Allocating resources to enhance resilience

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Disaster resilience is the ability to (Bruneau et al. 2003)
- Reduce the chances of a shock
- Absorb a shock if it occurs
- Recover quickly after it occurs

Nonlinear disaster recovery (Zobel 2014)


Quantifying disaster resilience

\[ R_*(\beta, X, T) = 1 - \frac{\beta X T}{T^*} \]
1. How should a decision maker allocate resources among the three factors in order to maximize resilience?

2. What are possible functions that determine effectiveness of allocating resources?

3. When is it optimal to allocate resources to reduce all three factors?

4. Does the optimal decision change when there is uncertainty?
Resource allocation model

\[ R_*(\beta, X, T) = 1 - \frac{\beta XT}{T^*} \]

Factor as a function of resource allocation decision

maximize \[ R_*(\beta(z_\beta), X(z_X), T(z_T)) \]

minimize \[ \beta(z_\beta) * X(z_X) * T(z_T) \]

subject to \[ z_\beta + z_X + z_T \leq Z \]

\[ z_\beta, z_X, z_T \geq 0 \]

Budget
• $\beta(z_\beta), X(z_X), \text{ and } T(z_T)$ describe ability to allocate resources to reduce each factor of resilience

• Requirements

  – Factor should decrease as more resources are allocated: $\frac{d\beta}{dz_\beta}, \frac{dX}{dz_X}, \text{ and } \frac{dT}{dz_T}$ are less than 0

  – Constant returns or marginal decreasing improvements as more resources are allocated: $\frac{d^2\beta}{dz_\beta^2}, \frac{d^2X}{dz_X^2}, \text{ and } \frac{d^2T}{dz_T^2}$ are greater than or equal to 0
Four allocation functions

1. Linear
2. Exponential
3. Quadratic
4. Logarithmic
Linear allocation function

\[
\beta(z_\beta) = \hat{\beta} - a_\beta z_\beta
\]
\[
X(z_X) = \hat{X} - a_X z_X
\]
\[
T(z_T) = \hat{T} - a_T z_T
\]

• Assume \( \hat{\beta} \geq a_\beta Z, \hat{X} \geq a_X Z, \) and \( \hat{T} \geq a_T Z \)

• Decision maker should only allocate resources to reduce one of the three resilience factors based on
\[
\max \left\{ \frac{a_\beta}{\hat{\beta}}, \frac{a_X}{\hat{X}}, \frac{a_T}{\hat{T}} \right\}
\]

• Focuses resources on the factor whose initial parameter is already small and where effectiveness is large
Exponential allocation function

\[
\beta(z_\beta) = \hat{\beta} \exp(-a_\beta z_\beta)
\]

\[
X(z_X) = \hat{X} \exp(-a_X z_X)
\]

\[
T(z_T) = \hat{T} \exp(-a_T z_T)
\]

- Decision maker should only allocate resources to reduce one of the three resilience factors based on \(\max\{a_\beta, a_X, a_T\}\)
- Decision depends only the effectiveness and not the initial values
\[ \beta(z_\beta) = \hat{\beta} - b_\beta z_\beta + a_\beta z_\beta^2 \]
\[ X(z_X) = \hat{X} - b_X z_X + a_X z_X^2 \]
\[ T(z_T) = \hat{T} - b_T z_T + a_T z_T^2 \]

Assume \( z_\beta \leq \frac{b_\beta}{2a_\beta}, z_X \leq \frac{b_X}{2a_X}, z_T \leq \frac{b_T}{2a_T} \) so that functions are always decreasing
Optimal to allocate to three factors
Logarithmic allocation functions

\[ \beta = \hat{\beta} - a_\beta \log(1 + b_\beta z_\beta) \]
\[ X = \hat{X} - a_X \log(1 + b_X z_X) \]
\[ T = \hat{T} - a_T \log(1 + b_T z_T) \]
Optimal to allocate to three factors
• $\hat{\beta} \sim U(0,1)$, $\hat{X} \sim U(0,1)$, $\hat{T} \sim U(0,30)$
• Effectiveness parameters: $a_\beta, b_\beta, a_X, b_X, a_T, b_T$
  – Uniform distribution
  – Allocation functions are not negative
  – Other requirements are met

<table>
<thead>
<tr>
<th>Percent of simulations</th>
<th>Quadratic allocation</th>
<th>Logarithmic allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sufficient conditions met</td>
<td>0.4</td>
<td>51</td>
</tr>
<tr>
<td>Optimal to allocate to all 3 factors</td>
<td>1.3</td>
<td>91</td>
</tr>
</tbody>
</table>
Uncertainty with independence

- Assume $\hat{\beta}, \hat{X}, \hat{T}, a_{\beta}, b_{\beta}, a_X, b_X, a_T, b_T$ have known distributions
- Assume independence
- Maximize expected resilience $E[R_{\star}(\beta, X, T)] = 1 - \frac{E[\beta]E[X]E[T]}{T^*}$
- Linear and quadratic allocation functions (same as with certainty)
- Logarithmic allocation function: more likely to allocate to reduce all three factors than with certainty
Exponential allocation, uncertainty

Always a convex optimization problem
• Assume dependence among uncertain parameters
• Linear: allocate to reduce one or all three factors
• Exponential
  – Convex optimization problem
  – May allocate to reduce one, two, or three factors
  – Allocation may be influenced by $\hat{\beta}$, $\hat{X}$, and $\hat{T}$
• Quadratic and logarithmic: no special properties
Uncertainty without probabilities

- Each parameter is bounded above and below, i.e. $\underline{\beta} \leq \hat{\beta} \leq \bar{\beta}$ and $\underline{a_\beta} \leq a_\beta \leq \bar{a_\beta}$

- Maxi-min approach

  \[
  \max_{\beta} \min \left( R^*_\beta \left( \beta(z_\beta), X(z_X), T(z_T) \right) \right)
  \]

- Same rules as the case with certainty but choose worst-case parameters to determine allocation, i.e. $\hat{\beta}$ and $\underline{a_\beta}$
### Summary

<table>
<thead>
<tr>
<th>Allocation function</th>
<th>Certainty</th>
<th>Uncertainty with independence</th>
<th>Uncertainty with dependence</th>
<th>Uncertainty with no probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Reduce 1 factor</td>
<td>Reduce 1 factor</td>
<td>Reduce 1 or 3 factors</td>
<td></td>
</tr>
<tr>
<td>Exponential</td>
<td>Reduce 1 factor</td>
<td>Reduce 1, 2, or 3 factors</td>
<td>Reduce 1, 2, or 3 factors</td>
<td>Same as case with certainty but use worst-case parameters</td>
</tr>
<tr>
<td>Quadratic</td>
<td>May reduce 3 factors but not likely</td>
<td>May reduce 3 factors but not likely</td>
<td>Reduce 1, 2, or 3 factors</td>
<td></td>
</tr>
<tr>
<td>Logarithmic</td>
<td>Often reduce 3 factors</td>
<td>Often reduce 3 factors</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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