in the LCP thin-filament approximation (Forest et al, 1997).

INTRODUCTION

The flow-driven molecular alignment of liquid crystalline polymers (LCPs) dominates various manufacturing processes. Various geometries and flows are involved in the different processes, so it is important to assemble the role of geometry, boundary conditions, and flow as they contribute to the orientation of LCP processing. Here we touch upon some of these contributing factors. Pure elongational flow, which stretches the LCP either along the axis of flow symmetry or radially in the orthogonal plane to the flow axis, approximates the interior flow away from boundaries of many extrusion and even film processes. LCP fibers are processed in a free surface elongational flow that leads to strong molecular alignment. Carefully designed flows alter the intrinsic isotropic to nematic ("I-N") phase transition of lyotropic LCPs, with advantages such as achieving high orientation at relatively low polymer concentrations, but also creating *biaxial nematic patterns* (Rey, 1995; Forest and Wang, 1998a). Steady fiber flows with draw down of the fiber radius have also been shown to generate biaxial behavior in the nematic orientation tensor (Forest et al., 1997), where the biaxiality enters weakly in the fiber cross section. Purely cylindrical stretching fibers are shown below to only support uniaxial nematic order. The main goal of this paper is to illustrate various flow-driven orientation phenomena that are deduced by special, idealized flows and geometries. These phenomena are part of, and meant to provide insight into, the more complex flow-microstructure interactions that take place in realistic confined and free surface processing of LCPs and polymers in general.

In a dynamical flow field, on experimental lengthscales for diffraction and birefringence measurements, LCPs exhibit either uniaxial or biaxial nematic symmetry depending on the nature of the flow field. Certain patterns are consistently observed in steady processes, presumably consisting in local spatial regions of stable equilibria that respond to the particular flow in that region. Textures that are realized on larger, even visible scales represent a patchwork of local orientation structure that is built from local equilibria of LCPs. These local patterns that are described by an average orientation tensor are the object of our studies to date. The longer lengthscale description of textures (Larson and Doi, 1991) is beyond this description, where defects are relevant and even necessary to mediate the different local patterns we describe here.

Bhave, Menon, Armstrong, and Brown (1993), extending work of Doi (1980;1981), developed a kinetic theory for flows of liquid crystalline polymers in a Newtonian solvent, subject to an anisotropic hydrodynamic drag and a polymer-polymer mean-field interaction with Maier-Saupe potential. An averaged version of this theory is adopted. See, Doi and Larson (1990), Bhave et al. (1993), and most recently Rey (1995) have applied the approximate Doi nematodynamic equations (hereafter called the Doi model) to predict spatially homogeneous nematic patterns, and phase transitions, in response to an imposed elongational flow. Analysis based on the Onsager theory of isotropic-nematic phase transitions is given by Khokhlov and Semenov (1982). The two bifurcation parameters in these studies and ours are an LCP density parameter (N) and the Peclet number, Pe, the ratio of elongation rate to LCP molecular relaxation rate. In (See et al., 1990; Bhave et al., 1993; Rey, 1995) special forms of the *orientation tensor* $\mathbf{Q}$ are posited in the Doi model, leading to a low-dimensional set of ordinary differential equations for the dynamical response of $\mathbf{Q}$ to elongational flow. These analyses presume spatially homogeneous nematic patterns, and apply rectangular coordinate representations for $\mathbf{Q}$. Rey (1995) used a biaxial parametrization of $\mathbf{Q}$ and explicitly characterized *biaxial steady state patterns* and their stability within the 2-D dynamical system for the two independent order parameters for biaxial nematics.

In both fiber free surface flows and our study of LCPs in imposed elongational flow, we have taken the point of view to begin with the most general representation for the orientation tensor, $\mathbf{Q}$, then understand how special forms are either forced by the general Doi nematodynamics or sit as special low-dimensional dynamics inside of a more general system of equations. The seminal analyses of (Khokhlov and Semenov, 1982; See et al., 1990), for example, presume a uniaxial representation of $\mathbf{Q}$ with one free order parameter. Rey generalized this analysis to allow for a second, biaxial, order parameter; these are simply the eigenvalues of the tensor $\mathbf{Q}$. There are three additional orientation tensor degrees of freedom suppressed in a pure order-parameter, or eigenvalue, representation; these are the directors, or eigenvectors, of $\mathbf{Q}$. The directors are parametrized by three director angles, whose dynamics are presumed absent in a pure order-parameter analysis. Absent of flow, we show (Forest and Wang, 1998) the director angles are necessarily constant, for both uniaxial and biaxial nematic tensors; *however, in the presence of flow, the biaxial director degrees of freedom are dynamic, and contain potential instabilities which we show are realized for many steady states.*

In the first part of this paper we summarize our latest results of a comprehensive study on the issues raised above. We use our own (Forest et al., 1997) cylindrical coordinate representations of $\mathbf{Q}$ based on an eigenvalue/eigenvector form, similar to that in (Vertogen and deJeu, 1988) and applied by (Rey, 1995) in Cartesian coordinates, to calculate all steady states of the elongational flow–driven Doi nematodynamic equation for the orientation tensor. We then exactly solve the linearized equations associated to each steady state within the 5-D orientation tensor space to investigate the stability of the underlying steady states.

Pure elongation is an idealized flow, convenient in that one can impose the kinematics, satisfy the mass and momentum equations, and extract a simple model for the orientation response to the posited kinematics. In realistic LCP flows, the flow and orientation are coupled. One of the flow-orientation coupled flows akin to the elongational flow is axisymmetric free surface thin fiber flow (Forest et al, 1997), which can be approximated by elongational flow near the fiber axis, away from the free surface. While similar in these respects, the hydrodynamics in these flows couples strongly with the orientation dynamics. It therefore provides an excellent model for a detailed analysis of this mutual interaction. The second part of this paper exploits this model to derive and study the stability of special contracting cylindrical LCP filaments.

3–DIMENSIONAL FORMULATION OF THE DOI MODEL

We briefly review the approximate macroscopic equations governing orientation and flow of LCPs from (Bhave et al., 1993).

Conservation of linear momentum:

$$\rho \frac{d}{dt}\mathbf{v} = \nabla \cdot \tau + \rho \mathbf{g}, \tag{1}$$

where ρ is the *density of the polymeric liquid*, $\mathbf{v}$ is the *velocity* vector, τ is the *total stress tensor*, $\rho\mathbf{g}$ represents the external force due to gravity and $\frac{d}{dt}$ denotes the *material derivative* defined by $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$.

Incompressibility:

$$\nabla \cdot \mathbf{v} = 0. \tag{2}$$

Constitutive equation for stresses:

$$\begin{aligned} &\tau = -p\mathbf{I} + \hat{\tau}, \\ &\hat{\tau} = 2\eta \mathbf{D} + 3ck\Theta[(1 - N/3)\mathbf{Q} - N(\mathbf{Q}\cdot\mathbf{Q}) + \\ &N(\mathbf{Q} : \mathbf{Q})(\mathbf{Q} + \mathbf{I}/3) + 2\lambda(\nabla\mathbf{v} : \mathbf{Q})(\mathbf{Q} + \mathbf{I}/3)], \end{aligned} \tag{3}$$

where $\mathbf{D} = \frac{1}{2}[\nabla\mathbf{v} + \nabla\mathbf{v}^T]$ is the *rate-of-strain tensor*, where $\nabla\mathbf{v}$ has i, j component $\frac{\partial v_i}{\partial x_j}$ in Cartesian coordinates, p is the *pressure*, η is the *solvent viscosity*, λ is a *relaxation time* associated with rotation of the dumbbell molecules, c is the *number of polymer molecules per unit volume*, N is the dimensionless *polymer concentration* which measures the strength of the intermolecular Maier-Saupe potential, $\mathbf{Q}$ is the *orientation tensor*, a traceless symmetric second order tensor defined by

$$\mathbf{Q} = \langle \mathbf{m} \otimes \mathbf{m} \rangle - \mathbf{I}/3, \tag{4}$$

where $\mathbf{m}$ is a unit vector in the LCP molecular direction, the average $\langle(\bullet)\rangle$ is with respect to a molecular probability density function consistent with rodlike molecules, k is the *Boltzmann constant*, and Θ is *absolute temperature*.

Orientation tensor equation:

$$\left\{ \begin{aligned} &\tfrac{d}{dt}\mathbf{Q} - (\nabla\mathbf{v}\cdot\mathbf{Q} + \mathbf{Q}\cdot\nabla\mathbf{v}^T) = \tfrac{2}{3}\mathbf{D} - \\ &2(\nabla\mathbf{v} : \mathbf{Q})(\mathbf{Q} + \mathbf{I}/3) - \sigma/\lambda\{(1 - N/3)\mathbf{Q} - \\ &N(\mathbf{Q}\cdot\mathbf{Q}) + N(\mathbf{Q}:\mathbf{Q})(\mathbf{Q} + \mathbf{I}/3)\}, \end{aligned} \right. \tag{5}$$

where $0 < \sigma \leq 1$ is the anisotropic drag parameter describing the anisotropic drag that a molecule experiences as it moves relative to the solution.

In order to describe the most general average internal orientation, a biaxial representation for $\mathbf{Q}$ is posited in (Forest et al., 1997):

$$\mathbf{Q} = s(\mathbf{n}_3 \otimes \mathbf{n}_3 - \mathbf{I}/3) + \beta(\mathbf{n}_2 \otimes \mathbf{n}_2 - \mathbf{I}/3), \tag{6}$$

where s and β are the two scalar order parameters defined by

$$\begin{aligned} s &= \langle(\mathbf{n}_3 \cdot \mathbf{m})^2\rangle - \langle(\mathbf{n}_1 \cdot \mathbf{m})^2\rangle, \\ \beta &= \langle(\mathbf{n}_2 \cdot \mathbf{m})^2\rangle - \langle(\mathbf{n}_1 \cdot \mathbf{m})^2\rangle, \end{aligned} \tag{7}$$

and $\mathbf{n}_i, i = 1, 2, 3$ are the three orthonormal eigenvectors of $\mathbf{Q}$ called directors. The range of (s, β) is a closed triangular region with vertices $(1, 0), (0, 1)$ and $(-1, -1)$ in the (s, β) plane. From the order parameters, we can calculate the average of the squares of the direction cosines with respect to the triad of eigenvectors $\mathbf{n}_i, i = 1, 2, 3$, respectively,

$$\begin{aligned} d_1 &= \langle(\mathbf{m}\cdot\mathbf{n}_1)^2\rangle = \tfrac{1-s-\beta}{3}, \\ d_2 &= \langle(\mathbf{m}\cdot\mathbf{n}_2)^2\rangle = \tfrac{2\beta-s+1}{3}, \\ d_3 &= \langle(\mathbf{m}\cdot\mathbf{n}_3)^2\rangle = \tfrac{2s-\beta+1}{3}. \end{aligned} \tag{8}$$

These quantities are the eigenvalues of the second order tensor $\mathbf{Q} + \frac{\mathbf{I}}{3}$ which characterize, in a nonlinear averaged sense, the *degrees of orientation* of the LCP molecules with respect to each director, respectively.

If $s\beta \neq 0$ or $s \neq \beta$, all the eigenvalues of $\mathbf{Q}$ are distinct and the nematic LCP is called *biaxial*; the triad of directors are equally important. The nematic liquid crystal is called *uniaxial* if two eigenvalues of $\mathbf{Q}$ are equal, in which case one of the directors is the distinguished orientation direction (or axis of nematic symmetry), called the uniaxial director and denoted as $\mathbf{n}$. The uniaxial limits have a generic representation

$$\mathbf{Q} = s_u(\mathbf{n} \otimes \mathbf{n} - \mathbf{I}/3), \tag{9}$$

where the scalar *uniaxial order parameter* s_u is defined by

$$s_u = 3/2\langle(\mathbf{m}\cdot\mathbf{n})^2\rangle - 1/2, \quad -1/2 \leq s_u \leq 1. \tag{10}$$

When $0 < s_u \leq 1$, the liquid crystal exhibits prolate uniaxial symmetry; when $-1/2 \leq s_u < 0$, there is oblate uniaxial symmetry; $s_u = -1/2$ corresponds to the LCP molecule aligned somewhere in the plane orthogonal to $\mathbf{n}$; $s_u = 1$ corresponds to parallel alignment of $\mathbf{n}$ and $\mathbf{m}$; $s_u = 0$ corresponds to an isotropic state in which molecular orientation information is lost.

The *major director* is defined as the director associated with the direction of highest degree of orientation (Wang, 1997). We note that the major director is not defined wherever a discontinuity, an isotropic phase, or an oblate uniaxial phase occurs. Therefore, we say that defects form at locations where the major director is not defined. Our proposed definition of defects on the scale of the averaged orientation tensor $\mathbf{Q}$ appears to be most natural with regard to experiments, since experimentally one searches for the primary optical axis either through index of refraction measurements or through diffraction patterns. In the cases we classify as defects, one should have experimental difficulty determining a primary optical axis. There are also larger scale phenomena associated with textures in which the patterns described here are pieced together, with defects arising as the necessary transition phases. We refer for example to Larson and Doi (Larson and Doi, 1991).

In order to completely represent the orientation tensor $\mathbf{Q}$, we need to parametrize the directors with respect to the coordinate basis $\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z$, where a cylindrical coordinate with its z-axis coincident with the axis of flow symmetry in both the pure and free surface elongational flow to be studied next. Here, we use three angle parameters $0 \leq \chi, \xi, \zeta < \pi$ to implement this:

$$\begin{aligned}
&\mathbf{n}_1 = (\cos\chi\sin\zeta + \sin\chi\cos\xi\cos\zeta, \\
&-\cos\chi\cos\zeta + \sin\chi\cos\xi\sin\zeta, -\sin\chi\sin\xi), \\
&\mathbf{n}_2 = (\sin\chi\sin\zeta - \cos\chi\cos\xi\cos\zeta, \\
&-\sin\chi\cos\zeta - \cos\chi\cos\xi\sin\zeta, +\cos\chi\sin\xi), \\
&\mathbf{n}_3 = (\sin\xi\cos\zeta, \sin\xi\sin\zeta, \cos\xi),
\end{aligned} \tag{11}$$

where ξ and ζ are the two spherical angles for the unit position vector $\mathbf{n}_3$ and χ is used to parametrize $\mathbf{n}_1$ and $\mathbf{n}_2$ in the plane orthogonal to $\mathbf{n}_3$.

Denoting $\mathbf{Q}$ in (6) by $\mathbf{Q}(\mathcal{O})$, where $\mathcal{O} = (s, \beta, \xi, \zeta, \chi)$, we remark that the correspondence between $\mathbf{Q}$ and $\mathcal{O}$ is not 1-1 globally. We refer the reader to (Forest and Wang, 1998a) for details.

STEADY STATE ELONGATIONAL FLOWS AND THEIR STABILITY

In this section, we describe the steady orientation patterns that are selected according to the Doi theory in response to a prescribed elongational kinematics with constant *elongational rate* ν:

$$\mathbf{v} = (-\frac{\nu r}{2}, 0, \nu z), \tag{12}$$

in the cylindrical coordinate. For $\nu > 0$ the flow is extensional along $\mathbf{e}_z$, termed axially stretching or axial elongation, while for $\nu < 0$ the flow stretches in the entire plane orthogonal to $\mathbf{e}_z$, termed planar stretching or planar elongation.

With this prescribed kinematics, the orientation tensor equation (5) can be decoupled from the other equations for two forms of $\mathbf{Q}$. First, in previous studies (Khokhlov and Semenov, 1982; See et al., 1990; Rey, 1995), the momentum equation is trivially satisfied by assuming $\mathbf{Q}$ depends only on the time coordinate, t. Second, the orientation tensor is allowed to depend on the angle θ explicitly (Forest and Wang, 1998a) in order to include radially symmetric, non-homogeneous equilibria. Due to the axisymmetric and torsionless nature of the imposed flow field, the orientation tensor equation is free of spatial derivatives with respect to θ. Thus, as far as the orientation tensor equation is concerned, it is indifferent to whether $\mathbf{Q}$ is a function of (θ, t) or t. However, we emphasize that $\mathbf{Q}(t)$ satisfies the momentum equation trivially, whereas $\mathbf{Q}(\theta, t)$ does so conditionally; this will lead to an interesting selection criterion for the non-homogeneous patterns.

When the orientation equation is decoupled from the momentum equation, and our biaxial representation for $\mathbf{Q}$ is employed, the variables s, β, χ, ξ are independent of the cylindrical polar angle θ, while the single director angle ζ (which parametrizes director orientation in the plane orthogonal to the flow axis of symmetry when $\xi = 0$) is a function of θ given by either

$$\zeta = \text{const}, \text{ or } \zeta = -\theta + \text{ const }. \tag{13}$$

If ζ =const, then the normal stress components τ_{rr} and $\tau_{\theta\theta}$ are constant with respect to θ, and the scalar pressure takes the explicit form:

$$p = -\frac{\rho\nu^2 r^2}{8} - \frac{\rho\nu^2 z^2}{2} + (\tau_{rr} - \tau_{\theta\theta} + \tau_{\theta\theta,\theta}) ln(r) + \tau_{rr} + p_0, \tag{14}$$

where p_0 is a constant and $(\bullet)_{,\theta}$ denotes the partial derivative with respect to θ.

Thus we conclude that for this special class of orientation tensors, the pressure has a singularity at $r = 0$ *except when* $\tau_{rr} = \tau_{\theta\theta}$*, which can be satisfied only if* $\zeta = \frac{\pi}{4}, \frac{3\pi}{4}$, or $\beta = 0$. In the uniaxial case $\beta = 0$, the value of ζ loses meaning and a simpler nonsingular form of the pressure exists. *If* $\beta \neq 0$*, to maintain regularity of the pressure at* $r = 0$*,* ζ *must be either* $\frac{\pi}{4}$ or $\frac{3\pi}{4}$. Thus, the regularity condition on pressure effectively serves as a selection criterion for non-homogeneous biaxial orientation patterns. If $\zeta = 0, \frac{\pi}{2}$, $\tau_{r\theta} = 0$ and the momentum equation is also satisfied. But, the pressure (14) has a singularity at the core $r = 0$.

If $\zeta = -\theta$+const, then $\tau_{r\theta,\theta} = -(\tau_{rr} - \tau_{\theta\theta})$ so that the balance of linear momentum is satisfied with a finite pressure given by

$$p = -\frac{\rho\nu^2 r^2}{8} - \frac{\rho\nu^2 z^2}{2} + \tau_{rr} + \text{const}. \tag{15}$$

These steady states correspond to the spatially homogeneous solutions, which are more easily described in rectangular coordinates. Rey has studied the order parameter dynamics of these steady states for planar stretching flows (Rey, 1995), which we (Forest and Wang, 1998a) then added to by resolving the director dynamics and considering uniaxial elongation as well. The remarkable coincidence is that *both classes of radially symmetric orientation tensors, homogeneous and nonhomogeneous, reduce the Doi nematodynamic equations to the exact set of* *nonlinear order parameter equations!* Thus, both types of biaxial patterns coexist, with the same phase transitions leading to their existence and changes in stability.

The full orientation tensor equation (5), represented in the order and angle parameters, consists of 5 coupled ordinary differential equations:

$$\begin{aligned}
&\beta_{,\tilde{t}} = -U(\beta) - \tfrac{2Ns\beta}{3}(s-\beta+1) - \tfrac{Pe}{4}[-\beta^2+ \\
&2s\beta+\beta+\cos 2\chi(3\beta^2-\beta-2+2s)+ \\
&3\cos 2\xi\beta(1+2s-\beta)+ \\
&\cos 2\chi\cos 2\xi(-3\beta^2+\beta+2-2s)], \\
&s_{,\tilde{t}} = -U(s) - \tfrac{2Ns\beta}{3}(\beta-s+1) - \tfrac{Pe}{4}[\beta-s\beta-1+ \\
&2s^2-s+3\cos 2\xi(\beta-s+2s^2-1-s\beta)+ \\
&\cos 2\chi(\beta+3s\beta-1+s)+\cos 2\chi\cos 2\xi(1-s \\
&-\beta-3s\beta)], \\
&\xi_{,\tilde{t}} = -\tfrac{Pe\sin 2\xi}{4s(\beta-s)}[\beta-2s-s^2-\beta^2+ \\
&s\beta+\cos 2\chi(\beta^2-\beta-2s\beta)], \\
&\zeta_{,\tilde{t}} = -\tfrac{Pe}{2s(\beta-s)}[\beta\sin 2\chi\cos\xi(-1+\beta-2s)], \\
&\chi_{,\tilde{t}} = \tfrac{Pe\sin 2\chi}{4s\beta(\beta-s)}[-\beta^2-s\beta+s^2-s^3+\beta^3+ \\
&3s^2\beta/2-5s\beta^2/2+\cos 2\xi(-\beta^2+s\beta \\
&+\beta^3-s^2+s^3-3s\beta^2/2-3s\beta^2/2)].
\end{aligned} \qquad (16)$$

where $\tilde{t} = \frac{t}{\lambda}$ is the dimensionless time, the Peclet number $Pe = \frac{\nu\lambda}{\sigma}$, and $\int U(s)ds$ corresponds to the bulk free energy for uniaxial phases with

$$U(s) = s(1 - \tfrac{N}{3}(1-s)(2s+1)). \qquad (17)$$

From these equations, in (Forest and Wang, 1998a) we are able to deduce previous uniaxial 1-D (See et al., 1990) and biaxial 2-D (Rey, 1995) order parameter equations, each of which corresponds to a special, nonlinear, low-dimensional subspace of the 5-D equations. Indeed, there is another so-called invariant subsystem of these equations, which is 3-dimensional and couples one director angle (ξ) to the order parameters s, β. Remarkably, each low-dimensional system sits inside of the higher dimensional equations, forming a nested structure of a 5-D system, which has 3-D invariant subsystems, which have 2-D invariant subsystems, which have a 1-D invariant subsystem. This structure allows us to characterize *all uniaxial and biaxial steady states*, but more remarkably, to *exactly solve the linearized equations about every family of steady states.* This means we completely characterize all equilibrium nematic patterns in the parameter space of LCP concentration N and normalized elongation rate (Peclet number), Pe; and, since we have exact formulas for all linearized eigenvalues of the 1-D, 2-D, 3-D, and 5-D equations, we explicitly characterize all bifurcation curves in N, Pe space. These bifurcation curves correspond physically to phase transitions, the boundaries in LCP concentration and elongational flow rate across which the number and types of equilibria change and/or the stability of individual patterns changes.

We have established a correspondence between steady states from our biaxial representation, using coordinates $\mathcal{O}$, and from standard tensorial components of $\mathbf{Q}$ (Forest and Wang, 1998a). This change of coordinates is faithful with regard to finding steady state solutions, but subtle with the stability analysis. Any nonlinear change of dependent variables strongly alters the linearized vector field, essentially by a factor given by the Jacobian of the transformation. The stability of the original physical variables is therefore recoverable when the Jacobian is non-singular, but must be carried out in original variables when the Jacobian is singular. As we discovered in (Forest and Wang, 1998a) the transformation is singular for all uniaxial steady states, or biaxial steady states wherever $\xi = 0$. Thus, (5) has to be used for stability analysis. The sign of the linearized eigenvalues in the linearized system determines the stability of the steady state: *linearly stable* if the real parts of all eigenvalues are negative ; *center-stable* if one is zero; *unstable* if one is positive.

For a complete existence and stability picture of the steady orientation patterns, readers are referred to (Forest and Wang, 1998a). Here, we focus only on the deformation of the steady orientation patterns with respect to the Peclet number and elaborate on stable orientation patterns in unidirectional and planar stretching flows, respectively.

Deformation of steady orientation patterns

We observe from (16) that when the flow is absent ($Pe = 0$), the director angles ξ, ζ, χ are necessarily constant, and the order parameter dynamics of s, β completely decouples from the directors. Then, the 5-D equations collapse to 2-D; this observation shows that absent of flow there is no information lost by positing a pure order parameter representation of $\mathbf{Q}$. However, note that the dynamics is 2-D, biaxial, and not 1-D, uniaxial. The steady states are then given by the simultaneous zeros of these two polynomials in s, β; an analysis shows **the only steady states reside in the three distinct uniaxial subspaces**, $s = 0, \beta = 0$, $s = \beta$, respectively, with the corresponding order parameter $\beta, s, -s$, respectively, taking discrete values given by the zeros of U, (17). Thus for *existence* of nematic steady states absent of an imposed flow, the 1-D uniaxial assumption for $\mathbf{Q}$ suffices; for *stability* of these uniaxial equilibria, however, the uniaxial equation misses information about the second, biaxial order parameter. In (Forest and Wang, 1998a), we show for example that the so-called oblate phase, $s_u < 0$, that exists for $N > 3$ with $Pe = 0$, is predicted to be stable in the 1-D uniaxial equation when indeed it is *unstable* in the second, biaxial degree of freedom!

As elongational flow is imposed ($Pe \neq 0$), the uniaxial equilibria with the uniaxial director axis parallel to the flow direction persist and deform, but more complex biaxial equilibrium patterns can also be supported by the flow (Rey, 1995; Forest and Wang, 1998a). If we identify $\mathbf{n}_3$ with the director parallel to the flow direction, the uniaxial family corresponding to $\beta = 0$ remains uniaxial while all other uniaxial equilibria deform into biaxial equilibria. Figure 1 depicts the uniaxial equilibria absent of flows and Figure 2 depicts the equilibria (uniaxial and biaxial) in uniaxially stretching elongation flows for a small value of Pe.

Figure 3 provides an atlas for all uniaxial and biaxial equilibria, in which the (N, Pe) plane is divided into *eleven distinct regions*; this extends Rey's atlas (Rey, 1995) to $Pe > 0$. Tables 1 and 2 list all coexisting equilibrium patterns in each region. The eleven regions are separated by bifurcation curves; the horizontal line $Pe = 0$ of Figure 1 corresponds to the flow-independent, pure orientation patterns; the upper half-plane $Pe > 0$ consists of the axially stretching patterns; and the lower half-plane $Pe < 0$ consists of patterns that exist in the presence of planar stretching flow (in the plane orthogonal to the flow axis of symmetry).

Stable orientation patterns

From Table 1, we observe that all stable orientation patterns exhibit uniaxial symmetries in unidirectional stretching flows. Specifically, a less aligned prolate equilibrium is the unique stable state in region I; when the parameters are varied into region II across the boundary AF in Figure 3, a highly aligned, stable prolate equilibrium is born out of a saddle node bifurcation so that two stable prolate states coexist in region II and III; when the parameters are further varied into region IV and V across the boundary ABC, the less aligned prolate equilibrium disappears due to a saddle-node bifurcation; the highly aligned prolate equilibrium remains the only stable state in both regions IV and V. All the other orientation patterns documented in Table 1 are unstable.

When the imposed elongational flow is planar stretching, the stable orientation patterns are more complex. In region XI, there exists a unique stable state that exhibits oblate uniaxial symmetry. When parameter values are varied into region X and VIII across the boundary FE, two simultaneous saddle node bifurcations yield two additional stable biaxial states, which coexist with the stable oblate uniaxial state. (Recall further that whenever we have biaxial states, there are co-existing homogeneous and non-homogeneous radially symmetric states.) When parameter values are further varied into region VI, VII and VIV across CK, only the pair of stable biaxial states coexist, which are born out of a pitchfork bifurcation along CK from the stable oblate state.

By examining the stability property of steady states in the full 5-D orientation tensor space, we also rule out some flow-induced orientation patterns which were mischaracterized as stable in previously reported studies (Rey, 1995; Bhave et al., 1993), because there are director instabilities that are simply missed in a pure order parameter analysis. Therefore, our analysis is definitive within the context of imposed kinematics, and for this physical model. It is likely that flow dynamics will change the picture further.

SPECIAL AXISYMMETRIC FREE SURFACE THIN FILAMENT LCP FLOWS AND THEIR STABILITY

Free surface axisymmetric filament flows have been widely used in modeling various industrial processes and natural phenomena. It is one of the most important flow types encountered in mathematical and engineering modeling. The governing equations for this flow problem consist of the equations listed in section 2 along with the kinematic boundary condition at the free surface, which dictates that the free surface convects with the flow, and the kinetic boundary condition, quantifying the stress jump across the free surface in the normal direction.

Under the assumption of axisymmetry, the velocity field for a torsionless flow is given by $\mathbf{v} = (v_r, 0, v_z)$ in cylindrical coordinates, with the z-axis in the direction of gravity. For an axisymmetric free surface, $r = \phi(z,t)$, the kinematic boundary condition is

$$\frac{d}{dt}(r - \phi(z,t)) = 0, \tag{18}$$

at $r = \phi(z,t)$. The kinetic boundary condition reads

$$((p_a - p)\mathbf{I} + \mathbf{T})\mathbf{m} = -\sigma_s \kappa \mathbf{m}, \tag{19}$$

where $\mathbf{m} = \frac{1}{\sqrt{1+\phi_{,z}{}^2}}(1, 0, -\phi_{,z})$ is the unit external normal to the free surface $r = \phi(z,t)$, p_a is a constant ambient pressure, κ is the mean curvature

$$\kappa = \frac{1}{\phi\sqrt{1+\phi_{,z}{}^2}} - \frac{\phi_{,zz}}{(\sqrt{1+\phi_{,z}{}^2})^3}, \tag{20}$$

and σ_s is the constant surface tension coefficient.

In order to derive asymptotic equations later, we nondimensionalize the governing equations and boundary conditions using a characteristic radial length scale r_0, axial length scale z_0, and time scale t_0, respectively. We scale the space variables r and z, time t, extra stress tensor $\mathbf{T}$, pressure p, velocity vector $\mathbf{v}$ and the radius of the cylindrical jet ϕ as follows:

$$\tilde{r} = \frac{r}{r_0}, \tilde{z} = \frac{z}{z_0}, \tilde{t} = \frac{t}{t_0}, \tilde{\mathbf{T}} = \frac{\mathbf{T}r_0^2}{f_0},$$
$$\tilde{p} = \frac{pr_0^2}{f_0}, \tilde{\mathbf{v}} = \frac{\mathbf{v}t_0}{z_0}, \tilde{\phi} = \frac{\phi}{r_0}, \epsilon = \frac{r_0}{z_0}, \tag{21}$$

where $f_0 = \frac{\rho r_0^2 z_0^2}{t_0^2}$ is a characteristic force scale, selected here as an inertial force, and ϵ is the aspect ratio of the radial length scale relative to the axial length scale. Then a host of dimensionless parameters arise

$$F = \frac{z_0}{gt_0^2}, W = \frac{\rho r_0 z_0^2}{\sigma_s t_0^2}, \Lambda = \frac{\lambda}{t_0},$$
$$Re = \frac{\rho z_0^2}{\eta t_0}, \alpha = \frac{3ck\Theta t_0^2}{\rho z_0^2}. \tag{22}$$

Here F, W, Re, Λ are the Froude number, Weber number, Newtonian Reynolds number, and Deborah number, respectively, and α measures the strength of the orientational stress relative to inertia.

Recently, Segur et al. discovered a new cylindrical transient solution for viscous free surface jet flows (Segur, 1997). Motivated by their work, we seek analogous solutions for LCP filaments. We seek solutions of the governing equations and boundary conditions in the form: $v_r = v_r(r,t), v_z = v_z(z,t), \mathbf{Q} = \mathbf{Q}(t), \phi = \phi(t), v_{z,z} \neq 0, v_{z,zz} = 0$. The solution found has the same velocity and geometric profile as that of the viscous jet, but different stress components and pressure:

$$v_r^{(1)} = -\frac{r}{2(t-t_0)}, v_z^{(1)} = \frac{z+z_0}{t-t_0} + \frac{t-t_0}{2F}, \phi^{(1)} = \frac{\Phi}{\sqrt{t-t_0}},$$
$$p^{(1)}(t) = T_{rr}(t) + \frac{1}{W\phi} + \frac{3}{8}(\phi^2 - r^2) + p_a,$$
$$\xi^{(1)} = 0, \chi^{(1)} = 0, \beta^{(1)} = 0,$$
$$T_{rr} = -\frac{{v_{z,z}}^{(1)}}{Re} + \frac{\alpha}{3}[-U(s^{(1)})/3+ 2\Lambda {v_{z,z}}^{(1)} s^{(1)}(1-s^{(1)}), \tag{23}$$
$$T_{\theta\theta} = T_{rr}, T_{r\theta} = 0, T_{rz} = 0, T_{\theta z} = 0,$$
$$T_{zz} = 2\frac{{v_{z,z}}^{(1)}}{Re} + \frac{2\alpha}{3}[U(s^{(1)}) + \Lambda s^{(1)}(2s^{(1)}+1){v_{z,z}}^{(1)}],$$

for $t > t_0$, where Φ is a constant determined by the initial radius of the filament and the uniaxial order parameter $s^{(1)}$ is determined by

$$s^{(1)}{}_{,t} = (1-s^{(1)})(2s^{(1)}+1)v_{z,z} - \frac{\sigma}{\Lambda}U(s^{(1)}). \tag{24}$$

We remark that the cylindrical profile of these special solutions does not accommodate the elongation-induced biaxial orientation patterns reported earlier; only uniaxial orientation is supported by this solution family.

This solution represents a uniform liquid filament under the influence of both inertia and gravity whose radius evolves. In the contracting uniform filament solution, there exists a time-dependent stagnation point along the filament, where the flow ceases. At opposing sides of the stagnation point, fluid particles move apart. If gravity is not included, the stagnation point is static. Otherwise, it is driven by gravity to move opposite to the direction of gravity in time. This set of solutions applies only to the case $t > t_0$. For $t < t_0$, we need to replace $\phi^{(1)}$ by $\phi^{(1)} = \frac{\Phi}{\sqrt{t_0 - t}}$ while retaining all other expressions. This set of solutions represents a uniform filament whose radius expands.

We note that the response of the orientation tensor for this velocity field resembles that of an imposed elongational flow with a time-dependent elongational rate $\frac{1}{t-t_0}$. When $t > t_0$, the flow field stretches axially; in contrast, for $t < t_0$ the flow is effectively planar elongation. The analogy is not complete; contrary to imposed elongational flows in unconfined flow, the cylindrical free surface fiber flow does not accommodate biaxial orientation patterns.

We next seek solutions of the 3-D governing equations in another form: $v_r = v_r(t), v_z = v_z(t), p = p(t), \mathbf{Q} = \mathbf{Q}(t), \phi(z,t) = \phi_0$. The solution found has a cylindrical shape with a constant radius and temporally varying velocity field and orientation patterns:

$$v_z^{(0)} = \frac{t}{F} + v_0, v_r^{(0)} = 0, \beta^{(0)} = 0, \xi^{(0)} = 0, \chi^{(0)} = 0,$$
$$p^{(0)}(t) - T_{rr}(t) - p_a = \frac{1}{W\phi_0}, \tag{25}$$

where v_0 is an arbitrary constant and the uniaxial order parameter is determined by

$$s^{(0)}{}_{,t} = -\frac{\sigma}{\Lambda}U(s^{(0)}). \tag{26}$$

Segur et al. studied the linearized stability property of the contracting uniform filament solution for inviscid and "very viscous" liquids (Segur, 1997). The analysis for the very viscous liquid is quite tedious, and generalizations to general viscous fluids or more complex rheology is not at all clear. For the special solution of a constant radius, Forest and Wang investigated the capillary instability of a liquid jet of LCPs while neglecting the influence of gravity and Newtonian viscosity (Forest and Wang, 1998b). Generalization of both studies to the general Doi model discussed here appears to be intractable at the full 3-D level.

On the other hand, for *slender fibers*, slender asymptotic models offer an approximate, but tractable analysis of the full 3-D stability results. Slender models, based on a small fiber aspect ratio ϵ, are particularly relevant because the slender asymptotic equations retain the nonlinear interactions of the flow and microstructure, with the primary approximations (or sacrifices of the full dynamics) consisting of an accurate description only of the longwave interactions, and for flows that are nearly elongational (often called plug flow). In particular, cylindrical solutions are preserved by a longwave approximation, so the asymptotic equations will preserve cylindrical solutions. (We note that $p^{(1)}$ in (23) evaluated at $r = \phi^{(1)}$ is the pressure solution for the asymptotic equation.) It happens further that many fiber instabilities are primarily longwave instabilities, such as capillary instability. For such phenomena, it then becomes very reasonable to approximate the full linearized stability analysis by first restricting to a slender asymptotic approximation of the full 3-D free surface equations, and then identifying the simple cylindrical solutions and performing stability of them in the approximate, longwave slender equations. This approach has been quite successful over the past 30 years, providing insight and estimates for slender fiber and filament stability when full analysis has been unattainable. A good survey is given in (Eggers, 1995).

Slender models for LCP filaments have been derived by Forest et al. (1997) in modeling of fiber spinning processes. While all physical effects are present at the leading order in terms of the small aspect ratio ϵ, the 1-D model consists of three partial differential equations for the radius ϕ , axial velocity v and the uniaxial order parameter s:

$$\begin{aligned}
&\phi_{,t} + v\phi_{,z} + \tfrac{\phi v_{,z}}{2} = 0, \\
&v_{,t} + vv_{,z} = \tfrac{1}{F} - \tfrac{1}{W}(\tfrac{1}{\phi} - \epsilon^2\phi_{,zz})_{,z} + \\
&\tfrac{1}{\phi^2}[\phi^2((3Re_{lcp}^{-1})v_{,z} + \alpha U(s))]_{,z}, \\
&s_{,t} + vs_{,z} = (1-s)(2s+1)v_{,z} - \tfrac{\sigma}{\Lambda}U(s),
\end{aligned} \tag{27}$$

where Re_{lcp} is defined as the LCP Reynolds number, consisting of a Newtonian and orientation contribution,

$$Re_{lcp}^{-1} = Re^{-1} + \frac{2}{3}\alpha\Lambda s^2. \tag{28}$$

The radial velocity and pressure at leading order are related to the above three variables algebraically. Next, we examine the linearized stability of the special cylindrical solutions using this 1-D model.

Solutions with constant radius

For this family of solutions, we write the physical variables as

$$v = v_z^{(0)} + \delta v, \phi = \phi^{(0)} + \delta\phi, s = s^{(0)} + \delta s. \tag{29}$$

Substituting these into the 1-D model and retaining the terms linear in the disturbances (the $\delta(\bullet)$ terms), we obtain the linearized equations. In the Lagrangian coordinates defined by

$$\zeta = z - v_0 t - \frac{t^2}{2F}, \tau = t, \tag{30}$$

the linearized system is a constant-coefficient partial differential equation system with spatial derivatives in ζ.

Treating the solution as infinitely long, we introduce Fourier modes in the linearized variables:

$$(\bullet)(\xi, t) = (\hat{\bullet})(\tau)e^{im\xi}, \tag{31}$$

where $(\bullet)$ represents ϕ, v, s, the 1-D model is then transformed into a constant coefficient ODE system. The real parts of the eigenvalues of the coefficient matrix are defined as the instantaneous growthrates, which measure the onset of temporal variation of the amplitude in the perturbed variables. If the maximum real part of the eigenvalues is negative, the base solution is called (instantaneously) stable. If the maximum real part is zero, the solution is called neutrally stable. Otherwise, the solution is said to be unstable.

The instantaneous growthrates for the linearized system are

$$\begin{aligned}
x_1 &= -\tfrac{\sigma}{\Lambda}U'(s^{(0)}), \\
x_2 &= -\tfrac{3Re_{lcp}^{-1}m^2 + h(s^{(0)})\frac{\Lambda}{\sigma}\alpha m^2 + \sqrt{\Sigma}}{2}, \\
x_3 &= -\tfrac{3Re_{lcp}^{-1}m^2 + h(s^{(0)})\frac{\Lambda}{\sigma}\alpha m^2 - \sqrt{\Sigma}}{2},
\end{aligned} \tag{32}$$

where

$$\begin{aligned}
\Sigma &= [3Re_{lcp}^{-1}m^2 + h(s^{(0)})\frac{\Lambda}{\sigma}\alpha m^2]^2 \\
&+ \frac{2\phi^{(0)}m^2}{W}(\frac{1}{{\phi^{(0)}}^2} - \epsilon^2 m^2) + 4\alpha m^2 U(s^{(0)}),
\end{aligned} \tag{33}$$

$$h(s^{(0)}) = (1 - s^{(0)})(2s^{(0)} + 1). \tag{34}$$

The first mode corresponding to x_1 is apparently independent of the hydrodynamics, thus, purely orientational. If $U'(s^{(0)}) \geq 0$, the mode is stable; otherwise, it is unstable. This is precisely the stability condition for uniaxial equilibria of the Doi nematodynamic equations (See et al., 1990) absent of flow; the only pitfall in this description is that biaxial instability of the oblate phase for $N > 3$ would be missed if the biaxial orientation equations went unrecognized.

The second and third modes, given by x_2 and x_3, are coupled. The effect of the orientation coupling on the growthrates is to lower their magnitudes. The cutoff wave number for instability is purely geometric, independent of the material properties, consistent with the 3-D calculation of the capillary stability of an LCP jet without gravity (Forest and Wang, 1998b).

Solutions with varying radius

We write the physical variables as follows

$$v_z = {v_z}^{(1)} + \frac{\delta v}{t-t_0}, \phi = \phi^{(1)} + \frac{\delta\phi}{\sqrt{t-t_0}}, \quad s = s^{(1)} + \delta s. \tag{35}$$

Then, we substitute them into the 1-D model and retain only the terms linear in $\delta(\bullet)$. We rewrite the equations in a new Lagrangian coordinate defined by

$$\xi = \frac{z+z_0}{t-t_0} - \frac{t-t_0}{2F}, \tau = t - t_0. \tag{36}$$

The resultant is a variable coefficient linear PDE system with time-dependent coefficients. We study the instantaneous stability properties of the underlying (base) solution by calculating the instantaneous growthrates of the linearized system, which are the eigenvalues of the coefficient matrix.

The eigenvalues of the coefficient matrix in the linearized system, after a single Fourier mode is introduced, are given by the zeros of the cubic polynomial equation

$$\begin{aligned} &-x^3 + (\tfrac{H'}{\tau} - \tfrac{\sigma}{\Lambda}U' - 3Re_{lcp}^{-1}M^2)\, x^2 + \\ &\Big[3Re_{lcp}^{-1}(\tfrac{H'}{\tau} - \tfrac{\sigma}{\Lambda}U') - \tfrac{4}{\tau}\alpha\Lambda s^{(1)}H + \alpha H U' \\ &+ \tfrac{\phi^{(1)}}{2W}(\tfrac{\sqrt{\tau}}{\phi^{(1)2}} - \tfrac{\epsilon^2 M^2}{\sqrt{\tau}}) + \tfrac{3}{\tau}Re_{lcp}^{-1} + \alpha U\Big]\, M^2\, x \\ &+ \Big[\tfrac{\phi^{(1)}}{2W}(\tfrac{\sqrt{\tau}}{\phi^{(1)2}} - \tfrac{\epsilon^2 M^2}{\sqrt{\tau}}) + \tfrac{3}{\tau}Re_{lcp}^{-1} + \alpha U\Big] \\ &(\tfrac{H'}{\tau} - \tfrac{\sigma}{\Lambda}U')\, M^2 = 0, \end{aligned} \tag{37}$$

where $M = \frac{m}{\tau}$, $H = (1-s^{(1)})(2s^{(1)}+1)$. The asymptotic behavior of the growthrates, for $|M| << 1$, is:

$$\begin{aligned} x_1 &\sim \tfrac{1-4s^{(1)}}{\tau} - \tfrac{\sigma}{\Lambda}U'(s^{(1)}), \\ x_2 &\sim M\sqrt{\tfrac{\sqrt{\tau}}{2W\phi^{(1)}} + \tfrac{3Re_{lcp}^{-1}(s^{(1)})}{\tau} + \alpha U(s^{(1)})}, \\ x_3 &\sim -M\sqrt{\tfrac{\sqrt{\tau}}{2W\phi^{(1)}} + \tfrac{3Re_{lcp}^{-1}(s^{(1)})}{\tau} + \alpha U(s^{(1)})}; \end{aligned} \tag{38}$$

for $|M| >> 1$, is:

$$\begin{aligned} x_1 &\sim \tfrac{1-4s^{(1)}}{\tau} - \tfrac{\sigma}{\Lambda}U'(s^{(1)}), \\ x_2 &\sim \tfrac{M^2}{2}\left[-3Re_{lcp}^{-1}(s^{(1)}) + \sqrt{(3Re_{lcp}^{-1}(s^{(1)}))^2 - \tfrac{2\phi^{(1)}\epsilon^2}{W\sqrt{\tau}}}\right], \\ x_3 &\sim -\tfrac{M^2}{2}\left[3Re_{lcp}^{-1}(s^{(1)}) + \sqrt{(3Re_{lcp}^{-1}(s^{(1)}))^2 - \tfrac{2\phi^{(1)}\epsilon^2}{W\sqrt{\tau}}}\right]. \end{aligned} \tag{39}$$

The rate x_1 is identified with the orientation mode and the other two, x_2 and x_3, correspond to the flow-orientation interaction modes. We denote the two zeros of $U'(s) = \frac{\Lambda(1-4s)}{\tau\sigma}$ by

$$\begin{aligned} s_1 &= \tfrac{1}{6} - \tfrac{\Lambda}{N\sigma\tau} - \sqrt{\Pi}, \\ s_2 &= \tfrac{1}{6} - \tfrac{\Lambda}{N\sigma\tau} + \sqrt{\Pi}, \end{aligned} \tag{40}$$

where $\Pi = (\frac{1}{6} - \frac{\Lambda}{N\sigma\tau})^2 - \frac{1}{2}(\frac{1}{N} - \frac{\Lambda}{N\sigma\tau} - \frac{1}{3})$. If $s_1 \le s^{(1)} \le s_2$, the orientational mode is stable; otherwise, it is unstable. The orientational instability is not limited to longwaves. From the asymptotic formulae for the two coupled modes, we observe that one is always negative and the other may be positive only in the range of longwaves, i.e., $|M| << 1$. In this case, the cutoff wave number is given by

$$M_{cutoff} = \frac{\sqrt{1 + \frac{2W\phi^{(1)}}{\sqrt{\tau}}\left[\alpha U(s^{(1)}) + \frac{3Re_{lcp}^{-1}(s^{(1)})}{\tau}\right]}}{\phi^{(1)}\epsilon}. \tag{41}$$

If, in addition to $x_1 < 0$,

$$\Delta = \frac{\sqrt{\tau}}{2W\phi^{(1)}} + \alpha U(s^{(1)}) + \frac{3Re_{lcp}^{-1}(s^{(1)})}{\tau} < 0, \tag{42}$$

the flow-orientation coupling modes are stable. Then, the solution is stable.

It is remarkable that orientation may stabilize the solution at certain moments for some parameter values. The discriminant (42) is in fact equal to the axial tension due to viscous stress, pressure and orientation stress. This result indicates that the liquid fiber in transient motion can be stabilized provided there is tension in the direction opposite to its movement. This effect is possible only if the material is non-Newtonian since, for a given orientation $s^{(1)}$, the effect of Newtonian viscosity is always destabilizing. For LCPs, the elastic stress is due completely to the molecular orientation. It is worthwhile to speculate how this result may be used to stabilize contracting filaments.

CONCLUSION

Results from (Forest and Wang, 1998a) are recalled relating to phase transitions and nematic patterns in response to pure elongational flow. For imposed unidirectional elongational flows, the stable orientation patterns are all uniaxial, usually unique except for a small region in the parameter space (N, Pe) where bistable patterns exist. When the imposed flow is planar elongation, the stable orientation pattern is uniaxial oblate and unique at low polymer concentration. Phase transitions then lead to a pair of stable biaxial patterns at sufficiently high concentration. The stable oblate pattern and the pair of biaxial patterns coexist only in a small region in the parameter space (N, Pe).

Moving to free surface flows, two special axisymmetric cylindrical solutions are given, each with uniaxial orientation. For the special solution of constant radius, the instantaneous instability occurs whenever $0 < M < \frac{1}{\epsilon \phi^{(0)}}$, which is consistent with the capillary instability obtained for constant equilibrium jets without gravity. For the special solution of varying radius, long wave instability may be suppressed completely by orientation provided the internal orientation produces a negative tension along the axial direction of the filament. This stabilizing criterion requires the first normal stress difference of the orientational stress not only to be negative, but also to dominate the surface tension and viscous stress. Otherwise, either a lone longwave instability due to surface tension and viscous effects, or an elastic instability which persists for all wave numbers, is predicted.

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Table 1: All steady states in uniaxially stretching flows

Region	Type of steady states	Dim of unstable manifolds, 5-D
I:	P^1	W_0^5
II:	P^1	W_0^5
	P^2	W_1^5
	P^3	W_0^5
III:	P^1	W_0^5
	P^2	W_1^5
	P^3	W_0^5
	B^1	W_1^5
	B^2	W_2^5
	B^3	W_2^5
	B^4	W_1^5
IV:	P^1	W_0^5
	B^1	W_1^5
	B^2	W_2^5
	B^3	W_2^5
	B^4	W_1^5
V:	O^1	W_5^5
	O^2	W_4^5
	P^1	W_0^5
	B^1	W_1^5
	B^2	W_2^5
	B^3	W_2^5
	B^4	W_1^5
OF	Iso	W_0^5
FC	Iso	W_0^5
	$P^1, \mathbf{n} = \mathbf{e}_i, i = z, r, \theta$	W_0^5
	$P^2, \mathbf{n} = \mathbf{e}_i, i = z, r, \theta$	W_1^5

Remark: We use B, Iso, O and P to represent biaxial, isotropic, oblate uniaxial and prolate uniaxial steady states, respectively. W_i^5 denotes the dimension of the unstable manifold is i in the 5-dimensional orientation tensor space.

Table 2: All steady states in planar stretching flows

Region	Type of steady states	Dim of unstable manifolds, 5-D
VI:	O^1	W_2^5
	P^1	W_2^5
	P^2	W_5^5
	B^1	W_0^5
	B^2	W_3^5
	B^3	W_3^5
	B^4	W_0^5
VII:	O^1	W_1^5
	P^1	W_2^5
	P^2	W_3^5
	B^1	W_0^5
	B^4	W_0^5
VIII:	O^1	W_0^5
	P^1	W_2^5
	P^2	W_3^5
	B^1	W_0^5
	B^{1u}	W_1^5
	B^4	W_0^5
	B^{4u}	W_1^5
VIV:	O^1	W_1^5
	B^1	W_0^5
	B^4	W_0^5
X:	O^1	W_0^5
	B^1	W_0^5
	B^{1u}	W_1^5
	B^4	W_0^5
	B^{4u}	W_1^5
XI:	O^1	W_0^5