The Nonhydrostatic Unified Model of the Atmosphere (NUMA): CG Dynamical Core

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## Overview of Existing Global Models

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Design Philosophy (2)

**Unified Dynamics**
- All limited-area models are nonhydrostatic. Resolutions of global models are approaching the nonhydrostatic limit (~10 km).
- Both limited-area and global models utilize the same equations.
- Engineer a common dynamical core for both models, then change grids, force, and boundary conditions.

**Unified Numerics**
- CG is more efficient for smooth problems at low processor counts.
- DG is more accurate for problems with sharp gradients and more efficient at high processor counts.
- Both EBGs utilize a common mathematical arsenal.
- NUMA allows the user to choose either CG or DG for the problem at hand.

**Unified Code**
- Code is *modular*, with a common set of data structures.
- New time-integrators, grids, basis functions, physics, etc. may be swapped in and out with ease.
- Code is portable: Successfully installed on Apple, Sun, Linux, and IBM.
Non-Conservative Form (2NC)

- Consider the compressible Euler in Cartesian coordinates (not spherical).
- Mass is conserved and energy can be conserved (up to time-truncation)

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \\
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho} \nabla P + g \hat{k} + \mathbf{f} \times \mathbf{u} = 0 \\
\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = 0 \\
P = P_A \left( \frac{\rho R \theta}{P_A} \right)^\gamma
\]

- The same model may be used for: global, limited-area, and urban-scale modeling (requires different grids, initial conditions, boundary conditions, and physics).
Example of 3D Grids

- NUMA runs in either Mesoscale (Limited-Area) or Global Mode.
- Currently, any (hexahedra-based) grid can be used including completely unstructured grids.
- Parallel Domain Decomposition handled by METIS.
Domain Decomposition via METIS

1: Decomposition of a 3D Cartesian domain using 64 spectral elements (fourth order).

2: Decomposition of a “cubed sphere” using 96 spectral elements (fourth order).

3: Decomposition of an “icosahedral sphere” using 96 elements (fourth order).
Rising Thermal Bubble: 3D
Orographic Flows: Linear Hydrostatic and Nonhydrostatic Mountain (3D)

- Flow of $U=20$ m/s in an isothermal atmosphere.
- LH Mountain: Solid of revolution of Witch of Agnesi: Mountain height = 1 m with radius 10 km.
- Radiating boundary condition implemented on lateral and top boundaries using an absorbing sponge (Rayleigh friction).
Linear Hydrostatic Mountain (3D)

u (m/s) t = 1200 s
v (m/s) t = 1200 s
w (m/s) t = 1200 s

u (m/s) t = 3600 s
v (m/s) t = 3600 s
w (m/s) t = 3600 s
3D Linear Hydrostatic Mountain (Verification)

- Mountain case tests orography and sponge BC.
- Analytical approximations for flow over a LHM available in Smith (1980) on mountain surface.
- Decent agreement between theory and numerical model for downstream and cross-stream velocity perturbations.
Results (Global): Pressure Wave

- Proposed by Tomita and Satoh (2004) to test nonhydrostatic global atmospheric models.
- A pressure perturbation is applied to a hydrostatic, isothermal atmosphere

\[ p' = \Delta p f(\lambda, \phi)g(z) \]

\[ f(\lambda, \phi) = \frac{1}{2} H(R - r) \left[ 1 + \cos \left( \frac{\pi r}{R} \right) \right] \]

\[ r = a \cos^{-1} (\cos \phi \cos \lambda) \]

\[ g(z) = \sin \left( \frac{\pi z}{z_T} \right) \]

- Top Level \( z_T = 10 \text{ km} \)
- Cubed Sphere Grid with 6 x 12 x 12 x 6 = 5184 elements using 4th order polynomials (~210 km horz. Resolution and 0.5 km vertical resolution)
- Model run using Semi-Implicit TI w/ 5 s time-step
Results (Global): Inertia-Gravity Wave

Potential Temperature after 48 hours
with (240x0.5) km resolution
Urban-Scale Modeling
CG: Scalability (Explicit + Semi-Implicit)

Moderate Processor Counts: 110592 elements

Large Processor Counts: 262144 elements

Time-Steps

Explicit RK35: $\Delta t = 0.001$ s

SI BDF2: $\Delta t = 0.01$ s
Summary and Outlook

• NUMA Dynamical Core is quite mature:
  – 3D and MPI.
  – Can use either CG or DG methods.

• We will continue testing the dry dynamics and improve the model to make it as user-friendly as possible.

• Future Projects:
  – We are implementing sub-grid scale parameterizations to NUMA
  – Need more resources added to implement sub-grid scale parameterization, data assimilation, testing, etc.
Conservative Form (2C)

• Mass is conserved but not energy.
• Conservation (or flux) form is required by DG machinery.

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{U} = 0
\]

\[
\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \left( \frac{\mathbf{U} \otimes \mathbf{U}}{\rho} + P\mathbf{I} \right) + \rho g \hat{k} + \mathbf{f} \times \mathbf{U} = 0
\]

\[
\frac{\partial \Theta}{\partial t} + \nabla \cdot \left( \frac{\Theta \mathbf{U}}{\rho} \right) = 0
\]

\[
P = P_A \left( \frac{R\Theta}{P_A} \right)^\gamma
\]

\[
\mathbf{U} = \rho \mathbf{u}
\]

\[
\Theta = \rho \theta
\]
2D Inertia-Gravity Wave

• Standard test suite for 2D mesoscale problems (Skamarock, Doyle, et. al.)

• Potential temperature perturbation after 3000 s for 250 m resolution (120 by 1 by 4 elements) and 10-th order polynomials.

• 2D and 3D results agree to 8 decimal places.

CG-NUMA run with 1 element in y-direction.

2D Density Current results for Giraldo and Restelli (2008).
Results (Global): Pressure Wave

- Acoustic (perturbation) field shown at central vertical level (5 km).
- Wave returns to initial state after about 32 hours, yielding a speed of sound of 348 m/s.
Non-hydrostatic Adaptivity Examples
(Müller, Behrens, Giraldo, Wirth 2010)

Rising Thermal Bubbles

Two (Warm/Cold) Thermal Bubbles
Rising Thermal Bubble
(Global)
## Overview of Existing Limited-Area Models

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Design Philosophy (1)

Numerical Methods

DG (Discontinuous Galerkin)

CG (Continuous Galerkin)

Limited-Area (Mesoscale)  Global Scale

MPI Dry core under development.

MPI Dry core developed. Physics will be added this year.
Performance of the NUMA Model
(3D Rising Thermal Bubble)

- Continuous Galerkin
- Discontinuous Galerkin

16 Million Grid Points