

# Maude-NPA: Status and Demonstration

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## Goal

- Crypto protocol analysis with the **standard free algebra model** (Dolev-Yao) well understood.
- **Extend** standard free algebra model of crypto protocol analysis to deal with **algebraic properties**
  1. Encryption-decryption,
  2. Diffie Hellman,
  3. Exclusive-or, etc.
- Provide **tool** that can be used to reason about protocols with these **algebraic properties** in the **unbounded** session model

## Our approach

- Use **rewriting logic** as general theoretical framework
  - crypto protocols are specified as **rewrite rules**
  - algebraic identities as **equational properties**
- Use narrowing modulo equational theories as a **symbolic reachability analysis method**
- Combine with state reduction techniques of NPA (grammars, optimizations, etc.)
- Implement in **Maude** programming environment
  - Rewriting logic gives us **theoretical framework** and understanding
  - Maude implementation gives us **tool support**

## Maude-NPA

- A tool to **find** or **prove the absence** of attacks using **backwards search**
- Analyzes **infinite state systems**
  - **Active intruder**
  - **No abstraction** or **approximation** of nonces
  - **Unbounded** number of sessions
- **Intruder** and **honest** protocol transitions represented using strand space model.
- Different algebraic theories included
- Uses **induction techniques** defined in terms of formal languages to cut down search space
- Uses **optimization** techniques to improve performance: only input messages, partial order, information from strand space model, lazy intruder, etc.

## Our Plans

- (1) Formalizing techniques in rewriting logic (**DONE**)
  - Initial version of Maude-NPA tool:
    - Grammar Generation
    - Backwards Narrowing Reachability Analysis
    - Soundness and completeness theorems
  - Paper published in TCS special issue on ARSPA
- (2) Include **state reduction** techniques present in **NPA and new** (**CURRENT**)
  - Optimizations for state reduction.
- (3) Extend model to different types of **equational theories** (**CURRENT**)
  - Bounded Associativity (SecReT 2006)
  - Diffie-Hellman Exponentiation (SecReT 2007)
  - AC version of Maude-NPA implemented via CiME tool and ported to Maude with AC Unification (**DONE**)
  - Extend grammars and state reduction techniques to equational theories (**CURRENT**)

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## Progress Since Last PX Meeting

### ① Unification Issues

- Ported Maude NPA to Steven Eker's Maude with AC unification
- Addressing nontermination of narrowing modulo AC with finite variant property

### ② Language Issues

- New specification and query language that human beings can use
- Approaches to Automatic Generation of Seedterms

### ③ Search Termination Issues

- Folding
- Grammars

### ④ Demos

- NSPK with "Never found any"
- Diffie-Hellman

## A Little Background on Unification

- Given a signature  $\Sigma$  and an equational theory  $E$ , and two terms  $s$  and  $t$  built from  $\Sigma$ :
- A **unifier** of  $s$  and  $t$  is a substitution  $\sigma$  to the variables in  $s$  and  $t$  such that  $\sigma s$  can be transformed into  $\sigma t$  by applying equations from  $E$  to  $s$  and  $t$  and their subterms
- Example:  $\Sigma = \{d/2, e/2, m/0, k/0\}$ ,  $E = \{d(K, e(K, X)) = X\}$ . The substitution  $\sigma = \{X/e(K, Y)\}$  is a unifier of  $d(K, X)$  and  $Y$ .
- The set of **most** general unifiers of  $s$  and  $t$  is the set  $\Gamma$  such that any unifier  $\sigma$  is of the form  $\rho\tau$  for some  $\rho$ , and some  $\tau$  in  $\Gamma$ .
- Example,  $\{X/e(K, Y), Y/d(K, X)\}$  is the set of mgu's of  $e(K, X)$  and  $Y$ .
- Given the theory, can have:
  - at most one mgu (empty theory)
  - a finite number (AC)
  - an infinite number (associativity)
- Problem in general undecidable, so different algorithms devised for different theories

## Narrowing

Let  $\sigma$  be a substitution,  $R$  a set of rewrite rules and  $E$  an equational theory

Narrowing:  $t \rightsquigarrow_{\sigma, R, E} s$  if there is

- a non-variable position  $p \in Pos(t)$ ;
- a rule  $l \rightarrow r \in R$ ;
- a unifier  $\sigma$  (modulo  $E$ ) such that  $\sigma(t|_p) =_E \sigma(l)$ , and  $s = \sigma(t[r]_p)$ .

Example:

- $R = \{ X \rightarrow d(k, X) \}$
- $E = \{ d(K, e(K, Y)) = Y \}$
- $e(k, t) \rightsquigarrow_{\emptyset, R, E} d(k, e(k, t)) =_E t$



## ***E*-Unification and Narrowing**

- Maude-NPA based on unification modulo equational theory defining the behavior of different operations used
- Two possible approaches:
  1. **Built-in unification** algorithms for each theory and combination of theories.
  2. **Hybrid** approach with  $\Delta$  and  $B$ 
    - $B$  is **built-in** unification algorithm
    - $\Delta$  confluent and terminating **rules modulo  $B$** 
      - \* Confluent: Always reach same normal form, no matter in which order you apply rewrite rules
      - \* Terminating: Sequence of rewrite rules is finite
    - Implement unification via narrowing with  $\Delta$  modulo  $B$ .
    - More readily extensible to different theories.
- Our Approach
  - Let  $B$  be the empty theory or AC
  - Old and new approaches
    - \* Old: Unification modulo  $B$  performed via calls to CiME unification tool
    - \* New: Unification modulo  $B$  provided by Maude
  - In both cases, narrowing with  $\Delta$  performed at Maude meta-level

## Order-sorted Unification

- In 2007, order-sorted unification and AC unification implemented in Maude by Steven Eker
- MaudeNPA ported to new AC version of Maude this summer
- Running time of Maude with CIME AC vs. AC Maude is approximately 1.3 on our benchmarks
  - External calls to CIME no longer necessary
  - Conversion from unsorted to sorted unification no longer necessary
  - True even for non-AC theories because because state is a set of strands, set obeys ACU
- We expect the ratio to go up as the number of AC operations increases.

## When Narrowing Doesn't Terminate: Finite Variant Property

- Problem: Even typed narrowing mod AC is **nonterminating** for several theories of interest
  - Example: exclusive-or
- Investigating the **finite variant property** of Comon and Delaune
- Finite variant property: have an algorithm that computes, from any term  $t$ , a finite set of substitutions  $\sigma_1, \dots, \sigma_n$  such that

$$\{t\sigma \downarrow \mathcal{R} \mid \sigma \in \Sigma\} = \bigcup_{i=1}^n \{t\sigma_i\theta \mid \theta \in \Sigma\}$$

- Finite variant property means that can compute a bound on the number of narrowing steps necessary to get a complete solution
- Even better, don't even need to narrow, just compute all finite variants, try to unification modulo AC using each one
  - Already have something like this for parts of MaudeNPA that require term to be in reduced form
- Allows us to maintain hybrid approach while expanding scope of theory
- Have begun experimental implementation, already have implementation of ACU

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## Parts of a MaudeNPA Specification

- Information About Message Components
- Strand Specifications
- Attack States
- Seed Terms

## Sorts

-----  
--- We modify only the relevant MAUDE-NPA modules  
-----

```
fmod PROTOCOL-EXAMPLE-SYMBOLS is
  --- Importing sorts Msg, Fresh, Public, and GhostData
  protecting DEFINITION-PROTOCOL-RULES .
```

-----  
--- Overwrite this module with the syntax of your protocol  
--- Notes:  
--- \* Sort Msg and Fresh are special and imported  
--- \* Every sort must be a subsort of Msg  
--- \* No sort can be a supersort of Msg  
-----

```
--- Sort Information
  sorts Name Nonce Key Enc .
  subsort Name Nonce Enc Key < Msg .
  subsort Name < Key .
  subsort Name < Public .
```

## Operations

--- Encoding operators for public/private encryption

op pk : Key Msg -> Enc [frozen] .

op sk : Key Msg -> Enc [frozen] .

--- Nonce operator

op n : Name Fresh -> Nonce [frozen] .

--- Principals

op a : -> Name . --- Alice

op b : -> Name . --- Bob

op i : -> Name . --- Intruder

--- ConcatenationBa operator

op \_;\_ : Msg Msg -> Msg [gather (e E) frozen] .

endfm

## Algebraic Theory

```
fmod PROTOCOL-EXAMPLE-ALGEBRAIC is  
  protecting PROTOCOL-EXAMPLE-SYMBOLS .
```

```
var Z : Msg .  
var Ke : Key .
```

```
*** Encryption/Decryption Cancellation  
eq pk(Ke,sk(Ke,Z)) = Z [nonexec] .  
eq sk(Ke,pk(Ke,Z)) = Z [nonexec] .
```

```
endfm
```



## Intruder Strands

```
fmod USER-INPUT is
  protecting PROTOCOL-EXAMPLE-SYMBOLS .
  protecting DEFINITION-PROTOCOL-RULES .
  protecting DEFINITION-CONSTRAINTS-INPUT .

var Ke : Key .
  vars X Y Z : Msg .
  vars r r' : Fresh .
  vars A B : Name .
  vars N N1 N2 : Nonce .

eq STRANDS-DOLEVYAO
  = :: nil :: [ nil | -(X), -(Y), +(X ; Y), nil ] &
    :: nil :: [ nil | -(X ; Y), +(X), nil ] &
    :: nil :: [ nil | -(X ; Y), +(Y), nil ] &
    :: nil :: [ nil | -(X), +(sk(i,X)), nil ] &
    :: nil :: [ nil | -(X), +(pk(Ke,X)), nil ] &
    :: nil :: [ nil | +(A), nil ]
[nonexec] .
```

## Protocol Strands

```
eq STRANDS-PROTOCOL
= :: r ::
  [ nil | +(pk(B,A ; n(A,r))), -(pk(A,n(A,r) ; N)), +(pk(B, N)), nil ] &
  :: r ::
  [ nil | -(pk(B,A ; N)), +(pk(A, N ; n(B,r))), -(pk(B,n(B,r))), nil ]
[nonexec] .
```

## Attack States

```
eq ATTACK-STATE(2)
= :: r ::
  [ nil, -(pk(b,a ; N)), +(pk(a, N ; n(b,r))), -(pk(b,n(b,r))) | nil ]
  || n(b,r) inI, empty
  || nil
  || nil
butNeverFoundAny
:: r' :: [nil | +(pk(b,a ; N)), -(pk(a, N ; n(b,r))), +(pk(b,n(b,r))), nil ]
        & :: nil :: [nil | -(N1 ; N2), +(pk(B, N1 ; N2)), nil] & S:StrandSet
  || K:IntruderKnowledge
  || M:SMsgList
  || G:GhostList
[nonexec] .
```

- **butNeverFoundAny** can be used to define insecure state and to rule out actions leading to infinite paths

## Seed Terms for Grammars

```
eq USER-GRAMMARS
  = (gr1 Y notInI => (X ; Y) inL . ! S1)
    |
    (gr1 X notInI => (X ; Y) inL . ! S1)
    |
    (gr1 Z notInI => pk(Ke,Z) inL . ! S1)
    |
    (gr1 Z notInI => sk(Ke,Z) inL . ! S1)
  [nonexec] .
```

## Quick and Dirty Suggestions for Automating Seed Term Generation

- Give  $X \in \mathcal{I}$  as a goal to MaudeNPA, where  $X$  is a variable
- Search back one step to find all terms  $T$  intruder must know to find  $X$
- For each such  $T$  containing  $X$  as a proper subterm, generate a seedterm

```
gr1 X notInI => T inL . ! S1
```

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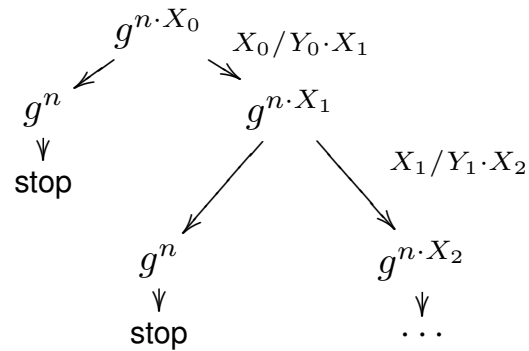
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## Infinite Behavior Not Captured by Grammars: 1



- Different from rewrite-rule based grammar behavior, because infinite behavior results from substitution
- Root term grows larger instead of leaf terms

## Infinite Behavior Not Captured by Grammars: 2

- Consider the following protocol
  1.  $A \hookrightarrow B : \text{mess}(A, r)$ .
  2.  $B \hookrightarrow A : \text{mess}(B, r')$ .
- If we ask how intruder can find  $\text{mess}(A_0, r_0)$  we run into the following:
$$\dots, -\text{mess}(A_2, r_2), +\text{mess}(A_1, r_1), -\text{mess}(A_1, r_1), +(\text{mess}(A_0, r_0), -\text{mess}(A_0, r_0))$$
- Possible to find much more complex examples
- Only a problem if have unbounded number of sessions



## Investigating Folding As a Solution

- While searching, collapse any state encountered into previously seen state if some criterion satisfied
  - Examples:  $=_E$ ,  $\approx_E$ ,  $\preceq_E$
- Basic idea is that reachability of old state implies reachability of new state, so no need for new state
- Original NPA did something very similar to this (although not as general), so we think this is likely to be successful
- Only drawback: Maude NPA will have to remember old states
- One possibility, only check  $k$  states back for some  $k$
- Reference: Santiago Escobar, Jos Meseguer Symbolic Model Checking of Infinite-State Systems Using Narrowing In proceedings of 18th International Conference on Rewriting Techniques and Applications (RTA 2007), LNCS volume 4533, pages 153-168, 2007.

## Grammar Termination

- Need to understand how grammars contribute to termination
- Question investigating now: to what degree can grammars contribute to termination in bounded session case
- It all depends on the exceptions:
  - `gr1 #0:Msg notInI, (#0:Msg notLeq pk( #2:Key, #3:Msg)) => pk( #7:Key, #0:Msg) inL .`  
leads to nontermination
  - `gr1 #0:Msg notInI, (#0:Msg notLeq #2:Name ; n(#2:Name, #3:Fresh) ) => pk( #7:Key, #0:Msg) inL` leads to termination
  - `gr1 #0:Msg notInI, (#0:Msg notLeq #2:Key, #3:Msg) => pk( #7:Key, #0:Msg) inL .`  
`gr1 #0:Msg notInI (#0:Msg notLeq n(#2:Name, #3:Fresh)) => #7:Msg, #0:Msg inL .` leads to termination
- Have defined the notion of a **exception rank** of a grammar, that can be used to compute the maximum size of a backwards search using intruder strands only
  - In above two cases, exception ranks were infinite, one, and two, respectively
- Exception rank gives a bound on the number of consecutive intruder strands that can be applied in a backwards search, also helps us to optimize search

## Questions

- How do we guarantee finite exception rank?
- How do we generalize exception rank for AC grammars?

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