A robust optimization approach to hybrid microgrid operation using ensemble weather forecasts

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HIGHLIGHTS

- Scenario-robust optimization provides superior solutions for microgrid operations.
- Ensemble weather forecasts provide valuable input data for planning models.
- Simulation techniques can generate large amounts of realistic data for testing.
- Longer planning horizons do not necessarily lead to better plans.

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ABSTRACT

Hybrid microgrids that use renewable energy sources can improve energy security and islanding time while reducing costs. One potential beneficiary of these systems is the U.S. military, which can seek to improve energy security when operating in isolated areas by using a microgrid rather than relying on a fragile (or nonexistent) commercial network. Renewable energy sources can be intermittent and unpredictable, making it difficult to plan operations of a microgrid. We describe a scenario-robust mixed-integer linear program designed to utilize ensemble weather forecasts to improve the performance of a hybrid microgrid containing both renewable and traditional power sources. We exercise our model to quantify the benefit of using ensemble weather forecasts, and we predict the optimal performance of a hypothetical grid containing wind turbines by using simulated realistic weather forecast scenarios based on data. Because forecast quality degrades with lead time, we perform a sensitivity analysis to determine which planning horizon results in the best performance. Our results show that, for day-ahead planning, longer planning horizons outperform shorter planning horizons in terms of cost of operations, but this improvement diminishes as the planning horizon lengthens.

1. Introduction

In recent years, the utilization of renewable energy sources (RESs) has become more widespread mainly due to the (1) world's growing population and respective increase in the energy requirement, (2) depletion of the world fossil energy resources, (3) need for more flexible and reliable energy sources at lower costs, and (4) smaller environmental footprint requirements [1]. For such reasons, energy policies often promote energy efficiency measures, such as combined heat and power, as well as increased utilization of RESs from distributed generation sources such as solar, wind, small-scale hydroelectric, and biomass [2]. Today, RESs supply around 14% of the total world energy demand [3,4] and this proportion is expected to increase to 30–80% by 2100 [3,5].

Among different types of RESs, wind power is a rapidly burgeoning energy technology with an annual growth rate of 34% as of 2010 [6]. Being a competitive alternative for the fossil fuel power plants, in 2012, wind power accounted for 39% of the world's renewable power capacity whereas hydroelectric and solar powers accounted for 26% [7].

A hybrid microgrid aims to ensure affordable and reliable energy by supplying power from (1) traditional power generation devices, (2) renewable power generation devices, and (3) energy storage systems. Enhancing the organization performance in terms of energy security and islanding time, hybrid microgrids containing RESs attract attention in the U.S. military. For security reasons, it is undesirable for U.S. military bases to rely heavily on fragile civilian (commercial) electrical grids. Therefore, hybrid microgrids

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are considered as a promising technology for answering the challenge of reliably supplying energy.

On the other hand, the potential benefits of including RESs in a microgrid are often difficult to realize due to their intermittent nature. The electricity supply of microgrids using RESs (e.g., wind, solar, hydro, geothermal) is strongly related to the fluctuation of renewable energy availability, which in turn generally depends on the local weather conditions or other extrinsic factors [8]. One way to overcome the intermittence challenge is to utilize energy storage devices, such as a pumped hydroelectric or compressed air energy storage system [9]. This would enable microgrid operators to store excess energy produced during peak renewable energy production times and use it at a later time. Another solution for the intermittence issue is to connect the microgrid to a commercial grid to provide an alternative energy source while allowing any excess production to be sold to the grid.

In general, however, the increased complexity of microgrids relative to traditional grids means that microgrids benefit greatly from an intelligent approach to operations management. A well-planned operating framework is critical for better grid efficiency and stability of hybrid systems [10,11]. Thus, a hybrid microgrid can benefit greatly from having an Energy Management Center (EMC) to act as the coordinator between a facility’s load side, generation side, and energy storage system. An effective EMC can partially overcome the issue of intermittence by anticipating any fluctuations in the renewable power output and responding accordingly.

One way to overcome the difficulties imposed by such uncertainties is to develop intelligent EMC designs capable of operating well under a variety of realizations of the environmental and operational parameters. The problem of accounting for uncertainties in energy systems has been addressed by a number of different techniques such as sensitivity analysis, fuzzy mathematical programming, stochastic programming, and robust optimization [12].

In this paper, we employ a scenario-robust optimization approach, where our scenarios are derived from ensemble weather forecasts. In contrast to existing approaches that use a deterministic weather forecast, our model takes as input an ensemble weather forecast as produced by most state-of-the-art weather models such as the Global Ensemble Forecasting System (GEFS) developed by the National Centers for Environmental Prediction (NCEP) [13]. We perform computational experiments demonstrating the superior operating schedules that result from planning over an ensemble weather forecast. However, there are various practical considerations to be made when using ensemble weather forecasts. One such consideration is the forecast quality, which deteriorates substantially at long lead times. To address this issue, we perform a sensitivity analysis aimed at determining an appropriate planning horizon for use with ensemble forecasts.

We now review a selection of the literature on microgrid operations and robust optimization.

1.1. Related work

A number of approaches have been suggested to optimize hybrid microgrid operational schedules considering various objectives, constraints, and ways of modeling uncertainty. Ref. [14] gives a detailed review of hybrid renewable systems describing various configurations, control strategies, storage needs, and energy management and control issues with applications in several locations of the world. Ref. [15] presents a review of optimization methods developed for determining operating plans for different types of RESs, operating modes, and objectives. In a more recent review, [16] provides an extensive survey of the stochastic modeling and optimization tools used to facilitate planning and control of microgrid operations.

There exist several papers which use mathematical programming and heuristic techniques to model and plan microgrid operations. Refs. [17,18,8] use a simulation-based optimization tool called HOMER (Hybrid Optimization Model for Multiple Energy Resources) to evaluate the performance of hybrid microgrids. In another example, [19] minimizes the annual energy cost of a microgrid using Lawrence Berkeley Laboratory’s Distributed Energy Resources Customer Adoption Model (DER-CAM), which implements a mixed integer linear program. The authors test the performance of their model on a San Francisco hotel and report 11% cost savings as well as 10% CO2 emission reductions. Ref. [20] presents the Smart Energy Management System (SEMS) tool to optimize the operation of a microgrid. The SEMS consists of a power forecasting module as well as an energy storage system (ESS) management module, and it determines a microgrid operation policy via a genetic algorithm. The authors show that the presented tool can lower energy prices for the consumers and reduce daily electricity costs by 28%. Ref. [21] minimizes the total fuel and operations cost by using a mixed-integer model, and they derive analytical closed-form expressions for the optimal commitment and dispatch solutions. The authors also introduce a probabilistic approach, called “probability of self-sufficiency (PSS),” to express the performance of a microgrid in terms of meeting local demand in a self-sufficient manner.

Ref. [22] considers the problem of jointly optimizing sizing schemes and operating strategies for islanded microgrids via a genetic algorithm. The optimization model includes three objectives: minimize lifecycle cost, minimize pollutant emissions, and maximize RES penetration. The authors implement their model at the Dongfushan microgrid project in China to validate it with actual operational data. In a similar study, [23] develops a mixed integer linear mathematical programming model to determine the optimal configuration and operating plan for a hybrid grid to minimize the total fuel and operations cost. Applying the model to a case study for a remote area, the authors demonstrate that the proposed approach can reduce costs while increasing energy supply reliability. Ref. [24] also uses a mixed-integer linear mathematical programming model to determine optimal schedules of the power generation units in an isolated microgrid. The proposed model aims to minimize the generation costs while taking into account the reliability of the system. Ref. [25] minimizes the fuel consumption rate of a microgrid while constraining it to fulfill a projected energy demand; the authors emphasize the importance of the communication infrastructure operating between the power sources. They also investigate the benefits of having an efficient power sharing scheme supported by a communication link which enables the coordination of microgrid elements. Ref. [26] employs a genetic algorithm to minimize the fuel cost of a microgrid and analyze the relationship between weather information errors and the operation results of the grid.

In other work, [27–36,1,37,38] develop optimization models with objectives such as minimizing emissions, fuel, or operations costs. Other proposed methods include multi-objective optimization [28,37,38,22], the Mesh Adaptive Direct Search Algorithm (MADS) [29,30], the Differential Evolution Algorithm (DEA) [31,34], genetic algorithms [32,20,22,26], Hybrid Inexact Stochastic-Fuzzy Chance-Constrained Programming (ITSFCCP) [33], Adaptive Chaos Clonal Evolutionary Programming (ACCEP) [39], the Dolphin Echolocation Optimization Algorithm (DEOA) [35], a receding horizon optimization strategy [36], and the Adaptive Modified Particle Swarm Optimization Algorithm (AMPSO) [1].

Many different approaches have been used to handle uncertainties in microgrid operations. For example, [39] uses fuzzy set theory to model the uncertainty of renewable production. The uncertainty of wind power is modeled based on dependable capacity concepts in [40]. [41] develops a stochastic model framework,
the stochastic security-constrained unit commitment (SCUC) problem, to manage microgrid operations which involve uncertain RES (wind and solar) outputs and demand load. To account for the uncertainty, the authors use a scenario generation method along with prediction intervals. The SCUC problem is solved via a genetic algorithm for five deterministic and four stochastic cases. The results show that the stochastic model is more robust than the deterministic ones. Ref. [42] also use stochastic programming for microgrid energy scheduling. The proposed model aims to minimize costs and power losses while accounting for RES power output uncertainties. Using a stochastic model, [43] handles the uncertainties by adopting a scenario-robust strategy in which each scenario is assigned a probability that reflects the likelihood of occurrence. Ref. [44] apply a point estimate method for modeling the wind power and solar power uncertainties and use a robust optimization technique to model demand uncertainty. Likewise, [45,46] employ point estimate methods to investigate the uncertainty of wind and solar energy and consider the optimal management of batteries. Ref. [34] model the uncertainties of renewable power production using various possible scenarios for wind speed and solar irradiation based on Weibull and beta probability distribution functions, respectively. Ref. [35] proposes a probabilistic framework based on a scenario-based method, where Monte Carlo simulation is used to generate the scenarios.

Within the field of robust optimization, there are several studies that address the problem of optimal microgrid management strategies. Ref. [47] develops a robust optimization methodology for generating optimal schedules for a microgrid in both grid-connected and isolated modes. The authors characterize the uncertainty in the wind power using a time series based Autoregressive Integrated Moving Average (ARIMA) model. The numerical results obtained from a case study reveal that significant cost reductions are possible through the implementation of the robust optimization based approach. Ref. [48] develops a robust mixed integer linear program to operate the power management system of a hybrid microgrid. The proposed robust optimization model minimizes operational costs while taking into account the uncertainties in power output. Ref. [49] proposes a robust optimization methodology for scheduling microgrid operations which considers uncertainties in both RESs and forecast demand loads. In this study, the authors first develop a deterministic mathematical model which minimizes operational costs of the microgrid. Next, the model is transformed using linear duality and the Karush-Kuhn-Tucker (KKT) optimality conditions. Simulations are used to prove that the proposed approach provides good solutions even for the worst-case realization of the uncertainty bounds.

The robust optimization approach in [50] accounts for the uncertainties in photovoltaic power production in the optimal management of distributed energy resources. Using the Non-dominated Sorting Genetic Algorithm II, the authors aim to minimize power losses, production costs, and CO2 emissions. Ref. [51] develops a two-stage robust mixed-integer linear programming model to determine optimal locations, sizes, and mixture of dispatchable and intermittent distributed generators with minimum cost while considering the probabilistic nature of diesel generator outputs and demand load. The problem is solved using a column and constraint generation methodology and its effectiveness is demonstrated on a case study. In a similar study, [52] aims to minimize costs by formulating an optimization model which schedules operations for microgrids which include RESs as well as combined heat and power generators. To cope with the uncertainty in demand, the problem is solved via a chance constraint approximation and a robust optimization approach.

Refs. [53,54] aim to minimize costs by implementing a robust microgrid scheduling approach which considers the intermittent nature of RESs. In contrast to previous work, the authors also consider adjustable loads. The problem is solved by applying a dual decomposition method. In a more recent study, [55] adopts a scenario-based robust energy management method which considers the uncertainties in RES outputs and load. A robust model is developed with the objectives of minimizing social benefits cost while getting a maximum total exchange cost at the same time. They use interval prediction techniques to describe the variation in uncertain components of the model and use Taguchi’s orthogonal array testing method to generate testing scenarios. They show that the proposed approach provides robust solutions against most scenario realizations. To determine efficient microgrid operation schedules, [56] formulate a mixed integer linear program and solve it over a rolling horizon window, applying a robust optimization approach to handle uncertainties in power production and load. In contrast to other studies that a adopt rolling horizon approach, [56] employs variable time steps in order to reduce computation time. Refs. [57,58] adopt a robust optimization approach which considers uncertainties in load, RES power generation, and market prices. They solve their model using Benders decomposition, decomposing it into an investment master problem and an operation subproblem. Ref. [59] studies the schedule optimization problem for resiliency-oriented microgrids and adopts a robust optimization approach which accounts for the uncertainty in demand load, power generation, and grid supply interruption time and duration. The authors develop mixed integer programming and linear programming models to solve the normal operation problem and the resilient operation problem, respectively.

Ref. [60] proposes a robust optimization framework to optimize the individual objectives of each stakeholders for a microgrid. The authors use an agent-based modeling framework in which each stakeholder is represented as an individual agent. Prediction intervals are used to describe the uncertainties in microgrid operations and RESs. Ref. [61] analyzes microgrid energy management operations using robust optimization based on prediction intervals and optimization based on expected values frameworks. They assess the impact of different levels and types of uncertainty on the cost and reliability of the microgrid. Ref. [62] studies the problem of determining optimal bidding strategies in the day-ahead market of a hybrid microgrid consisting of renewable and nonrenewable energy resources, energy storage systems, diesel generators, and price responsive loads. The problem is formulated as a hybrid stochastic/robust optimization model with the objective of minimizing total cost. Scenarios based on forecasts are used to model the uncertainties in energy outputs and the day-ahead market price. The authors show that the hybrid approach outperforms the pure stochastic approach as it exhibits a more robust output against fluctuations in market price.

1.2. Key features and contributions

Similar to previous studies, we adopt a robust optimization approach to microgrid management. In contrast to these studies, we use ensemble weather forecasts as input data for our model. An ensemble weather forecast consists of a finite sample of equally-likely weather scenarios; the advantage of using an ensemble is that it captures both the predicted outcome and the uncertainty inherent in this prediction. Although ensemble forecasts are produced as output by most state of the art global weather models, they are often converted to a single (deterministic) forecast before being promulgated to users. We propose using the ensemble forecasts directly, rather than adding uncertainty to the deterministic forecast or producing weather scenarios within the microgrid model, as previous studies have done.

We utilize a scenario-robust mixed integer linear program to prescribe day-ahead operating schedules for a hybrid microgrid, with the goal of minimizing expected cost while satisfying demand
in all scenarios. After describing our robust optimization model, we exercise it on a hypothetical microgrid using historical weather data produced by the Global Ensemble Forecasting System (GEFS) developed by the National Centers for Environmental Prediction (NCEP) [13]. In doing so we highlight the benefits robust optimization using ensemble forecasts, particularly relative to a deterministic forecast.

Such an optimization model must, in practice, be run in an iterative manner, since weather forecasts are periodically generated and updated by weather models. In order to run an optimization model iteratively, the user must specify the planning horizon over which the model will plan in each iteration. To address this practical issue, we perform a sensitivity analysis to determine the effect of the planning horizon length on the microgrid’s performance. Fig. 1 shows an example of an ensemble weather forecast. As the figure indicates, the forecast uncertainty grows substantially with the lead time. Thus, it is plausible that longer planning horizons do not necessarily lead to better performance, as in other models. Our work directly addresses this question. To perform our sensitivity analysis, we generate additional weather data that has similar properties to the existing forecast data. In contrast to previous studies, we use an algorithm originally developed in [63] that provides a set of simulated data using multivariate time-series based on a single set of real time-series data. This approach helps us to simulate realistic data to account for the uncertainties while ensuring that major qualitative aspects of the forecasts are preserved. Additionally, the proposed approach is flexible enough to incorporate data pertaining to other types of sources of uncertainty into the model.

The remainder of the paper is structured as follows: Section 2 describes our mathematical model of the hybrid microgrid, while Section 4 provides computational evidence of the superiority of a robust optimization approach. Section 3 discusses various implementation issues, including the need for a rolling horizon optimization approach, and it briefly explains the simulation methodology used to generate wind forecasts. Section 5 describes our approach to selecting an appropriate planning horizon, while Section 6 presents concluding remarks.

2. Microgrid model

Our optimization model is a variation of a mixed-integer linear optimization model originally described in [64]. The full model formulation appears in Appendix A; we now describe some of its high-level characteristics, as well as the components of our hypothetical microgrid.

2.1. Microgrid components

We consider a hypothetical hybrid microgrid consisting of (1) a number of fuel-based generators with different operating characteristics, (2) an energy storage device, (3) renewable energy sources, and (4) a connection to a power grid. In cases of power shortages or excess production, the commercial grid can be used to purchase and sell power at a cost, although doing so is generally less cost effective than utilizing power locally. For simplicity, we assume that the hourly demand is known and constant throughout the planning horizon and the uncertainty is introduced by the renewable power production.

2.1.1. Fuel-based generators

Our optimization model accounts for a mandatory warm-up period for the fuel-based generators. During this period the generator is running (and consuming fuel) without contributing to the total power production. In practice, this warm-up period is necessary for the generator to reach a safe operating temperature and to stabilize the power output before coupling it to the grid. Our model also incorporates the minimum and maximum operating speeds for each generator, and it restricts the total number of changes to each generator’s operating speed over the planning horizon.

The power produced and the fuel consumed by a generator are very nearly proportional to the square of the generator’s rotation speed. Thus, our optimization model uses the rotations per minute squared, $RPM^2$, as a decision variable and models power production, $P_{output}$, and fuel consumption, $Fuel_{consumption}$, using Eqs. (1) and (2).

![Fig. 1. Wind speed predictions from an 11-member ensemble forecast. In this example, note that although the ensemble members largely agree about their predictions for the near future, they differ substantially at long lead times. This higher degree of variability reflects higher uncertainty about the future.](image-url)
\[P_{\text{output}} = \text{ProdCoef} \times \text{RPM}^2\]  
\[\text{Fuel}_{\text{consumption}} = C_{\text{cons}} \times \text{RPM}^2\]

where \(\text{ProdCoef}\) and \(C_{\text{cons}}\) are the production and the fuel consumption coefficients, respectively.

### 2.1.2. Renewable energy source

In our optimization model we represent the total power output of renewable energy sources using a time- and scenario-indexed parameter \(\text{RenPow}_{k,s}\). In our numerical examples we assume that the renewable power is generated solely by a wind farm. Given a wind speed \(V\), the power output \(P(V)\) of a typical wind turbine in kilowatts (kW) is calculated as [65]:

\[
P(V) = \begin{cases} 
V < V_{ci} & 0 \\
V_{ci} < V < V_{n} & \frac{V^3 - V_{ci}^3}{V_{n}^3 - V_{ci}^3} P_n \\
V_{n} < V < V_{ca} & P_n \\
V > V_{ca} & 0
\end{cases}
\]

where \(V_{ci}, V_{n}\), and \(V_{ca}\) are the minimum required (cut-in), rated (nominal) and highest possible (cut-out) speeds, respectively and \(P_n\) is the nominal power in kW.

### 2.1.3. Energy storage

We model a generic energy storage device that allows our model to capture energy at a time when it is abundant and utilize it at a later time. We henceforth refer to this device as a “battery,” although in reality such a device might represent a traditional electrochemical battery or an alternative storage mechanism such as a compressed air energy storage device or a pumped hydroelectric device. Our battery is characterized by its efficiency \(\zeta\), which is the fraction of energy recuperated from the total energy input. The state of charge of the battery at time step \(k\), \(B_k\), is represented in terms of the initial charge \(B_0\), the efficiency \(\zeta\), and the energy used for charging and discharging in each time step \(k' \leq k\), denoted by \(C_{k'}\) and \(D_{k'}\), respectively:

\[B_k = B_0 + \sum_{k' \leq k} \zeta (C_{k'} - D_{k'}).\]

We do not consider degradation of the battery’s efficiency over time, as the timescales considered by our optimization model are too short for this such degradation to have an appreciable effect on performance.

### 2.2. Optimization model

We use an integer linear program (ILP) to prescribe an optimal operating schedule for the microgrid. It accounts for the uncertainty inherent in renewable energy production by utilizing an ensemble weather prediction. This ensemble of forecasts represents a finite sample of the countably infinite set of possible weather outcomes. The model then generates a time-indexed operating schedule for each component of the microgrid. This schedule includes: (1) the times which each generator should be turned on and off, and its operating speed(s), (2) the times when an energy storage device should be charged or discharged, as well as charge/discharge rates, and (3) the amount of energy to be purchased or sold to the commercial grid in each time step.

Decisions regarding generators are regarded as “first stage” decisions and are constant across all weather forecast scenarios; all other decisions are allowed to vary by scenario in order to reflect operational decisions that can be made in real time.

The objective function of our optimization model (Eq. (A.1)) calculates the expected total cost incurred over the planning horizon; this quantity is to be minimized. The first term measures the fuel consumption costs incurred during generator warm-up and production periods. The second term represents the cost of power purchased from the commercial grid. The next term measures the cost of using the energy storage device. Finally, the revenue obtained by selling the excess energy to the commercial grid is calculated in the fourth term.

The constraints used in our model are designed to capture both the physical (technical) limitations and the operational characteristics of the microgrid. They can be categorized under three types as the constraints reflecting the limitations on microgrid power production, diesel-generator operations, and energy storage system.

#### 2.2.1. Power production and demand satisfaction

Constraint (A.2) in our optimization model reflects the requirement associated with the balance between power production and demand. The constraint ensures that the total amount of energy available at any time step is at least as great as the total load. Energy can be produced by generators, wind turbines, purchases from the commercial grid, and discharge of the energy storage device. On the other hand, the load consists of the demand to be satisfied, sales to the commercial grid, and charging of the energy storage device.

#### 2.2.2. Generator operation

Constraints (A.3)–(A.10) in our model ensure that, each generator undergoes a warm-up period upon activation, as discussed in Section 2.1.1. This is achieved by enforcing that each generator spend a warm-up period for a certain length of time before they can actually start contributing to the grid. Moreover, the constraints ensure that each generator operates within the minimum and maximum operating speeds, and the total number of changes in the operating speed is limited.

#### 2.2.3. Energy storage

Constraints (A.11)–(A.16) ensure that, at each time step, the total amount of energy stored in the energy storage device is non-negative and does not exceed the storage device capacity. The constraints also ensure that the charge and discharge rates are within the minimum and maximum limits.

### 3. A simulation-based methodology for generating forecasts

In order to evaluate the performance of our optimization model and perform sensitivity analyses, we require a large amount of input data, specifically ensemble weather forecasts. Given limited sets of ensemble forecasts, we require a method of simulating weather data that has similar properties to the existing forecast data.

We use an algorithm developed in [63] that simulates multivariate time-series based on a single set of real-time-series data. Many time series simulation methods rely on parametrizing a time series of real data using methods like linear or autoregressive modeling. While these methods can be successful at replicating the overall statistical properties of the data, they often fail to replicate the specific dependence patterns in the data. In the case of weather data, we may not know the exact wind speed ahead of time, but may know the general trends that are expected in the short term. Thus, we require a method for simulating wind forecasts that keeps the overall major trends and patterns in the forecasts, while simulating variation to account for uncertainty in the forecasts.

The idea is by maintaining similar patterns of dependence as the original data and generating many simulated forecasts, we can test the robustness of the optimization model. The main
advantage of the method in [63] is that we can generate multiple forecasts from a single data set and retain qualitative structural properties of the original data realization without fitting a complicated parametric model. Our primary data set includes 11 forecasts from each day in 2012, each of which predicts the wind speed at an altitude of 80 m for up to 72 h. Each replication of the simulation algorithm generates 11 forecast predictions that follow the general trends of the original data. The algorithm also incorporates the dependence between these forecasts, so that if all the forecasts are predicting similar wind speeds, then the simulated forecasts are more likely to be closely distributed around the mean of the forecast speeds. If the real data has a high level of variation across forecasts, then the simulated data will also have a high variation.

The simulation algorithm works by mapping all the data forecasts into one multi-dimensional time series in a hypercube. Then, Algorithm 2 of [63] is used to generate paths that “follow” the data in the hypercube. We summarize this algorithm next. Suppose the original data time series consists of vectors $x_t$, $t = 1, 2, \ldots, n$, with each vector containing values mapped values from the 11 forecasts at time $t$ into the hypercube, and $n$ being the length of the forecast data series. Let the simulated path in the hypercube be vectors $y_t$, $t = 1, 2, \ldots, n$. Suppose we start the simulation with $y_1 = y(x_1, \sigma_1^2)$, where the simulated forecast at the first time step is sampled as a multivariate normal distribution around $x_1$. We then simulate the rest of the path in the hypercube $y_t$, $t = 2, \ldots, n$ using a recursive algorithm. The algorithm simulates the next step in the path $y_t$ based on $y_{t-1}$, and a linear combination of the vectors $x_t - y_{t-1}$ and $y_t - y_{t-1}$. The first vector is the difference between the original data forecast at time $t$ and the simulated forecast at time $t-1$, and captures the direction the simulated path needs to move to meet the actual forecast at time $t$. The second vector is the difference between the original forecasts at times $t$ and $t-1$, and represents the trajectory of the original forecast. Thus, the simulated forecast attempts to replicate the trajectory of the original data, and stay “close” to the original data values. Due to the initial randomness at $t = 1$ and an additional noise term added at each time $t$, a series of similar paths can be generated that follow the overall trajectory of the original mapped data series $x_t$.

The generated path in the hypercube is then reverted back to a forecast by way of an inverse of the mapping used to create the multi-dimensional time series in the hypercube. The recursive algorithm described above works better when values are scaled to be between 0 and 1, which is why we map the data to $x_t$ in the hypercube before simulating the paths. There are two main parameters in the algorithm that need to be chosen carefully to model the process of the simulated data following the real data time series. The first is a parameter that measures how closely the simulated data should follow the real data. The parameter controls how closely the simulated data follows the overall trends and dependence in the real data. The second parameter controls the variation in the simulation replications; higher variance means more variation around the mean forecast. These two parameters were chosen to generate simulated forecasts that had qualitatively similar structures to the real forecasts, with enough differences and variation to adequately test the robustness of the optimization model. For details on the algorithmic implementation for weather forecasts, see [66].

4. Scenario-robust optimization and its benefits

An important aspect of our optimization model is that it is capable of optimizing over an ensemble of weather forecasts rather than only a single forecast. This aspect of the model is reflected by the scenario index, $s$, that is present on the $RenPower_{s,k}$ parameter as well as on a number of decision variables reflecting real-time decisions. This modeling approach allows us to ensure that the resulting plan performs well in a variety of possible outcomes, rather than tailoring the plan toward a single forecast. To quantify the benefits of robust optimization in microgrid planning, we examine the grid operating costs that would have been incurred by performing single-day optimizations in each of a total of 10,614 simulated forecast days. For each day, we optimize over each of the following:

- An 11-member ensemble wind forecast, as generated by the GEFS. Given an ensemble of wind speed values $Wind_{s,k}$, we calculate $RenPower_{s,k}$ as in Eq. (3) and use all 11 scenarios in our robust optimization model.
- A single wind forecast, where the wind speed at each time step is equal to the expected wind speed over all ensemble members. That is, we calculate $\text{AvgWind}_s = \sum_{k} Wind_{s,k}$, then calculate $RenPower_{s=1,k}$ from $\text{AvgWind}_s$ according to Eq. (3). We optimize over only a single scenario, $s = 1$.
- A single renewable power forecast, calculated as the expected renewable power produced over all ensemble members. That is, we calculate $RenPower_{s,k}$ from $Wind_{s,k}$ according to Eq. (3), then calculate the expected renewable power produced in each time period as $\sum_{k=1}^{11} RenPower_{s,k}$. We optimize over only a single scenario, $s = 1$, reflecting this expected renewable power.
- Each single scenario from the 11-member ensemble, taken separately.

The first of these approaches leverages the scenario-robust nature of the model to account for the full spectrum of outcomes reflected in the ensemble forecast. The second approach most closely resembles the current practice of optimizing over only an expected weather outcome (in this case, expected wind speed). The third approach is slightly more sophisticated; instead of averaging over wind speed and then calculating renewable power production, it calculates renewable power production for each scenario and then uses the average renewable power produced in the optimization model. The fourth approach is another simple method for choosing a single scenario over which to optimize.

For each run of the optimization model, we compare the resulting cost to the expected cost of an “omniscient” solution, in which the planner has a perfect wind forecast for the next 24 h. This solution represents a best-case plan and its resulting cost for each day. Although one could not hope to achieve the best-case cost in practice, it provides a benchmark by which to compare the costs resulting from the various approaches. Because each ensemble member represents an equally likely weather outcome, we calculate the expected cost of an omniscient solution as the average of the optimal costs associated with each scenario. We solve the optimization model using CPLEX 12.2.0.2 in the General Algebraic Modeling System (GAMS) environment.

Figs. 2 and 3 show the results of our computational experiments. For each of the 10,614 days, we calculate the operating cost associated with implementing the solution resulting from the four approaches above, and we compare this cost to the lowest possible cost that could be achieved that day using the omniscient solution. As Fig. 2 shows, the robust optimization approach results in significantly more days with operating costs close to the omniscient cost than any other approach. Among the approaches that involve optimizing over only a single scenario, the lowest costs are achieved by optimizing over the expected renewable power production rather than the expected weather outcome, as is current practice. This result is not surprising, given that Eq. (3) is nonlinear: we obtain better results by taking the average of a number of values $P(V)$
than by evaluating $P$ at some average value $V$. Interestingly, optimizing over the expected weather outcome does not produce significantly different results than choosing an ensemble member at random. This is made more apparent in Fig. 3, which depicts the fraction of days (of the 10,614 considered) in which the cost associated with each approach exceeded the omniscient cost by a particular amount.

5. Implementation issues: rolling horizon implementation

We now examine two decisions that are important in the practical operation of a prescriptive model such as ours: the frequency with which the model should be run, and the time horizon over which it should optimize. Additional details on this work appear in [66].
5.1. Rolling horizon optimization

Weather forecasts typically span time frames ranging from 24 to 120 h, and their degree of accuracy varies over this time frame. In particular, their accuracy degrades as the lead time grows, as shown in Fig. 1. Although the wind speed prediction is fairly constant among the 11 ensemble members for short lead times, it begins to diverge at around 60 h. The ensemble members differ substantially in their wind speed predictions by about 96 h for this particular example. The exact time of this divergence varies from forecast to forecast, but qualitatively it is a consistent phenomenon in weather forecasting.

Because it is so difficult to accurately predict the weather far in advance, it is highly advisable to run a planning model such as ours in an iterative fashion. Such iterative execution is sometimes called rolling horizon optimization or receding horizon optimization. It proceeds as follows: first, the optimization model is solved over a time horizon known as the planning horizon. The optimal plan generated by this solve is then executed over a shorter time frame known as the execution horizon. For example, the model may optimize over a time horizon of 24 h, and then the first 6 h of the resulting plan may be executed. After the first execution horizon has elapsed, the user updates any input data that has changed (such as the weather forecast) and solves the model once more over the planning horizon. This process repeats ad infinitum.

There are a number of advantages to running an optimization model in this manner. One main advantage for our application is that we are less sensitive to imperfections in our input data, since we constantly incorporate improved information as it becomes available. But rolling horizon optimization is often used even in cases where perfect information is available due to its computational efficiency. If one wishes to optimize over a very long time horizon, it may be much more efficient to handle this horizon in “chunks.” The resulting solution will generally not be globally optimal, but it may be good enough, or it may be used to “warm start” a run of the optimization model using the full horizon. Simply providing the solver with a fairly good initial solution can significantly decrease computation time.

Two user-selected values determine how the rolling horizon iterations proceed: the length of the planning horizon and the length of the execution horizon. For our application, it is best to incorporate updated weather forecasts as they are generated. Thus, the interval between forecast updates is a natural choice for the execution horizon. We use weather forecasts that are generated daily, so in this paper our execution horizon is one day (24 h). While an appropriate execution horizon might be suggested by external factors, the user has more leeway in selecting a planning horizon. Of course, we are not able to plan over horizons that exceed the availability of our data; if we have a forecast for the next 120 h, we cannot plan more than 120 h. However, we may wish to plan over a shorter horizon than 120 h, due to the uncertainty of the forecast at long lead times. Although long planning horizons are often preferred because of their superior solution quality, this is not necessarily the case in our application. With a very long planning horizon, we run the risk of subordinating near-term decisions to account for possibilities expressed in highly uncertain future predictions. It may be better to base our near-term solutions only on near-term data, which has higher quality. To answer this dilemma, we perform a computational study comparing the performance of our model when using different planning horizons.

5.2. Impact of the planning horizon

To study the impact of the planning horizon on the solution quality, we use the simulation method described in Section 3 to generate thirty 72-h ensemble weather forecasts for each day in a 60-day period. We then use a rolling horizon approach to implement our optimization model. We use a 24-h execution horizon, and we compare the outcomes resulting from planning horizons of 24, 27, 30, 33, 36, 48, 60 and 72 h. Our figure of merit is the actual cost, i.e., the cost that would be incurred by using the prescribed plan under the observed weather conditions. Because we do not have observed wind speeds at an 80-m altitude in our historical database, it does not appear in our simulated forecasts either. As an alternative, we use a “most accurate” member of each forecast ensemble to represent the observed wind speed. In this approach, we determine which ensemble member’s wind speed prediction at 24 h is closest to the ensemble mean in the first time step of the next forecast. The first time step in a weather forecast generally contains the forecast model’s estimate of the current conditions, given the available observations. Thus, the ensemble mean in the first time step is a reasonable proxy for observed data, and we treat the forecast of this “most accurate” ensemble member as ground truth for the purposes of our study.

As in our analysis of the scenario-robust optimization approach, we again compute a “best-case” schedule and resulting cost for each day. Now, the best-case cost is the lowest cost that could be achieved over the entire 60-day period of interest by an omniscient planner with perfect knowledge of the wind speed for all future time and with the capability of running the optimization model with a 60-day planning horizon. One could not hope to achieve the best-case cost in reality, but it provides a useful reference point for comparing the costs obtained from our sensitivity analysis.

Fig. 4 compares the scheduled power production (left) and load satisfaction (right) resulting from a best-case plan (top) and a rolling horizon schedule generated with a 24-h planning horizon (bottom). In this example, the purchase and selling prices of energy are constant, and demand is also constant at 2000 kW. Production costs for the three generators vary with time, but continuing to run a generator is always less than purchasing from the commercial grid. (Recall that generators have a required warm-up period, making them inefficient to run for only short durations.) We see that Generator 3 is run fairly consistently in the best-case production schedule, whereas in the actual schedule it is started and shut down many times. For time steps in which Generator 3 is not running in the actual schedule, the deficit is addressed with costly purchases from the commercial grid. Perfect future knowledge allows the best-case schedule to continue running the generator at times when it is not strictly needed, with the excess production being stored in the battery for future use. In the actual schedule, the cost of continuing to run the generator (or start it, if it is not already running) cannot be justified due to a lack of knowledge of future demand. Thus, when such demand occurs, it must be met from the commercial grid. Overall, the best-case schedule uses the storage device more heavily and makes fewer transactions with the outside grid.

In addition to examining various planning horizons, we also consider three different microgrid configurations. Table 1 summarizes these configurations. Configuration 1 is our baseline configuration. In this configuration, the wind farm is capable of supplying approximately 40% of the demand at the average wind speed for the area. In Configuration 2 the commercial grid is less attractive: its purchase price is 50% higher, while its selling price is 25% lower. Configuration 3 represents a larger installation with a more significant renewable component: both demand and the wind farm production capacity are doubled. All three configurations use the same generator attributes: Generator 1 has a production cost of $0.10 per kW h (after warm-up), Generator 2’s production cost is $0.09 per kW h, and Generator 3’s is $0.14 per kW h. All generators must produce at least 490 kW while running and can produce at most 640 kW. For each of our rolling horizon iterations, we
terminate the optimization model when it obtains a solution with objective value that is proven to be within 1.5% of the optimal objective value.

Fig. 5 summarizes our results; the left side displays cost data (which our objective function sought to optimize), while the right side shows purchases from the grid. Along the horizontal axes we see the various planning horizons we seek to evaluate. Within the subfigures, blue dots represent values obtained for the individual simulated scenarios, and red dotted lines represent best-case values for each simulated scenario. Solid lines indicate average quantities over the 30 simulated scenarios. Note that a planning horizon of 24 h results in quite poor performance, both in terms of the cost incurred and the amount of energy purchased from the grid. This is to be expected, since the model has no information about future events that may occur beyond the 24-h execution horizon, and it is forced to execute the entire plan before seeing any additional information. Extending the planning horizon to 33 h results in significantly better solution quality across all configurations. A planning horizon longer than 33 h does not improve either figure of merit for any of the configurations considered. Our empirical results indicate that after a lead time of 33 h, the quality of the weather forecasts has deteriorated to the point that it is no longer beneficial to consider them for planning purposes. We predict that this qualitative behavior will occur in many systems that rely on weather predictions. The exact point at which it occurs will likely depend on many factors, including the adaptability of the system in question and the variability of the weather forecasts themselves.

It is interesting to note that although the optimization model does not explicitly minimize purchases from the commercial grid, we nonetheless see smaller amounts of energy purchased from the grid in the same planning horizons for which our overall costs are low. Because the commercial grid is always an unattractive option in terms of cost, large purchases from the grid generally reflect poor planning.

Comparing the results for Configurations 1 and 2, we see that when it is more expensive to purchase from the commercial grid and less profitable to sell to it, our total cost is higher. This is to be expected. However, the cost difference is much more pronounced for a 24-h planning horizon than for the longer planning horizons. Because we are better able to avoid interaction with the

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**Fig. 4.** An example of the optimal power production plan (left) and load schedule (right) resulting from a best-case plan (top) and a plan generated using a rolling horizon approach with a 24-h planning horizon (bottom). Power is produced using three generators, discharge from a storage device, and a wind power plant; it can also be purchased from the commercial grid. Power is consumed through demand satisfaction and charging of the storage device, or it can be sold to the commercial grid.

**Table 1.** Microgrid configurations.

<table>
<thead>
<tr>
<th>Configuration</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Demand (kW)</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Purchase cost ($ per kW h)</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>Selling price ($ per kW h)</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>Size of wind farm</td>
<td>Baseline</td>
<td>Baseline</td>
</tr>
</tbody>
</table>
commercial grid with a longer planning horizon, we are less sensitive to perturbations in its costs. This phenomenon is also clearly reflected in the right side of Fig. 5. (Note that the exponent on the vertical axis differs between Configuration 1 and Configuration 2.) Performing a similar comparison for Configurations 2 and 3, we see that the cost incurred more than doubles when we double both our demand and our renewable production capacity. Although this may seem counterintuitive, recall that the generator production capacity is the same in all configurations. In Configuration 3, the generators can satisfy a smaller proportion of the any remaining demand not satisfied by wind power. Moreover, a day with no wind produces no wind power, regardless of the side of the wind farm. In such a situation the only alternative is to purchase a larger fraction of energy from the commercial grid. Indeed, looking at the right side of Fig. 5, we see that this quantity approximately triples from Configuration 2 to Configuration 3. If renewable penetration increased further, one could speculate that this trend may continue, unless storage were made more efficient or cost effective.

6. Conclusions and future work

This paper has described the use of optimization and simulation techniques to assess the performance of a hybrid microgrid containing both traditional and renewable power generation devices, as well as energy storage systems. Our computational results indicate that scenario-robust optimization using ensemble weather forecasts produces substantially better results than planning based on a single forecast of any type. Furthermore, we explore an important implementation issue that must be addressed in operating such a model: the sequence of planning and execution horizons to be used. Our proposed rolling horizon optimization approach utilizes an ensemble of realistic weather forecast scenarios generated by a time series simulation method to develop more realistic and effective grid operation schedules. The proposed model is exercised on a hypothetical microgrid successfully. We also perform a sensitivity analysis to measure the impact of planning horizon length on solution quality. Our experimental results show that
Although longer planning horizons are superior to shorter planning horizons, the marginal benefit of increasing the planning horizon decreases substantially as the planning horizon increases. This is an important practical consideration, since longer planning horizons can significantly increase computational effort while resulting in only minor performance benefits, or perhaps even worsening performance.

In this study, we have assumed that the demand is deterministic and the uncertainty only arises from imperfect knowledge of wind power production. However, in many real-world applications, the demand is also uncertain and may be correlated with weather. A possible future work of our study extends to incorporate demand uncertainty.

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Appendix A

Sets

\( g \in G \quad \text{Fuel-based generators} \)
\( w \in W \quad \text{Wind turbines} \)
\( b \in B \quad \text{Batteries} \)
\( k \in K \quad \text{Time steps} \)
\( s \in \mathcal{S} \quad \text{Weather forecast scenarios} \)

Parameters

\( \Delta T \quad \text{Duration of time step [hours]} \)
\( N_{\text{max}} \quad \text{Maximum tolerable number of changes in the generator’s speed during the planning horizon} \)

Variables

\( \text{ProdCost}_g \quad \text{Production cost coefficient of generator } g \text{ [$/kW h]} \)
\( \text{ProdCoef}_g \quad \text{Production coefficient of generator } g \text{ [kW/} \text{RPM}^2] \)
\( \text{InitialRPM}_g \quad \text{Initial squared RPM of generator } g \text{ [RPM]²} \)
\( \text{InitialContrib}_g \quad \text{Contribution status of generator } g \text{ at initial step [binary]} \)
\( \text{MaxRPM}_g \quad \text{Maximum squared RPM of running the generator } g \text{ [RPM]²} \)
\( \text{MinRPM}_g \quad \text{Minimum speed of running the generator } g \text{ [RPM]²} \)
\( \text{warmup}_g \quad \text{Number of time steps } g \text{ must run before it can contribute power} \)
\( \text{Demand}_k \quad \text{Electricity demand at time step } k \text{ [kW]} \)
\( \text{RenPow}_{k,s} \quad \text{Power generated by renewable energy sources at time step } k \text{ from forecast scenario } s \text{ [kW]} \)
\( \text{PurchaseCost}_k \quad \text{Cost of purchasing power from the commercial grid at time step } k \text{ [$/kWh]} \)
\( \text{SellingPrice}_k \quad \text{Revenue from selling power to the commercial grid at time step } k \text{ [$/kWh]} \)
\( \text{MaxCharge}_b \quad \text{Maximum rate of charging battery } b \text{ [kW]} \)
\( \text{MinCharge}_b \quad \text{Minimum rate of charging battery } b \text{ [kW]} \)
\( \text{MaxDischarge}_b \quad \text{Maximum rate of discharging battery } b \text{ [kW]} \)
\( \text{MaxCapacity}_b \quad \text{Maximum storage capacity of battery } b \text{ [kW h]} \)

\( \alpha_b \quad \text{Fraction of power lost while charging battery } b \text{ (loss factor)} \)

\( \text{InitialStorage}_b \quad \text{Initial energy stored in battery } b \text{ [kWh]} \)

\( \text{StorageCost}_b \quad \text{Cost of storing electricity in battery } b \text{ [$/kWh]} \)

Decision variables

\( \text{ON}_g^k \quad \text{Binary} \) 1 if generator \( g \) is running at time step \( k \) and 0 otherwise
\( \text{RPM2}_g^k \quad \text{Continuous} \) \(( \geq 0 \) squared rotation speed of generator \( g \) at time step \( k \) [RPM²]
\( \text{CONTRIB}_g^k \quad \text{Binary} \) 1 if generator \( g \) is contributing at time step \( k \) and 0 otherwise
\( \text{PCONTRIB}_g^k \quad \text{Continuous} \) Power contributed by generator \( g \) at time step \( k \) [kW]
\( \text{PBUY}_k \quad \text{Continuous} \) Power purchased from the grid at time step \( k \) [kW]
\( \text{PSELL}_k \quad \text{Continuous} \) Power sold to the grid at time step \( k \) [kW]
\( \text{PCHARGE}_{b,k,s} \quad \text{Continuous} \) Rate of charging battery \( b \) at time step \( k \) in scenario \( s \) [kW]
\( \text{PDCHARGE}_{b,k,s} \quad \text{Continuous} \) Rate of discharging battery \( b \) at time step \( k \) in scenario \( s \) [kW]
\( \text{CHANGE}_{g,k} \quad \text{Binary} \) 1 if there is a change in generator \( g \)’s speed at step \( k \) and 0 otherwise

Objective function

\[
\min Z = \left[ \sum_g \sum_k \text{ProdCost}_g \times \text{ProdCoef}_g \times \text{RPM2}_g^k \times \Delta T \right] \\
+ \sum_k \text{PurchaseCost}_k \times \text{PBUY}_k \times \Delta T \\
+ \frac{1}{2} \sum_g \sum_k \sum_s \text{StorageCost}_b \times \text{PCHARGE}_{b,k,s} \times \Delta T \\
- \sum_k \text{SellingPrice}_k \times \text{PSELL}_k \times \Delta T \\
\]  

Subject to

\[
\sum_k \text{PCONTRIB}_g^k + \text{RenPow}_{k,s} \geq \sum_k \text{PDCHARGE}_{b,k,s} + \text{PBUY}_k \quad \forall k \in K, s \in S \\
\]  

\[
\text{PCHARGE}_{b,k,s} \leq \text{MaxRPM}_g^2 \times \text{ProdCoef}_g \times \text{CONTRIB}_g^k \quad \forall k \in K, g \in G \\
\]  

\[
\text{PCONTRIB}_g^k \geq \text{MaxRPM}_g^2 \times \text{ProdCoef}_g - \left( 1 - \text{CONTRIB}_g^k \right) \\
\times \text{MaxRPM}_g^2 \times \text{ProdCoef}_g \quad \forall k \in K, g \in G \\
\]  

\[
\text{PCHARGE}_{b,k,s} \leq \text{ProdCoef}_g \times \text{RPM2}_g^k \quad \forall k \in K, g \in G \\
\]  


that power production is high enough to satisfy demand at each time step $k$ while accounting for power bought from or sold to the commercial grid as well as power used to charge the battery. $RenPow_g$ is computed using (3). Constraint sets (A.3), (A.4) and (A.5) are used to create a linear model of power contribution by generator $g$ at time step $k$. Constraint sets (A.6) and (A.7) ensure that generator $g$ does not contribute to power production at time step $k$ unless it has been running sufficiently long or was contributing in its initial condition and has remained running since then. Constraint set (A.11) keeps track of the quantity of energy stored in every battery and forces it to be always less than the maximum storage capacity of the battery. Constraint set (A.12) ensures that the battery storage will not go below zero. Constraint sets (A.13) and (A.14) enforce the maximum and the minimum rate of discharging each battery. Constraint set (A.15) limits the maximum rate of discharging a battery. Constraint set (A.16) is used to ensure that we cannot charge and discharge a battery $b$ at the same time step $k$. Constraint sets (A.8), (A.9), and (A.10) calculate the number of changes in rotation speed for each generator and ensure that it does not exceed a prescribed maximum. In constraint sets (A.8) and (A.9), the binary variable $CHANGE_{g,k}$ is forced to be equal to 1 if $RPM_{g,k}$ differs from $RPM_{g,k-1}$. In constraint set (A.10) we enforce a maximum number of time steps $k$ in which $CHANGE_{g,k}$ can be equal to 1, for each generator $g$. Constraint set (A.17) and (A.18) define the maximum and the minimum values of $RPM_2$ for each generator, respectively. Constraint sets (A.19)–(A.29) declare decision variable domains.

References


