The Traveling (Christmas Tree) Salesman Problem

Like many of us, I feel as though I've been an analyst all my life. While it was always part of who I am, it was not my profession until relatively recently. One of the things I got to do on the way "here" was to sell Christmas trees. Since it's the November-December edition of the magazine, I thought I would take some time to think about the purchase of Christmas trees from an analytic point of view.

THE CHRISTMAS TREE EXPERIENCE

When a person goes to buy a Christmas tree, what are they really purchasing? It is not the tree itself but rather the experience of buying the tree. This is why Internet sales of real trees is unlikely to replace the traditional tree lot (but it does exist: www.atreejoyourdoor.com). Getting the Christmas tree is frequently a family affair and a family decision. Let's suppose that there is a particular tree that the salesman wants to sell, perhaps because he gets a better commission, or perhaps he has a bet with other salesmen about whether he can sell it. How does the analytical tree salesman approach this problem?

OPTIMAL SALES STRATEGY

The Christmas tree salesman, if he's an analyst, will quickly observe that the tree decision is a collaborative process. The decision is based on two factors: first, the quality, price, and suitability of a tree; and second, the comfort - read: coldness - of the coldest person. While the first factor is the one that everyone says they use, in practice, the second factor may be more important. So, frequently the purchase is made when one of the family's decision-makers reaches the critical level of discomfort and says "this one" in front of whichever one "this one" happens to be.

Now, family members typically come with the same "kit" - gloves, hat, scarf, coat, boots, but not the same "rig." It was my experience that one member of the family, for ease of exposition we'll call him "Dad," typically comes with a coat buttoned up, hat on, scarf around neck, gloves on, etc., while another family member who we'll call "Mom" has the same gear, but hat and gloves tucked uselessly in oversize pockets. Coat open, scarf fashionably - and uselessly - draped over shoulders.

Let us presume that body temperatures are a proxy for comfort and exposed skin experiences Newton's Law of cooling, i.e. 
\[ T = k(T - T_o) \]
where \( T \) is the outside air temperature. If we presume that the parameter \( k \) is dependent on the amount of exposed skin, with larger areas of exposed skin leading to high values for \( k \), fully dressed out family members will have values of \( k \) near zero.

Immediately, we see that the optimal strategy for family members is to have and properly use warm clothing. We should not be surprised that, as in other analytical problems, zero cost
The problem is not terribly difficult, because the salesman does not have to "hit" exactly the right spot, and he generally controls the speed of the walk.

Optimal strategies are not always adopted in the real world.

Now consider the salesman’s problem. He needs to do three things: First, estimate k for each of the family members; second, convert this into a time; and finally, determine a path through the tree lot that has the family shopping in front of a tree he wants to sell when this time occurs — and not make it obvious that he’s doing so.

The first two pieces of this are straightforward. The third problem might be hard (as in ‘NP’). It is a variation on the Travelling Salesman Problem. In the Travelling (Christmas Tree) Salesman Problem (TCTSP) the salesman needs to select an acyclic, directed path through the tree lot network which arrives at the selected tree at the time that the decision-maker is cold (Figure 1).

I said that the problem might be NP hard, but it turns out that in practice, it is not terribly difficult. This is because the arc times are not fixed, but have some variability; i.e. the salesman can adjust his speed as he walks through the lot, provided that he isn’t too obvious about it. This adds a new degree of freedom to the problem. In fact, the problem has two decision variables: First, the route to take, and secondly, the speed at which to travel. Because it’s an easier day for the tree salesman to walk slower, we’ll say that his objective is to minimize speed (Figure 2).

In practice, this problem is not terribly difficult, because the salesman does not have to “hit” exactly the right spot, and he generally controls the speed of the walk. For example, he will typically only have two or three realistic paths through the tree lot, which he may solve by enumeration (Figure 3).

Of course, running a tree lot is not only about selling the right tree for a commission. There’s also:

**Security.** Christmas trees generally have a high markup — typically around 100 percent. This is not pathological, however, as the trees are generally cheap, and selling them can be expensive. Should Christmas trees be guarded after hours?
Christmas trees are different from other commodities because they have a known, fixed shelf-life. Depending on the climate, the consumer's value of a Christmas tree is a function of the time of purchase.

The rate of "spillage" in our experience was around one to two trees per year. Again, for reasons discussed in the introduction, the customers are paying for the Christmas tree experience, and we figured — probably rightly — that people who were willing to steal a Christmas tree probably needed it so badly that we would have felt sorry for them anyway. The vigilance issues — what is the probability of catching a tree thief on a large lot at 3 a.m.? — combined with the legal/liability issues — what are you going to do about it even if you did? — made security in our particular situation not worth it. Others may, of course, come up with a different solution.

End of the season. Christmas trees are different from other commodities because they have a known, fixed shelf-life. Depending on the climate, the consumer's value of a Christmas tree is a function of the time of purchase. Suppose that the first day that you may buy a tree is Dec. 1. If you live in a cold climate, this brings the tree indoors and risks browning by Christmas day. However, if you live on Guam (as my wife and I did for two Christmases) you are better off to buy a tree immediately: first because of the danger of the lot selling out, but more importantly because your house is cooler than the tree lot, and this prolongs the trees' useful life.

We may agree that Christmas trees become worthless for retail on Dec. 26. Also, there are very few people who go out and buy a tree after Dec. 21. With this in mind, we deeply discounted our trees in the last few days of the season. While someone may wait until Dec. 21 to buy a tree at a discount, this doesn't often happen in practice, and trees left on the lot after Christmas becomes a liability — you have to figure out how to dispose of them. Therefore, at some point in the days immediately before Christmas, the expected value of the trees on the lot goes from positive to negative, and the proprietor would really prefer to give them away. However, there is a limited appetite for discount trees; therefore, the first tree salesman to "fold" as it were, receives a "bonus" in the form of getting rid of trees faster.

SUMMARY

I hope that you have fun shopping for your tree this year. As a retired tree salesman, my preference is Douglas Fir, single-wrap. They have just a little touch of "Charlie Brown" to them, and they smell wonderful in the house. Happy holidays to all and best of luck in 2013!

This article is dedicated to my father, Dale.