A Battery Depletion Risk Measure for Centralized Systems With Storage Capabilities.

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***Abstract*—This paper develops a new metric, inspired by the traditional loss of load probability, for evaluating a centralized energy storage system's performance on its ability to satisfy demand in electricity markets. While the loss of load probability can be used to estimate the probability of an outage during peak usage periods, our newly proposed battery depletion risk (or BDR) metric considers the cumulative probability of exhausting the centralized storage system across multiple consecutive time periods. The analytical expression for the BDR metric can be used in conjunction with an economic dispatch model to determine the appropriate design parameters for a centralized storage system needed to mitigate variation in demand across time periods, hence reducing the overall risk of an outage.**

***Index Terms*—Power systems reliability, energy storage systems, loss of load probability, power systems design and operation**

I. Introduction

As our society becomes more technologically advanced, it is critical for centralized power systems to provide reliable and uninterrupted access to electricity for consumers and businesses. This has become relevant as energy storage technologies have become more cost effective, thus leading to their increased adoption in recent years [1]. In the traditional sense, one of the most commonly used metrics to quantify the risk associated with an energy system is the loss of load probability (LOLP) [2]. For centralized networks, this metric quantifies the probability that the load exceeds the capacity during any given time period, or the proportion of time periods that might face a shortage. In practice, this metric is used as a reliability threshold or design requirement in assessing the appropriate reserve capacity requirements.

With this in mind, there is a need for a more robust performance metric to assess the design of a centralized network with energy storage in order to mitigate the risk of a capacity shortfall. To address this challenge, we introduce a new metric, which we call the battery depletion risk (or BDR). We define this metric as the probability that the system will experience at least one event over multiple consecutive time periods where the centralized energy storage resources are depleted, given that there exists the ability to store excess generation for use in future time periods. The value of BDR gives the probability that over a given time periods $t = 1, \dots, T$ the centralized energy system capacity will be exhausted at least once, and thus gives a longer-term perspective on risk and the success of the system over an extended period of time in addition to looking at risk in a single time period as LOLP does.

We develop a specific analytical formula for BDR in the case of short term energy storage being available to mitigate against demand variability. For example, a battery may be available to store excess energy from one time period for use in future time periods when there may be a shortage. This type of risk mitigation is often used in microgrids, whereby the goal is for a smaller unit to operate with self-sufficiency. Renewable energy resources, such as wind and solar, may also be combined with battery storage usage due to their intermittent output associated with uncertain weather patterns.

In Section II of this paper, we present a brief literature review that summarizes key applications and approaches to calculate and assess the risk of a shortfall of energy generation capacity, with particular emphasis on the LOLP and energy storage. In Section III, the newly proposed BDR metric is introduced in detail. Specifically, we derive an analytical upper bound for BDR under the assumption that load generation and demand are normally distributed, with the surplus (as the difference between these two variables) in any particular time period having a normal distribution with known mean and variance, i.e., $\mathcal{N}(\mu, \sigma^2)$. We calculate the probability that excess reserves (the current state of battery storage) drops below zero at least once during multiple time periods, with the drop below zero signifying that the existing system of generators is unable to satisfy demand. The BDR metric depends on the initial state of the charge in the first time period, SOC_0, the values of μ and σ^2, and the number of time periods T. We derive this metric using boundary crossing probabilities of Brownian motion. In developing the BDR metric we

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will demonstrate the ability to quantify the effect of the initial storage levels on reducing the probability of exhausting the centralized storage system's capacity over multiple time periods. Section IV evaluates this metric and illustrates its behavior via executing simulation trials of a day-ahead economic dispatch model and studying the resulting outputs. Key takeaways and conclusions are presented in Section V.

II. Literature Review

As outlined in the exhaustive literature survey conducted by [3], energy storage systems are usually evaluated by utilizing cost-based performance metrics. For instance, the levelized cost of storage (LCOS), proposed by [4], is analogous to its predecessor, the levelized cost of electricity (LCOE), in that it is used to compare different storage technologies by computing an average price during discharging hours over the lifetime of the unit in order to break even. Other researchers have proposed variants of these metrics, such as in [5] who apply traditional LCOE methods to lithium-ion and vanadium redox flow batteries in PV systems, or [6] who compute an annualized life cycle cost of storage (LCCOS) to evaluate potential investment options.

While metrics of this class are critical to understanding the value proposition of integrating these technologies into the portfolio, there is a need for a new class of operational performance metrics to evaluate the effectiveness of integrating an energy storage system into a centralized power system network. In a traditional sense, probabilistic methods, such as LOLP or the expected energy not served (EENS), are often applied to study the performance of an electric power grid. Moreover, relying on deterministic methods alone may result in poorly designed energy systems that struggle to satisfy network demand upon experiencing inherent levels of uncertainty. Additionally, in systems with centralized energy systems, from a design and operational perspective, it is critical for the battery to operate efficiently, in order to mitigate against uncertainty and peak demand levels. Inspired by the LOLP metric, our newly proposed BDR metric aims to bridge this gap, thus enabling engineers and decision makers to incorporate these technologies into a network without sacrificing the ability to satisfy system load. In light of this fact, we highlight the related literature on derivations of LOLP under different settings, which inform our development of the BDR metric for centralized energy storage systems.

The research in [7] is one of the earlier papers on methods for calculating LOLP, and develops a graphical capability for estimating the LOLP in terms of days of shortage per year based on adding new generating units to the model. In [8], uncertainty is modeled in wind-battery systems using probability and simulation modeling to estimate the reliability of the system using LOLP, and the authors note that using Monte-Carlo simulations can take a very long time for the LOLP estimate to converge. Thus, it is important to have analytical approximations available to quickly estimate the risk associated with a system, which can help further calibrate large-scale computational experiments.

One common approach is to incorporate LOLP metrics into constraints of an optimization model. The model in [9] incorporates a spinning reserve and develops a single-period hybrid deterministic/probabilistic approach which requires that an upper bound on the probability of the loss of load due to the random outage of one or two units is limited by some constraint. A second constraint is defined by the expected load not served. Bernoulli random variables are used to model the availability of the generators. The authors model the LOLP using binary variables for the outages which allows it to be used as a probability constraint. In [10], the authors consider the probability of random outage events using Bernoulli variables. The resulting nonlinear formulation is then converted to a linear formulation. The authors of [11] develop a method for calculating multi-order outages using an LOLP constraint, and describe how to linearize that constraint to approximate the LOLP at higher-level outages.

Uncertainty in demand or weather factors also play a major role in modeling system risk. The work in [12] develops an analytical framework for calculating LOLP from expected load duration curves, and incorporates the effects of uncertain demand and price changes, while [13] considers probabilistic simulations of weather which affects renewable energy supply, as well as stochastic availability of power plants. The focus is on evaluating LOLP using the residual load as the difference between the electricity load due to stochastic demand and those from renewable power. The authors then calculate aggregate LOLE (loss of load expectation) over a year, and do a thorough case study for Germany which seeks to reduce its nuclear and coal usage.

There are also multiple papers that justify the use of normally distributed random variables in modeling power generation and electricity demand. One attempt to explicitly model LOLP using probability is [14], who develop an analytical method using the surplus of generation as the difference between peak resource availability and peak load. The analytical approach to calculating LOLP in [14] treats the surplus energy value at peak times, S_t, like a random variable, which could be normally distributed as the number of generator units increases by the Central Limit Theorem. [15] use a normal distribution to model the growth in load in a given year and the output power associated with charging electric vehicles, while using a Weibull distribution to model wind speed and illumination intensity for solar sources. In [16], the authors argue that aggregate system power from multiple nodes in a network can be modeled as a normal distribution using the Central Limit Theorem. They then argue that if the demand distribution is known, the LOLP can be calculated explicitly using probability and translated to a deterministic model. Finally, [17] study the sensitivity of LOLP values to demand in different areas, and use a mixture of normal distributions to model demand from different regions.

III. Battery Depletion Risk

This section describes a proposed metric, battery depletion risk (BDR), and begins by relating it to the commonly used loss of load probability (LOLP) metric. The loss of load probability is defined as the probability that the demand load on a system is not met at a given point in time. Typically, this is estimated by comparing the maximum capacity of a system to the peak demand. A power system that is built to adequately manage risk will have a low LOLP such that the probability of demand exceeding the capacity is very small because there is adequate margin reserve built into the system. The LOLP can be estimated using simulation, or using a probability calculation that incorporates stochastic availability of a generator combined with varying demand levels.

One way to apply the LOLP to energy storage systems is to consider the probability that the battery is depleted at any point over a given time horizon. This implies that the network of generators, inclusive of the storage system, may have difficulty satisfying system demand. While it is certainly possible the amount of energy stored in the battery could approach zero while simultaneously satisfying system demand using the remaining generation capacity, upon experiencing peak demand instances or network uncertainties, battery depletion may be indicative of a potential loss of load event. Furthermore, while looking at battery depletion risk at a particular snapshot in time is an important metric, it does not take into account the risk associated with multiple time periods, whereby the main benefit of battery storage is to store excess from low demand periods to assist in time periods with unusually high demand. In this instance, there is a critical need for a cumulative measure, which quantifies the risk across dependent time periods.

We define battery depletion risk (BDR) as *the probability that the battery is depleted at least once in T time periods using a battery with an initial state of charge SOC_0*. Similar to [14], we treat the surplus energy S_t at each time period as a normally distributed random variable, where in that paper LOLP is estimated as the probability this surplus is smaller than zero, or $P(S_t < 0)$. Suppose D_t is the demand for energy in time period t and is normally distributed, while W_t is the energy production generated at each time period, also normally distributed. The surplus at each period $S_t = W_t - D_t$ is assumed to be $\mathcal{N}(\mu, \sigma^2)$, and Section IV will test this assumption on sample data.

As noted in Section II, a common assumption is that the generation and demand are normally distributed, hence the difference between them (the surplus) can also be assumed to be normally distributed. However, we note that our probability measure could also be employed in non-i.i.d. normal settings as the number of time periods increases towards infinity. This is the case for many probability results, in that even under weak dependence or non-normality, mean measures appear normally distributed over long time periods because of the Central Limit Theorem. However, for clarity in our derivation, and because our goal is to assess battery performance over finite time periods, we assume i.i.d. normal values of S_t. Of course, simulation methods can be used to estimate BDR if these assumptions do not apply, but the goal of this paper is to derive a formula that can be quickly used to assess battery risk.

We highlight the major difference between BDR and LOLP. BDR is the probability that a battery is exhausted in at least one time period during $t = 1, \ldots, T$ given charging/discharging schedules resulting from the output of an economic dispatch model. Thus, while storage is available to reduce the risk in a given time period, we calculate the probability that at least one shortage will occur during time periods $1, \ldots, T$, not just at a single peak period t as LOLP does. This provides a cumulative measure of shortfall risk occurring during a given day or week, etc, rather than simply looking at a single time period. Figure 1 shows an example of the current cumulative surplus stored in a battery over time, which is the state of the charge at time t, or SOC_t. BDR is the probability this surplus ever drops below zero, which upon experiencing such an event, implies that a loss of load event occurs assuming the remaining generators in the network cannot satisfy the load.

We begin the run with an initial battery reserve $SOC_0 > 0$ at the start of T time periods. We will quantify the effect of starting with this initial reserve on BDR, because having a starting buffer will help manage uncertainty due to high demands. If the state of charge at time t is defined as the cumulative surplus plus initial battery storage, then we have

$$SOC_t = SOC_0 + \sum_{i=1}^{t} S_i. \tag{III.1}$$

Define BDR as one minus the probability there is never a battery depletion during T time periods. There is never a battery depletion when $SOC_t \geq 0$ for all time periods

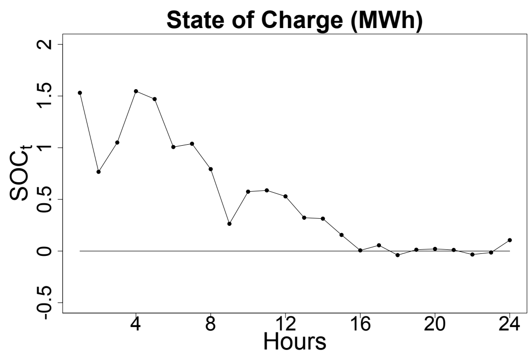

Fig. 1: Sample realization of the state of charge over one day. BDR is calculated as the probability that the surplus stored in the battery, SOC_t, ever goes below 0.

$t \in 1, \ldots, T$:

$$1 - \mathrm{BDR} = P\left(\bigcap_{1 \leq t \leq T} SOC_t \geq 0\right).$$

Then, using (III.1) this translates to:

$$1 - \mathrm{BDR} = P\left(\bigcap_{1 \leq t \leq T} \left\{SOC_0 + \sum_{i=1}^{t} S_i \geq 0\right\}\right). \tag{III.2}$$

We now present a formula for an upper bound on BDR.

Theorem 3.1. *Let Φ be the cumulative distribution function of the standard normal distribution, and $SOC_0 > 0$ be the initial battery reserve. Then, the upper bound on BDR over T time periods is*

$$\begin{aligned} &\exp\left(\frac{-2SOC_0\mu}{\sigma^2}\right)\Phi\left(\frac{\mu\sqrt{T}}{\sigma} - \frac{SOC_0}{\sqrt{T}\sigma}\right) \\ &+ \Phi\left(-\frac{\mu\sqrt{T}}{\sigma} - \frac{SOC_0}{\sqrt{T}\sigma}\right). \end{aligned} \tag{III.3}$$

The proof is presented in Appendix A. It relies on approximating the cumulative surplus (SOC_t) as Brownian motion, and employs boundary-crossing results to estimate the probability of a battery depletion. This type of probability result was first developed in [18], but did not allow for the ability to include an initial starting level for the cumulative process. We will see that having larger initial charge levels will significantly reduce BDR. The upper bound in the theorem comes from applying continuous Brownian motion results to a discrete system evaluated at time increments t, and this bound becomes tight as T increases, i.e., as $T \to \infty$ then BDR is equal to (III.3). Additionally, as T become large the assumptions on strict independence and normality of the S_t values can be relaxed.

Numerical tests applied to (III.3) reveal what we would expect. As the starting reserve SOC_0 increases, BDR decreases because there is more of a buffer to smooth over unexpected shortfalls. As the mean surplus μ increases, BDR decreases since we don't expect as many discharges from the battery, while when the surplus variability σ^2 increases, BDR increases due to the added uncertainty in the surplus.

IV. Implementation Example

In this section, we show how the BDR metric can be used to evaluate performance in a centralized network. Given a set of sequential demand forecasts, an optimization model can be constructed to find the optimal dispatch schedule, inclusive of the corresponding charge and discharge frequency of a centralized energy storage system. However, the guaranteed success of this plan assumes the demand forecasts are accurate without any uncertainty.

For a given optimal dispatch plan with an initial state of charge SOC_0, we can estimate the distribution of the output surplus values S_t to the battery as i.i.d. $\mathcal{N}(\mu, \sigma^2)$ using the generation output. Then using SOC_0, μ, and σ^2 we calculate an upper bound for BDR using (III.3). This estimates the risk of battery depletion given uncertainty in the sequential surplus energy charged to (or discharged from) the battery over time. By replicating this calculation using simulated demands, we can estimate a range of potential BDR values, and will find that battery depletion risk can be significant given demand uncertainty.

In the following subsections, to illustrate the BDR metric, we briefly present a day-ahead, hourly economic dispatch model from [19]. Specifically, we outline the model's assumptions, define key input parameters and decision variables, present the mathematical model, and display numerical results from applying the optimization model across multiple simulated inputs.

A. Model Formulation

Tables 1, 2, and 3 define the model paramaters and decision variables. In addition, we make the following assumptions listed below.

1) All dispatch and curtailment decisions are made by a centralized planner who manages all decisions made within the network.
2) All decisions are made on an hourly basis over a 24-hour time horizon.
3) All thermal units have deterministic characteristics and cost parameters.

TABLE I: Model Parameters and Cost Definitions

J	Number of generating units (indexed by j)
T	Number of time periods (indexed by t)
a_j	Quadratic cost coefficient for thermal generation from unit j (\$/MWh^2)
b_j	Linear cost coefficient for thermal generation from unit j in (\$/MWh)
c_j	Constant cost coefficient for thermal generation from unit j (\$)

TABLE II: Operational Parameters

$PMIN_j$	Minimum generation from unit j (MWh)
$PMAX_j$	Maximum generation from unit j (MWh)
RUR_j	Periodic ramp-up rate for unit j (MWh)
RDR_j	Periodic ramp-down rate for unit j (MWh)
$SOCMIN_t$	Minimum available charge required in period t (MWh);
$SOCMAX_t$	Maximum available charge required in period t (MWh);
SOC_0	Starting available charge in period t; (user defined)
$PCMIN_t$	Minimum charge in period t (MWh)
$PCMAX_t$	Maximum charge in period t (MWh)
$PDMIN_t$	Minimum discharge in period t (MWh)
$PDMAX_t$	Maximum discharge in period t (MWh)
α	Charging efficiency (percentage)
β	Discharge efficiency (percentage)
D_t	Electricity demand in period t (MWh)

4) No costs are incurred upon charging or discharging from the energy storage system.

Next, the mathematical formulation for the model from [19] is presented. The model is formulated as a nonlinear program (NLP) and is solved using the open-source solver COIN-OR via GAMS. The objective function of the model is the total dispatching costs over the planning horizon in (IV.1):

$$z = \sum_{j=1}^{J} \sum_{t=0}^{T} \big(a_j x_{j,t}^2 + b_j x_{j,t} + c_j \big). \tag{IV.1}$$

TABLE III: Decision Variables

$x_{j,t}$	Electricity generated by thermal unit j in period t (in MWh)
chg_t	Excess electricity sent to charge the battery in period t (MWh)
$dchg_t$	Electricity discharged from the battery in period t (MWh)
SOC_t	Battery “state of charge” in period t (MWh)
W_t	Minimum output from all thermal units at time t (MWh)

We now present the constraints. First, (IV.2) is the load balancing constraint which implies that total generation and discharge from the storage unit must satisfy total demand:

$$\sum_{j=1}^{J} x_{j,t} + dchg_t \geq D_t + chg_t, \ \forall t. \tag{IV.2}$$

Additionally, all excess generation is used to charge the battery. Minimum thermal generation output requirements from all thermal units and individualized minimum and maximum thermal generation limits are given in (IV.3) and (IV.4) respectively:

$$\sum_{j=1}^{J} x_{j,t} \geq W_t, \tag{IV.3}$$

$$PMIN_j \leq x_{j,t} \leq PMAX_j, \ \forall j, \ \forall t. \tag{IV.4}$$

Ramping rates are provided in (IV.5) and (IV.6):

$$x_{j,t+1} - x_{j,t} \leq RUR_j, \ \forall j, \ \forall t, \tag{IV.5}$$

$$x_{j,t-1} - x_{j,t} \leq RDR_j, \ \forall j, \ \forall t. \tag{IV.6}$$

Next, the state of the storage unit's charge is presented in (IV.7):

$$SOC_t = SOC_0 + SOC_{t-1} + \alpha chg_t - \frac{dchg_t}{\beta}, \ \forall t. \tag{IV.7}$$

The minimum and maximum limits for charging levels, discharging levels, and the state of charge are provided in Equations (IV.8), (IV.9), and (IV.10) respectively:

$$PCMIN_t \leq chg_t \leq PCMAX_t, \ \forall t, \tag{IV.8}$$

$$PDMIN_t \leq dchg_t \leq PDMAX_t, \ \forall t, \tag{IV.9}$$

$$SOCMIN_t \leq SOC_t \leq SOCMAX_t, \ \forall t. \tag{IV.10}$$

Finally, (IV.11) displays the nonnegativity constraints:

$$x_{j,t}, \ chg_t, \ dchg_t, \ SOC_t, \ W_t \geq 0, \ \forall j, \ \forall t. \tag{IV.11}$$

B. Numerical results

In this section, the optimization model is applied to a test case to evaluate the impact of system characteristics on the BDR metric as well as an hourly dispatch schedule. The test case in this section is adapted from [19], where a system with four thermal generators and a centralized energy storage system are dispatched hourly to satisfy the demand. To demonstrate our framework, we run the model with 11 different values of SOC_0, which is varied from 0 MWh to 1000 MWh in increments of 100 MWh. For each tested value of SOC_0, we execute the optimization model 100 times employing common random numbers to test the same set of 100 simulated hourly demand sequences against all values of SOC_0. The demand values are simulated from an i.i.d. normal distribution with a mean of 647.5 MWh and a standard deviation of 94.9 MWh as calibrated from the example in [19].

Operational details of the energy storage system are given in Table IV. In this numerical study, we allow essentially unlimited values for SOC_t with $SOCMAX_t$ being set very high since our BDR metric does not place limits on excess surplus. Similarly, we do not place limits on charging and discharging amounts to align with our assumptions calculating the metric. These relaxations allow the battery to operate with as few restrictions as possible to store and discharge electricity as required. Thus, in reality, the BDR values could be worse than those calculated here if there are limits placed on how much can be stored in a given time period.

TABLE IV: Energy Storage System Operational Details

Parameter	Value
$SOCMIN_t$	0 MWh
$SOCMAX_t$	1200 MWh
$PCMIN_t$	0 MWh
$PCMAX_t$	$SOCMAX_t$
$PDMIN_t$	0 MWh
$PDMAX_t$	$SOCMAX_t$
α	95%
β	90%

As evidenced by Figure 2, as the initial state of charge increases, the total output from thermal units in the system decreases in a nearly linear fashion. In Figure 3, as SOC_0 increases, the mean surplus (or difference between total thermal output and demand) decreases accordingly. This relationship appears to be linear in nature, and is indicative of the fact that the system becomes increasingly dependent on the energy storage system to satisfy demand, because less thermal generation is needed when the system starts with a high level of initial charge.

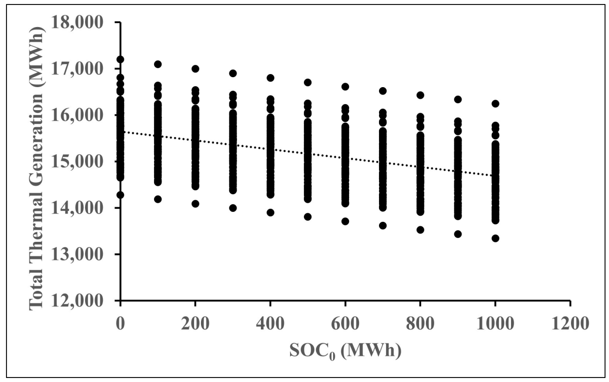

Fig. 2: SOC_0 vs. Total Thermal Output

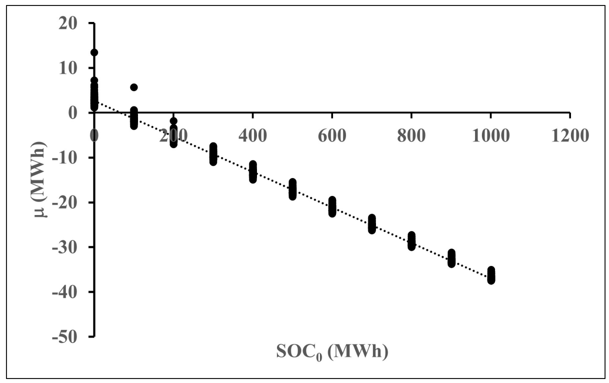

Fig. 3: SOC_0 vs. μ

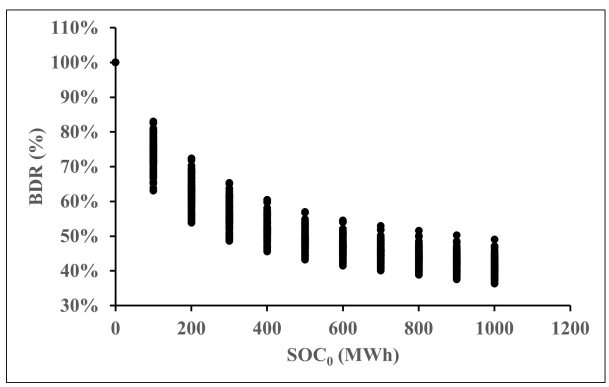

Fig. 4: SOC_0 vs. BDR

Finally, we calculate the BDR metric for each replication using the estimated mean and variance of the surplus values derived from the optimal dispatch schedule. As depicted in Figure 4, as the SOC_0 values increase, the corresponding BDR metric declines at a decreasing rate. This implies that the risk of depleting the storage system can be mitigated by increasing the initial state of charge, and numerical tests reveal that increasing SOC_0

to infinity will lead to a BDR of zero. In studying these results in tandem with the output displayed in Figure 3, at smaller SOC_0 values, the thermal generation output exceeds the demand in efforts to charge the battery for future energy requirements. Additionally, taking the results from Figure 2 into consideration, as the SOC_0 values increase in magnitude, the system becomes less reliant on the existing thermal generation capabilities and more reliant on the energy storage system to meet system demand. Ultimately, from a design perspective, increasing the initial state of charge of the energy storage system yields tangible performance benefits with respect to the electricity supply network's ability to satisfy demand, even under volatile demand realizations.

C. Normality Tests

We test the assumption of normality on our optimization model output. Many probability results rely on the assumptions of normally distributed data, and as mentioned above, past research has assumed normally distributed demand, generation and surplus values. In reality, these values can be non-normal and dependent across time periods and it is highly unlikely that they come from a true normal distribution. However, the absence of true normality and dependence can still be managed. As the sample size approaches infinity, approximate normality in mean values can sometimes be achieved. Otherwise, simulation can be used to estimate BDR rather than using the result of Theorem 3.1.

Using the output from the optimization runs, we test our assumption that the surplus $S_t = W_t - D_t$ is approximately normally distributed. For each of the 1,000 scenarios run, we test the normality of the 24 generated S_t values over the course of a day using the Shapiro-Wilks test. We find the mean p-value is 0.05, implying that many of the optimally generated surplus values would be close to being rejected as normal at the 95% confidence level. However, we note that the p-values can be much lower for data that is clearly nonnormal. For example, testing 1,000 simulated sets of 24 random exponential variables yields a mean p-value of 0.02, so exponential data is much more likely to be rejected than the S_t data. Additionally, we note that many of the optimally generated values of S_t are 0, because the model is designed to generate just enough power to cover demand to minimize costs. The presence of multiple zeros in a run will likely decrease the appearance of normality, whereas in reality system uncertainty makes it unlikely for W_t and D_t to exactly coincide in practice.

V. Conclusion

Inspired by the LOLP metric, the newly proposed BDR metric outlined in this paper estimates the probability that the capacity energy storage system will be exhausted over the planning horizon. While an hourly electricity dispatch plan can be developed using demand forecasts, uncertainty in demand projections increase the risk of a shortage. We show how increasing the initial state of charge in the energy storage system can serve as a mitigation strategy to ensure that the generation network is able to fully satisfy demand over a sequence of time periods, and derive an analytical formula which can be used to quickly assess the storage system's performance in the generation portfolio.

The BDR metric allows for power systems planners to evaluate the efficiency of centralized storage systems with respect to their ability to mitigate against demand variability by controlling the battery capacity and starting charge levels at the beginning of a high demand period. While the current state-of-the-art energy storage metrics are critical in evaluating the value proposition of their integration, the BDR metric is the first quantitative operational performance measure for this class of problems. However, in order to fully assess system performance in its entirety, this metric must be considered in tandem with traditional reliability metrics, such as LOLP and EENS. Furthermore, while the newly proposed BDR metric is a quantitative measure of risk, from a managerial perspective, more research is required to translate its numerical output into a qualitative scale and actionable insights for practitioners.

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References

[1] W. Cole and A. Frazier, "Cost Projections for Utility-Scale Battery Storage," National Renewable Energy Laboratory, Tech. Rep. NREL/TP-6A20-73222, June 2019.

[2] G. Gambirasio, "Computation of Loss-of-Load Probability," *IEEE Transactions on Reliability*, vol. R-25, no. 1, pp. 54–55, 1976.

[3] N. Günter and A. Marinopoulos, "Energy Storage for Grid Services and Applications: Classification, Market Review, Metrics, and Methodology for Evaluation of Deployment Cases," *Journal of Energy Storage*, vol. 8, pp. 226 – 234, 2016. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S2352152X16301141

[4] A. Belderbos, E. Delarue, K. Kessels, and W. D'haeseleer, "Levelized Cost of Storage — Introducing Novel Metrics," *Energy Economics*, vol. 67, pp. 287 – 299, 2017. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S014098831730289X

[5] C. S. Lai and M. D. McCulloch, "Levelized Cost of Electricity for Solar Photovoltaic and Electrical Energy Storage," *Applied Energy*, vol. 190, pp. 191 – 203, 2017. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S030626191631933X

[6] M. H. Mostafa, S. H. Abdel Aleem, S. G. Ali, Z. M. Ali, and A. Y. Abdelaziz, "Techno-Economic Assessment of Energy Storage Systems Using Annualized Life Cycle Cost of Storage (LCCOS) and Levelized Cost of Energy (LCOE) Metrics," *Journal of Energy Storage*, vol. 29, p. 101345, 2020. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S2352152X19316925

[7] L. L. Garver, "Effective Load Carrying Capability of Generating Units," *IEEE Transactions on Power Apparatus and Systems*, no. 8, pp. 910–919, 1966.

[8] A. Roy, S. B. Kedare, and S. Bandyopadhyay, "Optimum Sizing of Wind-Battery Systems Incorporating Resource Uncertainty," *Applied Energy*, vol. 87, no. 8, pp. 2712–2727, 2010.

[9] F. Bouffard and F. D. Galiana, "An Electricity Market with a Probabilistic Spinning Reserve Criterion," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 300–307, 2004.

[10] J. Aghaei, N. Amjady, A. Baharvandi, and M.-A. Akbari, "Generation and Transmission Expansion Planning: MILP–Based Probabilistic Model," *IEEE Transactions on Power Systems*, vol. 29, no. 4, pp. 1592–1601, 2014.

[11] S. A. Rashidaee, T. Amraee, and M. Fotuhi-Firuzabad, "A Linear Model for Dynamic Generation Expansion Planning Considering Loss of Load Probability," *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 6924–6934, 2018.

[12] M. Joung and J. Kim, "Assessing Demand Response and Smart Metering Impacts on Long-Term Electricity Market Prices and System Reliability," *Applied Energy*, vol. 101, pp. 441–448, 2013.

[13] L. Nolting and A. Praktiknjo, "Can We Phase-Out All of Them? Probabilistic Assessments of Security of Electricity Supply for the German case," *Applied Energy*, vol. 263, p. 114704, 2020.

[14] K. Dragoon and V. Dvortsov, "Z-Method for Power System Resource Adequacy Applications," *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 982–988, 2006.

[15] Z. Liu, F. Wen, and G. Ledwich, "Optimal Siting and Sizing of Distributed Generators in Distribution Systems Considering Uncertainties," *IEEE Transactions on Power Delivery*, vol. 26, no. 4, pp. 2541–2551, 2011.

[16] T. Jin, Y. Yu, and E. Elsayed, "Reliability and Quality Control for Distributed Wind/Solar Energy Integration: A Multi-Criteria Approach," *IIE Transactions*, vol. 47, no. 10, pp. 1122–1138, 2015.

[17] E. Tómasson and L. Söder, "Coordinated Optimal Strategic Demand Reserve Procurement in Multi-Area Power Systems," *Applied Energy*, vol. 270, p. 114984, 2020.

[18] D. Singham and M. Atkinson, "Boundary Crossing Probabilities for the Cumulative Sample Mean," *Probability in the Engineering and Informational Sciences*, vol. 32, no. 2, pp. 275–295, 2018.

[19] A. Soroudi, *Power System Optimization Modeling in GAMS*. Springer International Publishing, 2017.

[20] L. Schruben, "Confidence Interval Estimation Using Standardized Time Series," *Operations Research*, vol. 31, no. 6, pp. 1090–1108, 1983.

[21] D. Singham and M. Atkinson, "Construction of Cumulative Mean Bounds for Simulation Output Analysis," Naval Postgraduate School, Tech. Rep. NPS-OR-15-004, May 2015.

[22] M. Abundo, "Some Conditional Crossing Results of Brownian Motion over a Piecewise-Linear Boundary," *Statistics & Probability Letters*, vol. 58, no. 2, pp. 131–145, 2002.

[23] T. Scheike, "A Boundary-Crossing Result for Brownian Motion," *Journal of Applied Probability*, pp. 448–453, 1992.

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Appendix A
Proof of Theorem 3.1

We derive an expression for BDR using (III.2) which is repeated next, but using notation R as the initial reserve SOC_0 for brevity in what follows:

$$1 - \text{BDR} = P\left(\bigcap_{1\le t\le T}\left\{R + \sum_{i=1}^{t} S_i \ge 0\right\}\right). \quad \text{(A.1)}$$

Equation (A.1) is the probability that for all days $t \in 1, \ldots T$ there will nonnegative energy stored in the battery. It will be easier to work with cumulative averages to invoke the translation to Brownian motion. Rearranging terms and dividing by t yields:

$$1 - \text{BDR} = P\left(\bigcap_{1\le t\le T}\left\{-\frac{1}{t}\sum_{i=1}^{t} S_i \le \frac{R}{t}\right\}\right). \quad \text{(A.2)}$$

Adding the average surplus to both sides

$$1 - \text{BDR} = P\left(\bigcap_{1\le t\le T}\left\{\frac{1}{T}\sum_{i=1}^{T} S_i - \frac{1}{t}\sum_{i=1}^{t} S_i \le \frac{R}{t} + \frac{1}{T}\sum_{i=1}^{T} S_i\right\}\right),$$

and multiplying by $t/\sqrt{T}$ yields

$$1 - \text{BDR} = P\left(\bigcap_{1\le t\le T}\left\{\frac{t}{\sqrt{T}}\left(\frac{1}{T}\sum_{i=1}^{T} S_i - \frac{1}{t}\sum_{i=1}^{t} S_i\right) \le \frac{R}{\sqrt{T}} + \frac{t}{T\sqrt{T}}\sum_{i=1}^{T} S_i\right\}\right). \quad \text{(A.3)}$$

Under mild conditions on the S_i (weak dependence is acceptable, i.i.d. normality not required), then

$$\frac{1}{T}\sum_{i=1}^{T} S_i - \frac{1}{t}\sum_{i=1}^{t} S_i \quad \text{(A.4)}$$

can be converted to the skeleton of a standardized time series as defined by [20]. Essentially, the cumulative mean process $\frac{1}{t}\sum_{i=1}^{t} S_i$ can be rescaled to converge to Brownian motion as T increases. In [20], the author defines standardized time series as continuous functions over $u \in [0, 1]$:

$$X_T(u) = \frac{\lfloor Tu\rfloor\left(\frac{1}{T}\sum_{i=1}^{T} S_i - \frac{1}{\lfloor Tu\rfloor}\sum_{i=1}^{\lfloor Tu\rfloor} S_i\right)}{\sigma\sqrt{T}}, \quad u \in [0, 1].$$

The process $X_T(u)$ converges to a Brownian bridge $B(u)$, which is a stochastic process that is Brownian motion over $u \in [0, 1]$ that takes the value of zero at both endpoints. In our context, the time periods $t = \lfloor Tu\rfloor$ are the discrete realizations of the process, while $X_T(u)$ is a continuous process. [21] demonstrates that when S_i is i.i.d. normal, the discrete process (A.4) has the same joint distribution as the corresponding skeleton points of a continuous Brownian bridge. Thus converting the left hand side of the inequality in (A.3) to a standardized time series yields

$$1 - \text{BDR} = P\left(\bigcap_{1\le t\le T}\left\{\sigma X_T(t/T) \le \frac{R}{\sqrt{T}} + \frac{t}{T\sqrt{T}}\sum_{i=1}^{T} S_i\right\}\right).$$

Following a similar type of calculation from Eqns. (2.2)-(2.5) of [18] means we can translate the probability calculation to one involving Brownian bridges using $X_T(u) \to B(u), u \in [0, 1]$ as $T \to \infty$. However, for S_i as an i.i.d. normally distributed variable, we don't need to take T to infinity and can directly apply Brownian bridge results since the distributions at the skeleton points are the same. Setting $u = t/T$ and converting to a Brownian bridge in continuous time, $B(u), u \in [0, 1]$, we have

$$1 - \text{BDR} \ge P\left(\bigcap_{u\in[0,1]}\left\{\sigma B(u) \le \frac{R}{\sqrt{T}} + u\left(\frac{1}{\sqrt{T}}\sum_{i=1}^{T} S_i\right)\right\}\right). \quad \text{(A.5)}$$

Note that (A.5) is the probability that a standard Brownian bridge rescaled by σ crosses a linear boundary with a positive intercept of $R/\sqrt{T}$ at 0. The inequality leading to the bound $1 - \text{BDR} \ge$ in (A.5) exists because a continuous process is more likely to cross the bounds than its skeleton which is a discrete process. Thus the probability of not having a battery depletion will be smaller under a continuous Brownian bridge than the corresponding discretized process, but this discrepancy decreases to zero as $T \to \infty$.

Next, let $X = \sum_{i=1}^{T} S_i/\sqrt{T}$ and note that this term has distribution $\mathcal{N}(\mu\sqrt{T}, \sigma^2)$. Define $f_X(x)$ as the density of this normal random variable X at value x. Continuing and writing (A.5) in terms of an upper bound for BDR yields

$$
\begin{aligned}
&\text{BDR} \le \\
&1 - P\left(\bigcap_{u\in[0,1]}\left\{\sigma B(u) \le \frac{R}{\sqrt{T}} + u\left(\frac{1}{\sqrt{T}}\sum_{i=1}^{T} S_i\right)\right\}\right)\\
&= P\left(\bigcup_{u\in[0,1]}\left\{\sigma B(u) \ge \frac{R}{\sqrt{T}} + u\left(\frac{1}{\sqrt{T}}\sum_{i=1}^{T} S_i\right)\right\}\right)\\
&= \int_{-\infty}^{\infty} P\left(\bigcup_{u\in[0,1]}\left\{\sigma B(u)\ge\frac{R}{\sqrt{T}} + ux\right\}\right) f_X(x)dx.
\end{aligned} \tag{A.6}
$$

Equation (A.6) is the probability of a Brownian bridge crossing above a linear boundary $R/\sqrt{T}+ux$ probability weighted over the random variable X taking values x. We know that the Brownian bridge starts below the boundary because $B(0) = 0 < R\sqrt{T}$ and we assume $R = SOC_0 > 0$. Consider two options for the endpoint location. If the endpoint of the Brownian bridge ends above the linear boundary, then $B(1) = 0 > \frac{1}{\sqrt{T}}\left(R + \sum_{i=1}^{T} S_i\right)$ where $u = 1$. This means the battery has failed (so BDR is 1 in this case) since it does not have any energy in reserves at the end of time T if

$$
R + \sum_{i=1}^{T} S_i < 0, \text{ or } X < -R/\sqrt{T}.
$$

The probability of this happening is

$$
\begin{aligned}
P(X < -R/\sqrt{T}) &= \Phi\left(\frac{-R/\sqrt{T} - \mu\sqrt{T}}{\sigma}\right)\\
&= \Phi\left(\frac{-R}{\sqrt{T}\sigma} - \frac{\mu\sqrt{T}}{\sigma}\right)\\
&:= V,
\end{aligned} \tag{A.7}
$$

where we define V as (A.7), which is the probability that the cumulative generation at T plus starting battery reserve is not enough to cover demand. The second option is that the Brownian bridge ends below the linear boundary, so $B(1) = 0 \le \frac{1}{\sqrt{T}}\left(R + \sum_{i=1}^{T} S_i\right)$, meaning

$$
R + \sum_{i=1}^{T} S_i \ge 0, \text{ or } X \ge -R/\sqrt{T}
$$

which implies that there is excess charge in the battery at the end of T time periods. In this case, we need to compute BDR to find the probability that there was a depletion during some intermediate time period. The probability that a standard Brownian bridge crosses a linear boundary given that the starting and ending points are both below the boundary is calculated in [22] and [23]. Thus we can replace the probability in the integral of (A.6) (the $P(\cup\{\})$ term) with

$$
\exp\left(\frac{-2R^2 - 2Rx\sqrt{T}}{T\sigma^2}\right).
$$

Then we can rewrite (A.6) as the probability $X < -R/\sqrt{T}$ (which is V) plus the probability of crossing conditioned on all values $X \ge -R/\sqrt{T}$. Plugging in the normal density function for $f_X(x)$ yields

$$
\begin{aligned}
\text{BDR} \le V + \int_{-R/\sqrt{T}}^{\infty}\Bigg[&\exp\left(\frac{-2R^2 - 2Rx\sqrt{T}}{T\sigma^2}\right)\\
&* \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-(x-\mu\sqrt{T})^2}{2\sigma^2}\right)\Bigg] dx\\
= V + \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-R/\sqrt{T}}^{\infty}\Bigg[&\exp\left(\frac{-4R^2/T - 4Rx/\sqrt{T}}{2\sigma^2}\right)\\
&* \exp\left(\frac{-(x^2 - 2x\mu\sqrt{T} + \mu^2 T)}{2\sigma^2}\right)\Bigg] dx.
\end{aligned} \tag{A.8}
$$

Multiplying together the two exponential terms in the integral in (A.8) and simplifying by completing the square yields

$$
\begin{aligned}
&\exp\left(\frac{-\left(x^2 + x(4R/\sqrt{T} - 2\mu\sqrt{T}) + \mu^2 T + 4R^2/T\right)}{2\sigma^2}\right)\\
&= \exp\left(\frac{-\left[\left(x + \left(2R/\sqrt{T} - \mu\sqrt{T}\right)\right)^2 + 4R\mu\right]}{2\sigma^2}\right).
\end{aligned} \tag{A.9}
$$

Plugging (A.9) back into (A.8) and pulling out the terms from the integral that do not depend on x yields:

$$
\begin{aligned}
&\text{BDR} \le V + \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-2R\mu}{\sigma^2}\right) *\\
&\int_{-R/\sqrt{T}}^{\infty}\exp\left(\frac{-\left[\left(x - \left(\mu\sqrt{T} - 2R/\sqrt{T}\right)\right)^2\right]}{2\sigma^2}\right) dx.
\end{aligned}
$$

Letting $Y = \mu\sqrt{T} - \frac{2R}{\sqrt{T}}$, defining $\mathcal{N}(x, \mu, \sigma^2)$ as the normal density function with mean μ and variance σ^2 evaluated at x, and using $\mathcal{N}(0, 1)$ as a standard normal random variable, we have

$$
\begin{aligned}
&\text{BDR} \le V + \exp\left(\frac{-2R\mu}{\sigma^2}\right) *\\
&\qquad \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-R/\sqrt{T}}^{\infty}\exp\left(\frac{-\left[(x - Y)^2\right]}{2\sigma^2}\right) dx
\end{aligned}
$$

$$= V + \exp\left(\frac{-2R\mu}{\sigma^2}\right) * \int_{-R/\sqrt{T}}^{\infty} \mathcal{N}(x, Y, \sigma^2) dx$$
$$= V + \exp\left(\frac{-2R\mu}{\sigma^2}\right) P\left(Y + \sigma\mathcal{N}(0,1) > -R/\sqrt{T}\right)$$
$$= V + \exp\left(\frac{-2R\mu}{\sigma^2}\right) \left(1 - \Phi\left(\frac{-R/\sqrt{T} - Y}{\sigma}\right)\right).$$

Substituting for V and Y, and using $1 - \Phi(z) = \Phi(-z)$ yields the result:

$$\text{BDR} \le \Phi\left(-\frac{\mu\sqrt{T}}{\sigma} - \frac{R}{\sqrt{T}\sigma}\right) + \exp\left(\frac{-2R\mu}{\sigma^2}\right) \Phi\left(\frac{\mu\sqrt{T}}{\sigma} - \frac{R}{\sqrt{T}\sigma}\right). \quad \square$$