Homomorphic Encryption in Maude-NPA

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Beyond the Dolev-Yao Model

Crypto protocol analysis in the standard model free algebra (Dolev-Yao) well understood. But, it fails to account for algebraic identities of crypto operations:

- Diffie-Hellman,
- exclusive-or,
- homomorphism (one-sided distributivity)

used to break a protocol or to specify modern protocols. These operations are beginning to be understood in the bounded sessions case

- Decidability results for exclusive-or, exponentiation, homomorphisms, etc.

What is lacking:

(1) more general understanding, especially for unbounded sessions,

(2) tool support.

Our approach

- Use rewriting logic as general theoretical framework
  - protocols and intruder rules specified as rewrite rules
  - crypto properties as oriented equational properties and axioms
- Use narrowing modulo equational theories in two ways
  - as a symbolic reachability analysis method
  - as an extensible equational unification method
- Combine with state reduction techniques (grammars, optimizations, etc.)
- Implement in Maude programming environment
  - Rewriting logic gives us theoretical framework and understanding
  - Maude implementation gives us tool support

Maude-NPA

- A tool to find or prove the absence of attacks using backwards search
- Analyzes infinite state systems:
  - Active Dolev-Yao intruder
  - No abstraction or approximation of nonces
  - Unbounded number of sessions
- Intruder and honest protocol transitions using variant of strand space model: strands with a marker denoting the current state
  - Searches backwards through strands from final state.
  - Set of rewrite rules governs how search is conducted
  - Sensitive to past and future

Our Plans

(1) Start by formalizing NPA techniques in rewriting logic (done)

- Prove soundness and completeness theorems (done)
- Implement in Maude (the Maude-NPA tool) (done)

(2) Include state reduction techniques present in NPA and new (done)

(3) Document and distribute the tool (done)

(4) Extend model to different types of equational theories

- Explicit Encryption and Decryption (done)
- Bounded Associativity (done)
- AC-unification (done)
- Diffie-Hellman Exponentiation (done)
- Exclusive-or (done)
- Homormophism (one-sided distributivity) (current)

Explicit Encryption and Decryption

- Most formal models lack explicit decryption operator
- If a principal knows an encrypted message and a key, assume principal can decrypt message
  - Implicit assumption that principal never decrypts a message that wasn't encrypted in the first place
  - Usually justified by assumption that principals can check format of decrypted message
- What if format checking isn't implemented? Or what if it is, but you are trying to verify that it works properly?
- In that case, need to model both encryption and decryption explicitly, plus their cancellation, e.g. $d(K, e(K, Y)) = Y$.

Modular Exponentiation in Diffie-Hellman

- Basic DH protocol (each nonzero residue mod P is a power of g)
  1. $A \rightarrow B : g^{N_A}\ mod\ P$

     B computes $(g^{N_A})^{N_B}\ mod\ P$
  2. $B \rightarrow A : g^{N_B}\ mod\ P$

     A and B compute $(g^{N_B})^{N_A} = (g^{N_A})^{N_B}\ mod\ P$ and get a shared secret key.
- Properties:

$$(g^X)^Y = g^{X*Y} = g^{Y*X} = (g^Y)^X$$

$$(X * Y) * Z = X * (Y * Z) \qquad X * Y = Y * X$$

of modular exponentiation in order to faithfully represent this protocol

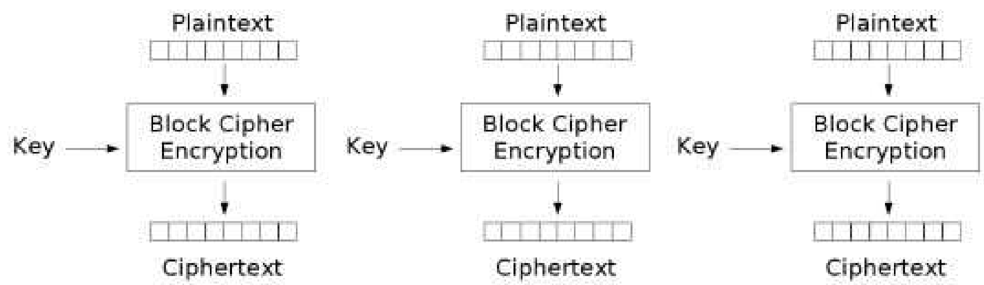
Exclusive-Or

- Cheap and has provable security properties
  - If we send $X \oplus R$, where R a random secret, observer learns no more about X than before it saw message
- On the other hand, commutativity and cancellation properties make it tricky to reason about

$$
\begin{array}{ll}
X \oplus Y = Y \oplus X & X \oplus X = 0 \\
(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) & X \oplus 0 = X
\end{array}
$$

Homomorphism

- The electronic codebook (ECB) encryption splits a message into blocks and cyphers the blocks using the same key



Electronic Codebook (ECB) mode encryption

- Identical plaintext blocks are encrypted into identical ciphertext blocks (does not hide data patterns well). Sensitive to the property:

$$e(K, X; Y) = e(K, X); e(K, Y)$$

Outline

① Introduction to Rewriting Logic and Narrowing
② Equational Unification
③ Maude-NPA Integration & Demo
④ Conclusions & Future Work

Rewriting Logic in a Nutshell

A rewrite theory $\mathcal{R}$ is a triple $\mathcal{R} = (\Sigma, E, R)$, with:

- (Σ, R) a set of rewrite rules of the form $t \to s$ (i.e., protocol transitions)
  e.g. $e(K, N_A; X) \to e(K, X)$
- (Σ, E) a set of equations of the form $t = s$ (i.e., cryptographic properties)
  e.g. $d(K, e(K, Y)) = Y$

Intuitively, $\mathcal{R}$ specifies a concurrent system,
whose states are elements of the initial algebra $T_{\Sigma/E}$ specified by (Σ, E), and
whose concurrent transitions are specified by the rules R.

R, E-rewriting

Let R a set of rewrite rules and E an equational theory

Rewriting: $t \rightarrow_{R,E} s$ if there is

- a non-variable position $p \in Pos(t)$;
- a rule $l \rightarrow r \in R$;
- a matching σ (modulo E) such that $t|_p =_E \sigma(l)$, and $s = t[\sigma(r)]_p$.

Example:

- $R = \{X \oplus X \rightarrow 0, X \oplus 0 \rightarrow X, X \oplus X \oplus Y \rightarrow Y\}$
- $E = \{X \oplus Y = Y \oplus X, (X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)\}$
- $(a \oplus b) \oplus a \quad \rightarrow_{R,E} \quad b$

Narrowing and Backwards Narrowing

Narrowing: $t \rightsquigarrow_{\sigma,R,E} s$ if there is

- a non-variable position $p \in Pos(t)$;
- a rule $l \to r \in R$;
- a unifier σ (modulo E) such that $\sigma(t|_p) =_E \sigma(l)$, and $s = \sigma(t[r]_p)$.

Example:

- $R = \{\ e(K, N_A; X) \to e(K, X)\ \}$
- $E = \{\ d(K, e(K, Y)) = Y\ \}$
- $$\begin{array}{lll} e(k,\ X) & \rightsquigarrow_{\{X \mapsto N_A;X'\},R,E} & e(k, X') \\ d(k,\ X) & \rightsquigarrow_{\{X \mapsto e(k,e(K,N_A;X'))\},R,E} & e(K, X') \end{array}$$

Backwards Narrowing: narrowing with rewrite rules reversed

Narrowing Reachability Analysis

Narrowing can be used as a general deductive procedure for solving reachability problems of the form

$$(\exists \vec{x})\; t_1(\vec{x}) \to t'_1(\vec{x}) \;\wedge \ldots \wedge\; t_n(\vec{x}) \to t'_n(\vec{x})$$

in a given rewrite theory.

- The terms t_i and t'_i denote sets of states.
- For what subset of states denoted by t_i are the states denoted by t'_i reachable?
- No finiteness assumptions about the state space.
- Sound and complete for topmost rewrite theories.

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A Little Background on Unification

- Given a signature Σ and an equational theory E, and two terms s and t built from Σ:
- A unifier of s and t is a substitution σ to the variables in s and t such that σs can be transformed into σt by applying equations from E to s and its subterms
- Example: $\Sigma = \{d/2, e/2, m/0, k/0\}, E = \{d(K, e(K, X) = X\}$. The substitution $\sigma = \{X/e(K, Y)\}$ is a unifier of $d(K, X)$ and Y.
- The set of most general unifiers of s and t is the set Γ such that any unifier σ is of the form $\rho\tau$ for some ρ, and some τ in Γ.
- Example, $\{X/e(K, Y), Y/d(K, X)\}$ is the set of mgu's of $e(K, X)$ and Y.

- Given the theory, can have:
  - at most one mgu (empty theory)
  - a finite number (AC)
  - an infinite number (associativity)
- Problem in general undecidable, so different algorithms devised for different theories
- Compare to syntactic unification:
  - $f(a, X) = f(Y, b)$ has solution $X \mapsto b, Y \mapsto a$
  - $f(a, X) = f(b, Y)$ has no solution
  - $f(a, X) =_{AC} f(b, Y)$ has solution $X \mapsto b, Y \mapsto a$
  - $X + 0 = X$ has no solution
  - $X + 0 =_{ACU} X$, where 0 is the identity, has solution id

Equational Unification

- Equational unification is the solving of formulas $\exists \vec{x}\ t =_E t'$
- When E convergent TRS, narrowing provides a complete E-unification procedure [Hullot80] e.g. cancellation $d(K, e(K, Y)) = Y$
- When $E = \Delta \uplus B$ and Δ convergent and coherent modulo B, narrowing provides a complete E-unification procedure [Jouannaud-Kirchner-Kirchner-83] e.g. exclusive-xor

$$(g^X)^Y = g^{X*Y} = g^{Y*X} = (g^Y)^X$$

$$(X * Y) * Z = X * (Y * Z) \qquad X * Y = Y * X$$

- Narrowing provides semi-decidable E-unification procedure, since it may not terminate even for simple cases

Equational Unification - Strategies

- Narrowing is very inefficient. Strategies have been studied in the literature.
- When E convergent TRS, basic narrowing strategy [Hullot80] is complete for normalized substitutions.
- Cases where basic narrowing terminates have been studied in order to provide decidable E-unification procedures.
- When $E = \Delta \uplus B$ and Δ convergent and coherent modulo B, no many strategies have been studied in practice and AC-narrowing is highly non-terminating.
- Variant-narrowing [Escobar-Meseguer-Sasse] is the most promising strategy for equational unification. It is partially implemented in Maude-NPA.

Variant-Narrowing

1. Complete narrowing strategy modulo axioms B with smaller search space than full B-narrowing.
2. Decidable narrowing-based E-unification procedure

- Based on E-*variant* of [Comon-Delaune-RTA05], we define variant narrowing strategy:
  1. it only uses substitutions in normal form
  2. complete under very general assumptions on B and Δ
  3. if Δ has the finite variant property of [Comon-Delaune-RTA05],
     (a) compute all E-variants of a term in a space-effective way
     (b) obtain a finitary E-unification procedure

Equational Theories & Finite Variant Property

1. [Escobar-Meseguer-Sasse-TechRep07]
   $Lhs \to Rhs$ where Rhs is a variable or a constant (bound 1)
   plus some extra conditions
2. [Comon-Delaune-RTA05]
   Exclusive Or (max. bound 1)
3. [Comon-Delaune-RTA05]
   Abelian group (max. bound 2)
4. [Comon-Delaune-RTA05]
   Diffie-Hellman (max. bound 4)
5. [Comon-Delaune-RTA05]
   Homomorphism (NOT)

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Protocol Verification: Maude-NPA

- Maude-NPA uses backwards search from an insecure state to find attacks or to prove unreachability of cryptographic protocols $(\Sigma, \Delta \uplus B, R)$
- Narrowing at two levels in Maude-NPA
  1. a theory $(\Sigma, \Delta \uplus B, R)$: ($\Delta \uplus B$-narrowing with rules R)
  2. for $\Delta \uplus B$-unification (B-narrowing with rules Δ)
- $\Delta \uplus B$-unification for each backwards step using R
  1. Built-in unification algorithms desirable
  2. Our hybrid approach: built-in algorithms for B, and a generic algorithm (variant narrowing) for Δ.

Demo

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Conclusions

- Equational unification is critical for cryptographic protocol analysis
- Equational unification can be:
  - unitary (empty theory),
  - finitary (AC),
  - infinitary (associativity)
- Equational unification in Maude-NPA:
  - built-in unification algorithms
    (AC –Core Maude–, homomorphism –Meta Level–),
  - our hybrid approach: built-in algorithms for B, and a generic algorithm (variant narrowing) for Δ.
- When a theory E has the finite variant property (modulo AC), variant narrowing provides an efficient equational unification algorithm

Future work

- Prototype implementation developed (already used in Maude-NPA)
- Homomorphism algorithm (from Narendran-Lynch) does not support AC properties but Maude-NPA relies on that for states $\Rightarrow$ infrastructure for combining different unification procedures (AC & homomorphism)
- Homomorphism algorithm (from Narendran-Lynch) does not support sorts (order-sorted) $\Rightarrow$ order-sorted filter (Hendrix-Meseguer)
- Is there variant-narrowing modulo homomorphism possible??? (e.g. homomorphism with exclusive-or)