

# Lanchester models with Discontinuities: An application to Networked Forces

Harrison C. Schramm



Operations Research Department  
Naval Postgraduate School, Monterey, CA  
80<sup>th</sup> MORS  
13 June 2012

# Overview

- In this talk I plan to:
  - Provide a general overview of Lanchester Models
  - Discuss how they might be adapted to consider Network Effects
  - Show some details about how effectiveness is determined
  - Provide an historically motivated example.

# Why do this?

- To provide a 'bite sized' analytic model that incorporates the effect of network loss with the kinetic battle. In short, it takes 'kinetic-like' and 'cyber-like' inputs and provides a 'kinetic' output.
- This may prove be useful for fast, transparent verification of a complex simulation model

# Lanchester Models

Pit two sides, Blue and Red, against each other, and analyze the resulting combat as a deterministic model. In their most general form,

$$\frac{dR}{dt} = -\gamma_B$$
$$\frac{dB}{dt} = -\gamma_R,$$

where the gammas represent arbitrary functions. We explore specific choices, and their consequences subsequently

# Common Lanchester Model 'Flavors'

- For *Aimed fire*

$$\frac{dB}{dt} = -\beta_R R$$

$$\frac{dR}{dt} = -\beta_B B$$

- For *Area fire*

$$\frac{dB}{dt} = -\beta_R RB$$

$$\frac{dR}{dt} = -\beta_B RB$$

- For *Ambush situations*

$$\frac{dB}{dt} = -\beta_R R$$

$$\frac{dR}{dt} = -\beta_B BR$$

# Solving the Lanchester Models

- Cumbersome, explicit time-dependent solutions exist,
- ... but numerical integration via *Euler's Method* or others is fast, accurate, and easily implemented in a spreadsheet
- ... While the explicit solutions are messy, the equations may be easily rewritten to find the time-independent state equation; for example, *Aimed fire* obeys the law:

$$\frac{B_0^2 - B_t^2}{R_0^2 - R_t^2} = \frac{\beta_R}{\beta_B}$$

# Shock Action - modification

- Our contribution is to consider a model in which the dynamics of combat change suddenly and irrevocably at a deterministic time,  $t^*$ .
- Our solutions to follow are implicit in the corresponding variables, which we call  $B^*$  or  $R^*$

$R^*$

$$\frac{dR}{dt} = -\gamma_{BN} \quad t < t^*$$

$$\frac{dR}{dt} = -\gamma_B \quad t \geq t^*$$

$$\frac{dB}{dt} = -\gamma_R \quad \forall t$$

# Aimed Fire → Aimed Fire

## Model and results

- In this situation network loss causes us to go from highly effective aimed fire to less accurate aimed fire. The model is specified as:

$$\frac{dR}{dt} = -\beta_{BN} B \quad t < t^*$$

$$\frac{dR}{dt} = -\beta_B B \quad t \geq t^*$$

$$\frac{dB}{dt} = -\beta_R R \quad \forall t$$

$$B_* = \sqrt{\frac{\beta_{BN} B_0^2 - \beta_B B_f^2 - \beta_R (R_0^2 - R_f^2)}{\beta_{BN} - \beta_B}}$$



# Aimed Fire → Area Fire: Model and Result

- Conversely, in this situation, network reduction causes us to go from aimed fire to area fire

$$\frac{dR}{dt} = -\beta_{BN} B \quad t < t^*$$

$$\frac{dR}{dt} = -\beta_B BR \quad t \geq t^*$$

$$\frac{dB}{dt} = -\beta_R R \quad \forall t$$

$$R_* = \frac{\beta_{BN}}{\beta_B} + \sqrt{\frac{\beta_{BN}^2}{\beta_B^2} + R_0^2 - \frac{\beta_{BN}}{\beta_R} [B_0^2 - B_f^2] - 2 \frac{\beta_{BN}}{\beta_B} R_f}$$

# The effect of the Network on Targeting

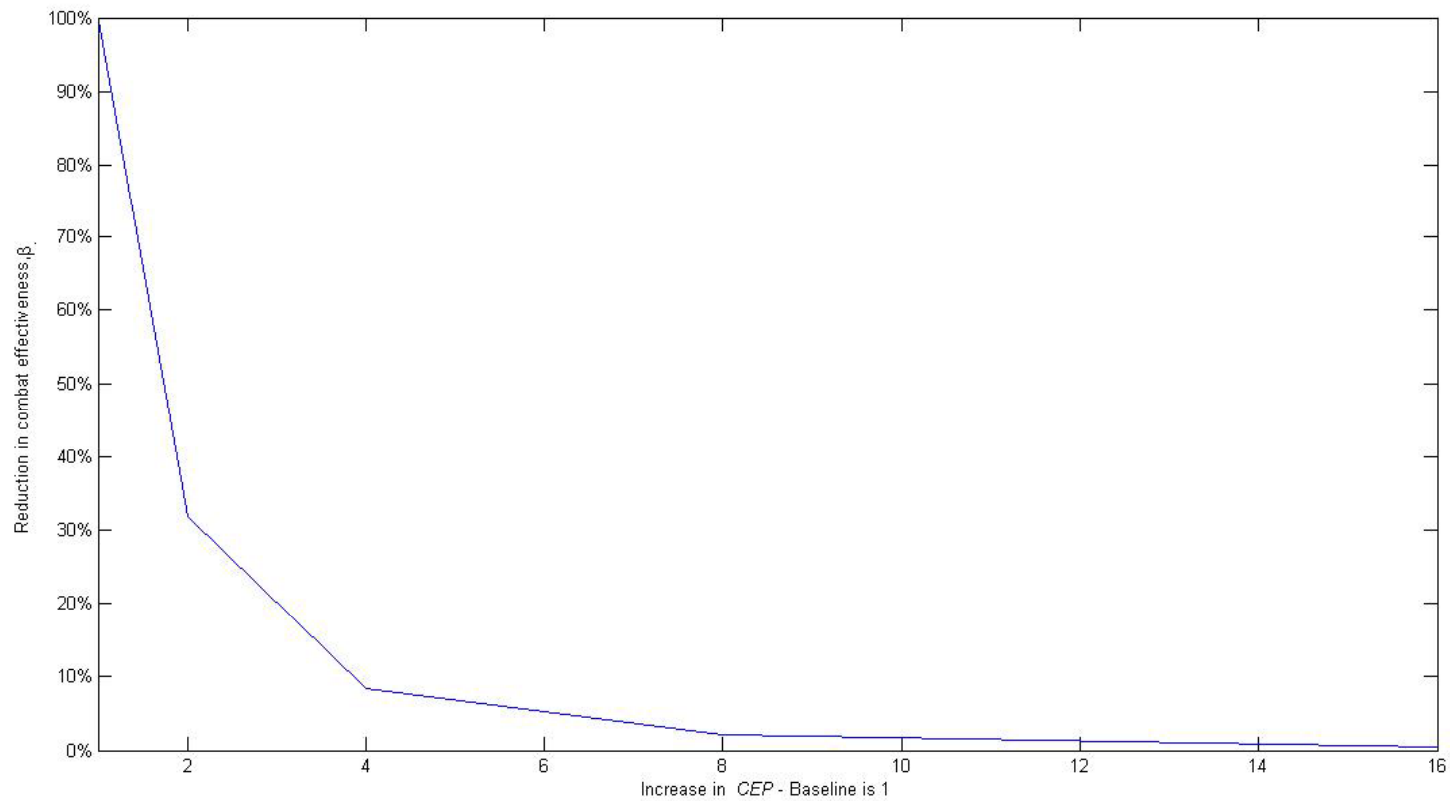
- If ordnance errors are equal and uncorrelated, we may say that they are *circularly distributed*, and

$$\Pr\{R < r\} = 1 - e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2}$$

Where the common unit of error is *Circular Error Probable* (The radius that encloses  $\frac{1}{2}$  of the rounds fired), which may be converted by:

$$\sigma = \frac{CEP}{\sqrt{\ln 4}} \approx \frac{CEP}{1.177}$$

# Reduction in $\beta$ as a function of CEP



# When should we just switch from Aimed to Area fires?

- Let  $\eta$  be the firing rate. For Aimed fire:

$$E[k(h)] = \eta p_{kill|hit} p_{hit} B h$$

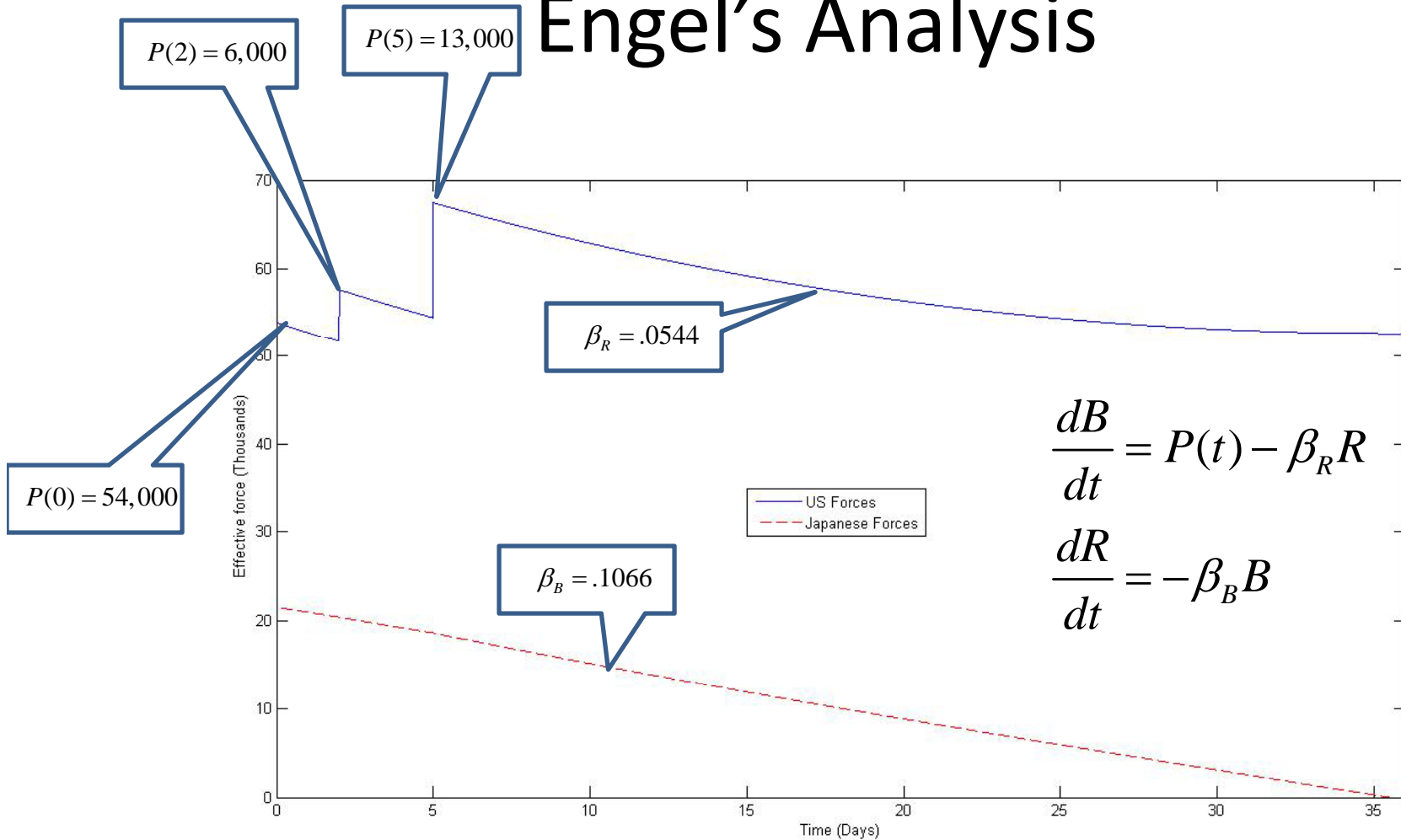
- For Area fire:  $E[k(h)] = \eta p_{kill|hit} \frac{A_L}{A_T} B R h$

- We should prefer aimed fire iff:

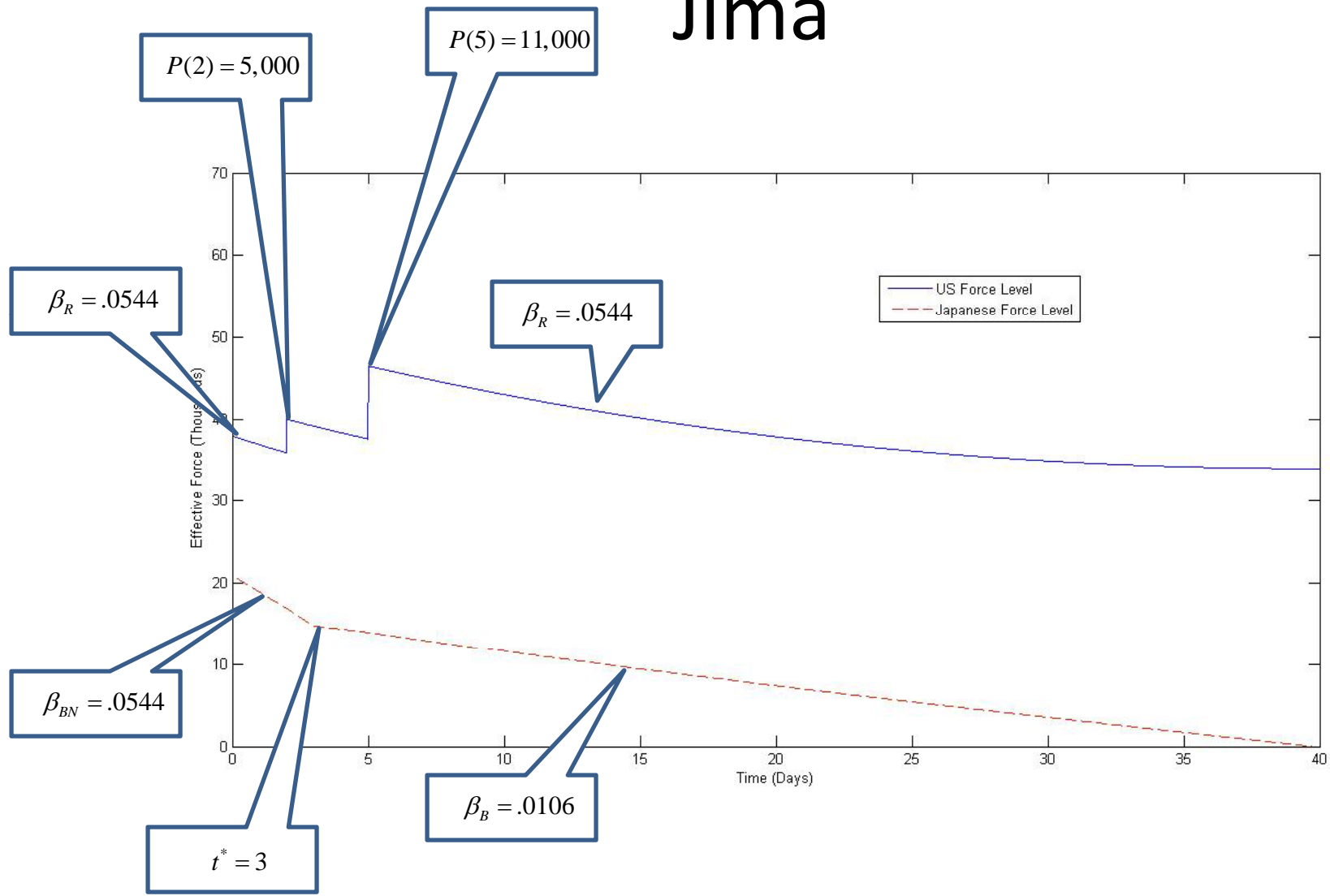
$$\frac{A_L}{A_T} R > p_{hit}$$

# Case Study: The battle of Iwo Jima

## Engel's Analysis



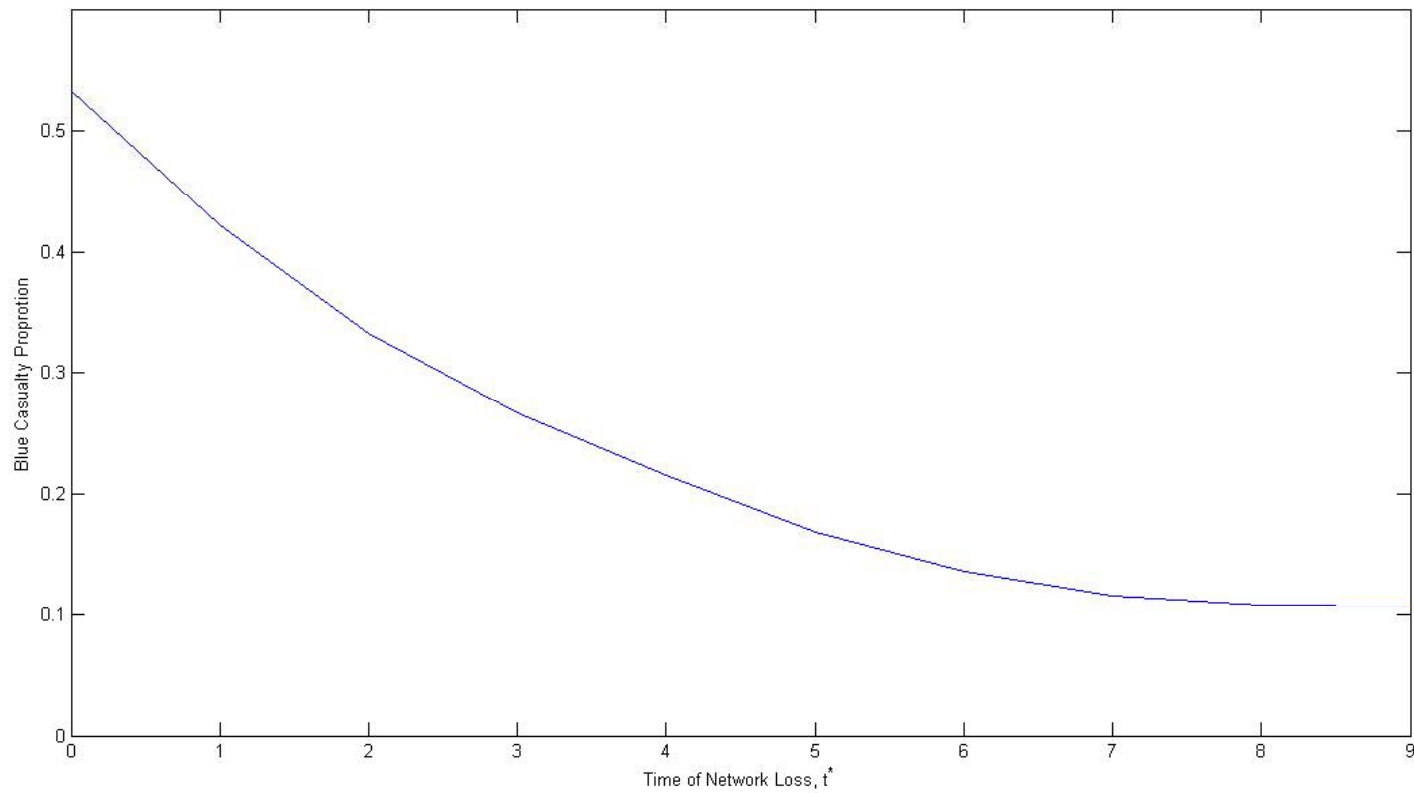
# Case Study II: Networked Battle of Iwo Jima



## We may ask...

- Suppose that Blue has a vulnerable network, but plans like his network was invulnerable, uses Lanchester for his planning and plans for a 10% casualty rate.
- Suppose further that the quality of his network gives him parity with the advantage for being 'dug in'
- We may ask: What's the impact of having a his network fail?

# The Impact of Network Failure





# Conclusion

- Our aim was to:
  - Demonstrate an analytical approach to incorporate the effects of network loss
  - Show that planning for combat with a networked force, and then losing said network, may lead to negative results
  - Demonstrated how a Service may compare the value of network reliability to capability.