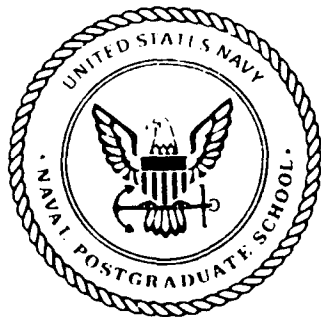


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**THESIS**

PREDICTION OF ATTITUDE STABILITY OF  
ASYMMETRIC DUAL-SPIN STABILIZED SPACECRAFT  
USING IMPROVED LIQUID SLOSH MODEL

by

Michael J. Szostak

June 1991

Thesis Advisor:

Prof. Brij Agrawal

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22a Name of Responsible Individual <b>B. Agrawal</b>	22b Telephone (Include Area code) <b>(408) 646-3338</b>	22c Office Symbol <b>AA/Ag</b>
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Prediction of Attitude Stability  
of Asymmetric Dual-Spin Stabilized Spacecraft  
Using Improved Liquid Slosh Model

by

Michael J. Szostak  
Lieutenant Commander, United States Navy  
B.S., United States Naval Academy, 1979

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN ASTRONAUTICAL ENGINEERING

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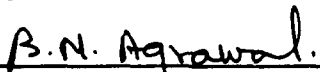
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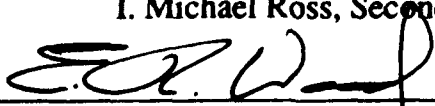
Author:

  
Michael J. Szostak

Approved By:

  
Brij N. Agrawal, Thesis Advisor

  
I. Michael Ross, Second Reader

  
E. R. Wood, Chairman, Department of Aeronautics and  
Astronautics

## ABSTRACT

The "rigid slug" method for modelling sloshing liquid fuel aboard dual-spin stabilized spacecraft has been shown to be inadequate by recent flight data. This "rigid slug" model and a uniform gravity model put forth by Abramson is examined in detail. The Abramson model is incorporated into a computer simulation written specifically to predict spacecraft attitude. An analysis is performed with both the modified and unmodified versions of this simulation to determine the boundaries of stability for rotor and platform asymmetries. The results show that the improved model is better able to predict spacecraft attitude.



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## I. INTRODUCTION

### A. BACKGROUND

Spin stabilized spacecraft have been in existence almost from the very start of man's artificial satellite program. This type of attitude control for satellites uses an inherently simple principle, that is, to provide gyroscopic stiffness for inertial pointing by rotation of the whole or part of the satellite. "Dual spin" satellites are a subset of spin stabilized spacecraft and have two bodies, a rotor and a platform, that spin about a common axis. The rotor spins at a high angular rate to provide inertial pointing while the platform generally spins at a rate of one revolution per orbit period to be earth pointing.

Under the current trend of using liquid fuelled apogee motors, liquid propellant constitutes almost two thirds of rotor mass in transfer orbit. With fuel loads now comprising such a high fraction of spacecraft mass, it is critical that fuel slosh effects be better understood. Just as flexible elements on the spacecraft can interact stably or unstably, so can sloshing fuel interact stably or unstably.

Recent flight data from the first two LEASAT communications satellites have raised questions about the current modelling of sloshing liquid fuel. Both satellites experienced instability during transfer orbit that was not predicted by the computer model used. The simulation that was used models liquid propellant as spherical pendulums attached to the centers of the fuel tanks. The principals behind this model need to be examined.

## **B. OBJECTIVE**

The primary objective of this study is to improve the modelling of liquid propellant aboard dual spin stabilized spacecraft. This will be accomplished by using an analytical model of sloshing liquid put forth by Abramson [Ref. 1]. Abramson's model, called the uniform gravity model, treats liquid as a pendulum whose parameters such as mass of the pendulum, length of the pendulum's arm and frequency of oscillation are determined by fill fraction of the tank. This model is different than most in that only a portion of the liquid in the tank is modeled as a pendulum and that the rest of the fuel is an addition to the dry moment of inertia of the rotor. Abramson's model will be incorporated into a computer simulation authored by Chung [Ref. 2]. Chung's computer program was originally intended to model the attitude of INTELSAT VI, but it can be adapted to model the attitude of almost any dual-spin spacecraft with large liquid mass. The liquid model that Chung used for his simulation is nearly identical to the one used for the LEASAT flights. This model, named the "rigid slug" model, depicts liquid fuel as spherical pendulums with two degrees of freedom and assumes that 100 percent of the liquid propellant is in motion. The goal of this study is to incorporate the more accurate Abramson model into Chung's simulation.

The second objective of this study is to explore the boundary between stability and instability of dual-spin stabilized spacecraft with asymmetric rotors and platforms. Specifically, it is highly desirable to know whether liquid slosh is stabilizing or not for various degrees of rotor and platform asymmetry, inertia ratios and fuel loads. Knowing this point of stability/instability, engineers will know the upper limit in asymmetry of the platforms or the rotors, and satellite controllers can more accurately predict attitude control requirements.

To perform the rotor asymmetry analysis, Chung's computer simulation with the "rigid slug" model of liquid fuel will be used along with a version of the simulation modified with Abramson's model. Chung's simulation, using equations of motion to model spacecraft attitude as a function of time, can predict stability. Starting with a stable spacecraft, rotor asymmetry will be increased until instability occurs. This point will be called the stability cutoff point. For the study, inertia ratio is the ratio of the spin axis moment of inertia to the average of the transverse moments of inertia. Instability is defined where the spacecraft nutation angle does not decrease passively. The entire analysis will be performed with both the modified and unmodified versions of Chung's simulation.

To perform the platform asymmetry analysis, only Chung's modified simulation will be used. The objective of this portion of the study will be the determination of the stability boundary limits of an asymmetric platform and an axisymmetric rotor.

### **C. LITERATURE REVIEW**

In 1963, W.T. Thomson [Ref. 3] presented a paper on the motion of a rotating asymmetric body with internal dissipation. Noting that this moment free motion resulted in elliptical functions, Thomson presented the results in a form very similar to the symmetric case. The solution stated the nutation angle rate is a function of kinetic energy rate, angular momentum, and the moments of inertia.

Peter Likins [Ref. 4] in 1967, was the first to extend initial spin stability analysis to non axisymmetric vehicles. All previous work to that point involved systems with symmetrical platforms and rotors. By Routhian analysis though, Likins showed that a system with an asymmetric despun platform could be made

asymptotically stable. Energy losses in the rotor were found to cause instability if rotor spin inertia was less than the average total transverse inertia. This is the famous energy sink stability criteria, that is, the ratio of the axial moment of inertia of the rotor to the average of the transverse moments of inertia of the spacecraft must be greater than one.

T.M. Spencer [Ref. 5] in 1973 challenged Likins on several points and questioned his assumptions concerning the energy sink derivation. Specifically, Spencer believed that using the algebraic energy sink equation is an oversimplification for asymmetric satellites. Spencer stated that the kinetic energy dissipation rate in the rotor is a time varying factor, and that variations must be determined before averaging can be used to get a stability result. Spencer concluded by proposing that the geometric mean transverse moment of inertia is a crucial stability parameter, rather than the algebraic mean as forwarded by Likins.

Chung, in 1985, stated that previous works applying the energy sink equation to dual spin spacecraft made certain assumptions that are not applicable for spacecraft with a high liquid fraction. Specifically, the general equation for angular momentum does not take into account accurately the effects of liquid slosh. He added that the equation for kinetic energy does not always decrease and concluded with a warning that the energy sink results be used with caution for spacecraft with large liquid fuel loads and high inertia asymmetries. Chung authored a computer simulation for spacecraft attitude based on the "rigid slug" model at this time.

In 1987, Slafer [Ref. 6] showed that the "rigid slug" model like the one used by Chung to model dual-spin spacecraft attitude was not accurate for spacecraft with high liquid fuel loads. Slafer analyzed the interaction between the control system and the large amount of liquid propellant aboard the first two LEASAT

synchronous orbit communications satellites. Slafer's post flight analysis included several simulations and compared them with actual flight data. The "rigid slug" model that had been previously used at Hughes for static stability analysis, failed to predict the observed flight instability. Abramson's model however, correctly predicted the observed flight instability.

Myers [Ref. 7], in 1990, used Chung's simulation to show that an inertia ratio greater than one was not alone criteria for passive stability, but that the inertia ratio had to be greater than one by a certain amount. This stability point was dependent on fuel load and platform asymmetry. Myers also showed that increasing platform asymmetry increased stability.

A simple algebraic expression like the energy sink equation has shown itself useful for first order stability approximations. This is especially true if a dual spin satellite can be modelled as a rigid, rotating body. To use this simple equation as a definitive stability parameter for high liquid fuel spacecraft though, is circumspect. The energy sink equation therefore, will only be used as an initial starting point in the rotor and platform asymmetry analysis portion of this study. Chung's simulation program will be used to further define the boundaries of passive stability for rotor and platform asymmetry as a function fuel load and inertia ratio. Slafer's results bring into question Chung's "rigid slug" method of modelling liquid and therefore several enhancements to Chung's simulation will be proposed based on the uniform gravity model.

#### **D. ORGANIZATION OF STUDY**

Chapter II of this study will further define dual spin satellites and also provide a derivation of the energy sink equation. A description of Chung's computer simulation as well as the required inputs and output for the program are provided in Section C of this chapter. The various satellite configurations

used in the study are described in Section D. Abramson's model and the proposed changes to Chung's simulation are provided in Section E.

Chapter III presents a description of both the "rigid slug" and the uniform gravity model. Detailed graphics, and equations and curves for the computation of parameters are included. Lastly, Slafer's results comparing the two models with the actual LEASAT flight data are presented.

Chapter IV contains the procedures to generate the input data and the procedures to run the main simulation program. The spreadsheet written to generate platform and rotor moments of inertia for a given inertia ratio and fuel load is described in Section A. Also described in Section A is a program written by Chung to convert INTELSAT VI parameters to a format readable by the main simulation program. Section B describes the procedures used for the rotor and platform asymmetry analysis and Section C describes how Abramson's model is incorporated into Chung's simulation.

Chapter V presents the results of the rotor asymmetry analysis for both the modified and unmodified computer simulations. Plots are provided for each of the models to show the location of the stability cutoff points. For the platform asymmetry analysis, the minimum inertia ratio for zero, five and ten percent platform asymmetry is determined. An analysis is performed to explain the curves.

Chapter VI presents the summary of the findings and the conclusions.

## II. BACKGROUND

This chapter will clarify terms and definitions of the analysis of rotor and platform asymmetry of dual-spin stabilized spacecraft. A derivation of the energy sink equation and a description of the simulation program authored by Chung will be provided. The method used to derive the various satellite configurations, as well as several improvements to Chung's simulation will also be discussed.

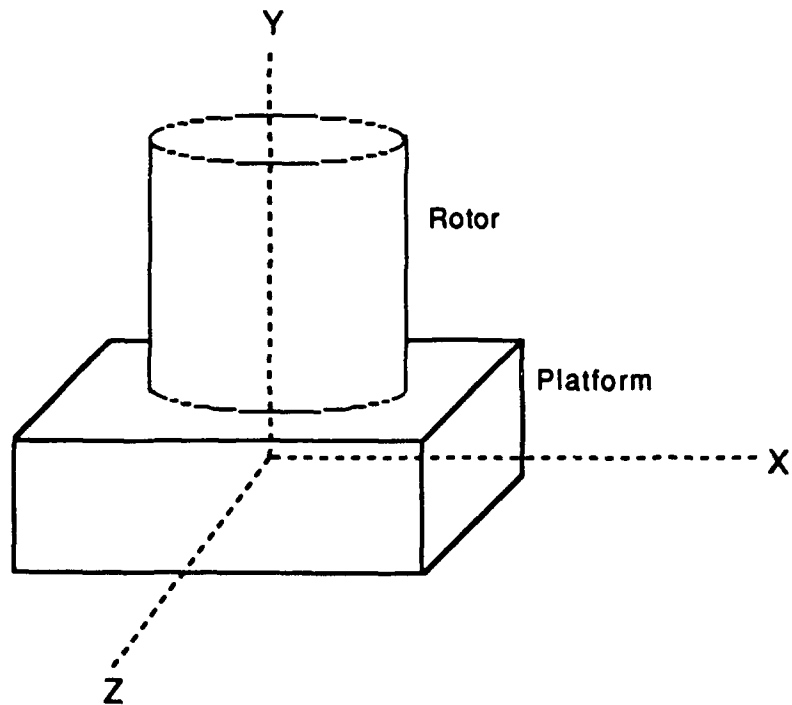
### A. DUAL-SPIN SATELLITES

"Dual-spin" spacecraft are comprised of two primary bodies capable of unlimited relative rotation about a common axis. Typically, the spun section is called the rotor while the despun body the platform. A motor is usually mounted on the platform which is used to drive the rotor so that it maintains a constant rotation rate relative to the platform. The rotation speed of the rotor is dependent on the moments of inertia and required accuracy in orientation of the spacecraft. The platform can be either completely despun or rotate at a rate as required by the payload. The utility of such a combination of bodies allows the simplicity and reliability of spin stabilization to be used with the unidirectional pointing of components like sensors and antennas. Dual-spin satellites vary in size from the 70 kg INTELSAT I to the 12,000 kg LEASAT.

As the size of dual-spin satellites has increased over the years, so has the amount of fuel these satellites are required to carry. In addition, extended lifetimes also have necessitated larger fuel loads. Spacecraft fuel is normally located in spherical tanks on the rotor that are extended out from the axis of



rotation. The number and size of the tanks is a function of the mission and lifetime of the satellite. An idealized dual-spin system is depicted in Figure 1.



**Figure 1. Idealized Dual-Spin Stabilized Spacecraft**

## **B. ENERGY SINK DERIVATION**

The derivation of the stability condition is provided using Likins' model of an idealized system depicted in Figure 1. The figure depicts a platform P of arbitrary inertia properties and an axisymmetric rotor R. The center of mass of both bodies lies along the axis Y, the axis of rotation. As a starting point the equation of motion governing the entire system S is

$$\mathbf{M} = \mathbf{h} \quad (1)$$

where  $M$  is the applied moment about the system mass center  $O$ , and  $h$  the angular momentum about  $O$ . The dot over the vector  $h$  depicts time rate of change in inertial space. The resulting linearized Euler equations of motion are

$$M_1 = I_1 \dot{\omega}_1 + (I_3 - I_2) \omega_p \omega_2 + I_r \omega_r \omega_2 = 0 \quad (2)$$

$$M_2 = I_2 \dot{\omega}_2 + (I_1 - I_3) \omega_p \omega_1 - I_r \omega_r \omega_1 = 0 \quad (3)$$

$$M_3 = I_3 \dot{\omega}_p + I_r \dot{\omega}_r + I_2 \omega_1 \omega_2 - I_1 \omega_1 \omega_2 = 0 \quad (4)$$

$$M_r = I_r (\dot{\omega}_p + \dot{\omega}_r) \quad (5)$$

where  $M_1$ ,  $M_2$  and  $M_3$  are the resulting moments about each of the primary axes and  $M_r$  the moment applied about the rotor by the platform.  $I_1$  and  $I_2$  are the transverse moments of inertia of the system while  $I_3$  is the system's axial moment of inertia.  $I_r$  is the axial moment of inertia of the rotor. The transverse angular rates of the system are  $\omega_1$  and  $\omega_2$  while  $\omega_p$  is the rotation rate of the platform and  $\omega_r$  the relative rotation rate of the rotor with respect to the platform. The four unknowns,  $\omega_1$ ,  $\omega_2$ ,  $\omega_p$  and  $\omega_r$  can now be solved for using the four equations.

Equations (2) and (3) can be further simplified under the assumption that the system consists of a rigid asymmetric body with a rigid axisymmetric rotor attached, and no external moments where

$$\lambda_1 = ((I_3 - I_2) \omega_p + I_r \omega_r) / I_1 \quad (6)$$

$$\lambda_2 = ((I_3 - I_1) \omega_p + I_r \omega_r) / I_2 \quad (7)$$

Eqs. (2) and (3) become

$$\dot{\omega}_1 + \lambda_1 \omega_2 = 0 \quad (8)$$

$$\dot{\omega}_2 - \lambda_2 \omega_1 = 0 \quad (9)$$

Changes in rotation rates of the two bodies about their common axis are ignored assuming small motor and bearing friction torques. Assuming the initial conditions

$$\omega_2(0) = \omega_0 \quad \omega_1(0) = 0 \quad (10)$$

the solutions to (8) and (9) become

$$\omega_1 = -\omega_0 \sqrt{\frac{\lambda_1}{\lambda_2}} \sin(\sqrt{\lambda_1 \lambda_2} * t) \quad (11)$$

$$\omega_2 = \omega_0 * \cos(\sqrt{\lambda_1 \lambda_2} * t) \quad (12)$$

The resulting necessary condition for stability is

$$\lambda_1 * \lambda_2 > 0 \quad (13)$$

The kinetic energy for this dual spin system is

$$T = 1/2 I_1 \omega_1^2 + 1/2 I_2 \omega_2^2 + 1/2 I_3 \omega_p^2 + 1/2 I_r \omega_r^2 + I_r \omega_r \omega_p \quad (14)$$

and the square of the system angular momentum about O is

$$h^2 = I_1^2 \omega_1^2 + I_2^2 \omega_2^2 + (I_3 \omega_p + I_r \omega_r)^2 \quad (15)$$

The angular momentum magnitude of the nominal motion is

$$h_0 = I_3 \omega_p + I_r \omega_r \quad (16)$$

The time derivative of Eqs. (14) and (15) yields the following equations. Note that the kinetic energy rate is negative because of an assumed energy dissipating device on the system and that the angular momentum is constant

$$d/dt (T) = I_1 \omega_1 \dot{\omega}_1 + I_2 \omega_2 \dot{\omega}_2 + I_3 \omega_p \dot{\omega}_p + I_r \omega_r \dot{\omega}_r + I_r \omega_r \dot{\omega}_p + I_r \omega_p \dot{\omega}_r < 0 \quad (17)$$

$$1/2 d/dt (h^2) = I_1^2 \omega_1 \dot{\omega}_1 + I_2^2 \omega_2 \dot{\omega}_2 + (I_3 \omega_p + I_r \omega_r)(I_3 \dot{\omega}_p + I_r \dot{\omega}_r) = 0 \quad (18)$$

The solutions to the Euler equations (Eqs. 11 and 12) are no longer valid under the just mentions conditions. However, if the effects of the energy dissipating device are felt over time, the form of Eqs. (11) and (12) is correct with  $\omega_0$  taking the form of  $\omega_0 = \omega_0(t)$ , a slowly varying function of time. Averaging over the period  $\tau = 2\pi/2\lambda_2(\lambda_1/\lambda_2)^{0.5}$  and assuming slow time variations of  $\omega_0$ , the approximation for the kinetic energy becomes

$$\begin{aligned} \bar{T} = & 1/2 I_1 \omega_0 \dot{\omega}_0 (\lambda_1/\lambda_2) + 1/2 I_2 \omega_0 \dot{\omega}_0 + I_3 \omega_p \dot{\omega}_p \\ & + I_r \omega_r \dot{\omega}_r + I_r \omega_r \dot{\omega}_p + I_r \omega_p \dot{\omega}_r \end{aligned} \quad (19)$$

and the moment equation

$$0 = 1/2 I_1^2 \omega_0 \dot{\omega}_0 (\lambda_1 / \lambda_2) + I_2^2 \omega_0 \dot{\omega}_0 + (I_3 \dot{\omega}_p + I_r \dot{\omega}_r)(I_3 \dot{\omega}_p + I_r \dot{\omega}_r) \quad (20)$$

To obtain the kinetic energy rate in terms of the platform and rotor angular rates, it is first necessary to solve Eq. (20) for  $\omega_0 \dot{\omega}_0$ . This results in the following

$$\omega_0 \dot{\omega}_0 = -2\lambda_2 (I_3 \dot{\omega}_p + I_r \dot{\omega}_r)(I_3 \dot{\omega}_p + I_r \dot{\omega}_r) / (I_1^2 \lambda_1 + I_2^2 \lambda_2) \quad (21)$$

Substituting this into Eq. (19) yields

$$\begin{aligned} \bar{T} = & -I_p \dot{\omega}_p (-\dot{\omega}_p + (I_3 \dot{\omega}_p + I_r \dot{\omega}_r) * (I_1 \lambda_1 + I_2 \lambda_2) / (I_1^2 \lambda_1 + I_2^2 \lambda_2)) \\ & - I_r (\dot{\omega}_r + \dot{\omega}_p) * (-(\dot{\omega}_r + \dot{\omega}_p) + (I_3 \dot{\omega}_p + I_r \dot{\omega}_r) * (I_1 \lambda_1 + I_2 \lambda_2) / (I_1^2 \lambda_1 + I_2^2 \lambda_2)) \end{aligned} \quad (22)$$

where  $I_p$  is the axial moment of inertia of the platform. Recalling the definition of the nominal angular momentum magnitude, the expression for  $\bar{T}$  can be further simplified using the following substitution

$$\lambda_o = h_o * (I_1 \lambda_1 + I_2 \lambda_2) / (I_1^2 \lambda_1 + I_2^2 \lambda_2) \quad (23)$$

which results in

$$\bar{T} = -I_p \dot{\omega}_p (\lambda_o - \dot{\omega}_p) - I_r (\dot{\omega}_r + \dot{\omega}_p) (\lambda_o - (\dot{\omega}_r + \dot{\omega}_p)) \quad (24)$$

Using the following definitions in order to simplify

$$\lambda_p = \lambda_o - \dot{\omega}_p \quad \text{and} \quad \lambda_r = \lambda_o - (\dot{\omega}_r + \dot{\omega}_p) \quad (25)$$

the approximation for kinetic energy rate becomes

$$\bar{T} = -I_p \dot{\omega}_p \lambda_p - I_r (\dot{\omega}_r + \dot{\omega}_p) \lambda_r \quad (26)$$

The two terms on the right side of Eq. (26) can be written as

$$P_p = -I_p \dot{\omega}_p \lambda_p \quad \text{and} \quad P_r = -I_r (\dot{\omega}_r + \dot{\omega}_p) \lambda_r \quad (27)$$

which are the first approximations for the average energy dissipation rates for the platform and the rotor. These equations can be rewritten in order to be substituted into Eq. (21).

$$I_p \dot{\omega}_p = -P_p / \lambda_p \quad I_r (\dot{\omega}_r + \dot{\omega}_p) = -P_r / \lambda_r \quad (28)$$

The substitution yields

$$\omega_0 \dot{\omega}_0 = \left( \frac{P_D}{\lambda_p} + \frac{P_r}{\lambda_r} \right) \frac{(2h_0 \lambda_2)}{(I_1^2 \lambda_1 + I_2^2 \lambda_2)} \quad (29)$$

As a necessary and sufficient condition for stability

$$\omega_0 \dot{\omega}_0 < 0 \quad (30)$$

and since  $\lambda_1 \lambda_2 > 0$  was shown to be necessary for stability, and  $h_0 > 0$  by convention, then Eqs. (29) and (30) together result in

$$\frac{P_D}{\lambda_p} + \frac{P_r}{\lambda_r} < 0 \quad (31)$$

Since both the energy dissipating rates of the platform and the rotor must be negative, both  $\lambda_p$  and  $\lambda_r$  must be positive or if one is negative, the respective energy dissipating rate must be large enough so that the condition of Eq. (31) is satisfied.

There are several special cases to study, but for the purposes of this analysis we will consider the case of a despun platform, for which the rotation speed of the platform  $\omega_p$ , will be zero. With this assumption Eqs. (6) and (7) simplify to

$$\lambda_1 = I_r \omega_r / I_1 \quad (7)$$

$$\lambda_2 = I_r \omega_r / I_2 \quad (8)$$

Substitution into Eq. (20) will simplify the value for  $\lambda_0$  and result in the following equation

$$\lambda_0 = \frac{2I_r \omega_r}{(I_1 + I_2)} \quad (20)$$

Substituting this value into Eq. (23)

$$\lambda_r = \frac{2I_r \omega_r}{(I_1 + I_2)} - \omega_r \quad (25)$$

Factoring out an  $\omega_r$  results in

$$\lambda_r = \omega_r \left( \frac{2I_r}{(I_1 + I_2)} - 1 \right) \quad (25)$$

For this case recall that  $\omega_p$  is zero and assume that  $P_p$  also equals zero. Eq. (31) then becomes

$$\frac{P_r}{\lambda_r} < 0$$

If  $P_r < 0$ , that is assume damping is on the rotor (ie liquid fuel), then  $\lambda_r$  must be positive. Therefore Eq. (25) then becomes

$$\omega_r \left( \frac{2I_r}{(I_1 + I_2)} - 1 \right) > 0 \quad (25)$$

Dividing both sides by  $\omega_r$  and adding one to both sides results in the very familiar energy sink equation

$$\frac{2I_r}{(I_1 + I_2)} > 1 \quad (32)$$

Likins' derivation cited above does not take into account asymmetric rotors, but it can be shown that the form is the same as Eq. (32). The conclusion drawn from this fact is that it does not matter in which body the damping mechanism is located. In addition, Likins proved that a body could rotate about its axis of least inertia and be asymptotically stable, but fell short of proving that the energy sink equation had application for large liquid loads (because of the simplifying equations made). Because of its widespread use in industry, the criteria put forth in Eq. (32) will be used in this analysis to as an initial stability parameter.

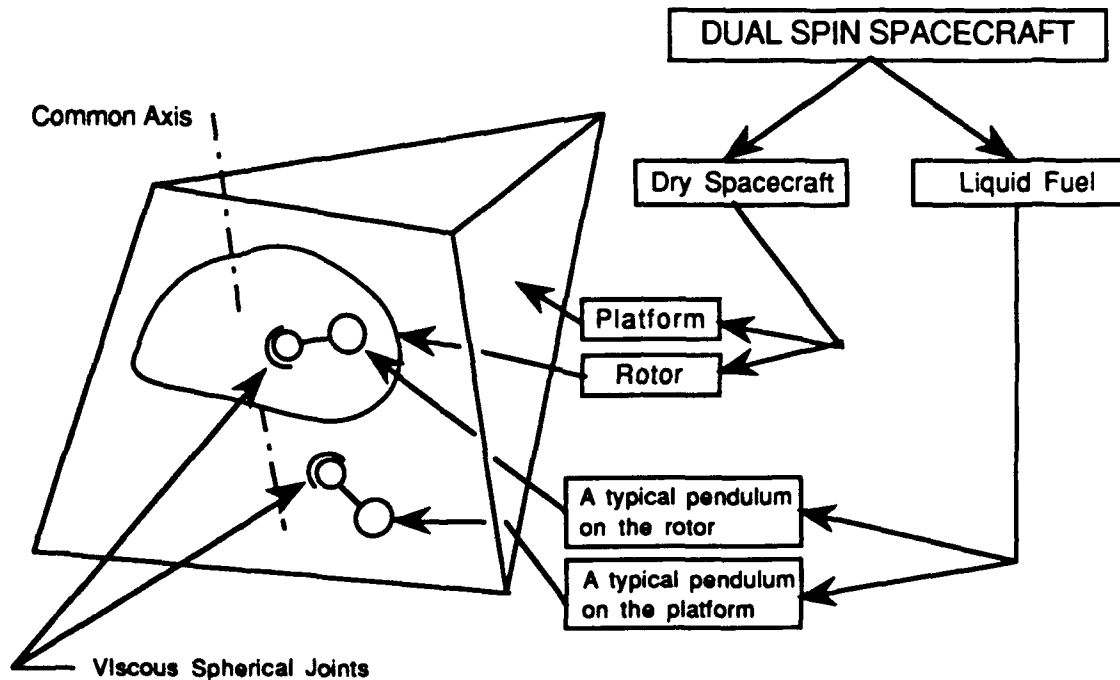
### C. SIMULATION DESCRIPTION

The simulation program authored by Chung can model the motion of almost any dual-spin spacecraft. The actual modeling consists of determining certain time dependent variables given initial conditions for these variables. The simulation can accommodate fuel tanks on either the platform or the rotor. The size, location and number of tanks is determined by the simulation user. A schematic representation of the dual spin stabilized satellite used by Chung is shown in Figure 2.

Using the model above, differential equations governing generalized coordinates and generalized speeds that characterize all motions of the spacecraft are written. The differential equations of motion are of the form.

$$\dot{x}_i = f_i(x_1, \dots, x_n, T) \quad \text{where } i = 1, \dots, n$$

where  $f_1, f_2, \dots, f_n$  are in general nonlinear functions of their arguments. These functions are assumed to be continuous and to have continuous derivatives. The equations are solved numerically in a subroutine of the main simulation program.



**Figure 2. Dual Spin Spacecraft with Liquid in Spherical Tanks**

Kane's method is used to obtain the exact equations of motion for the model spacecraft. This method involves the following three steps

1. The derivation of expressions for generalized inertia forces.
2. The derivation of expressions for generalized active forces.
3. Substitution into the equation

$$F_r + F_r^* = 0 \quad (r = 1, 2, \dots, N_u)$$

where  $F_r$  and  $F_r^*$  are the generalized active forces and the generalized inertia forces and  $N_u$  is the total number of generalized speeds. In addition to the  $N_u$  equations of motion, kinematical equations governing generalized coordinates are also required.

The generalized active forces of the system are computed by summing all active forces on each part of the system. The parts of the system that are considered are the platform, rotor, and the spherical pendulum of the platform and/or rotor. It is assumed that there are no external forces acting on any of the spherical pendulums and that the resultant external forces on the platform and on the rotor are zero.

The generalized inertia forces of the system are also computed by considering contributions from the platform, rotor and the spherical pendulums on the platform and/or rotor.

The kinematical equations of motion are the equations governing the generalized speeds. These equations are primarily first order differential equations governing the motion of the spacecraft.

### **1. Input**

The required inputs into the simulation include mass and dry moments of inertia of the platform and the rotor, as well as their center of mass position from a reference point. Fuel tank data required is number, location, spherical damping coefficients and transverse and axial moments of inertia of the liquid assumed rigid. This data is input for the main simulation program via two input files.

### **2. Output**

Other quantities besides generalized speeds and generalized coordinates are produced as output from the simulation program. The quantities which are also useful in studying spacecraft motion include central angular momentum of

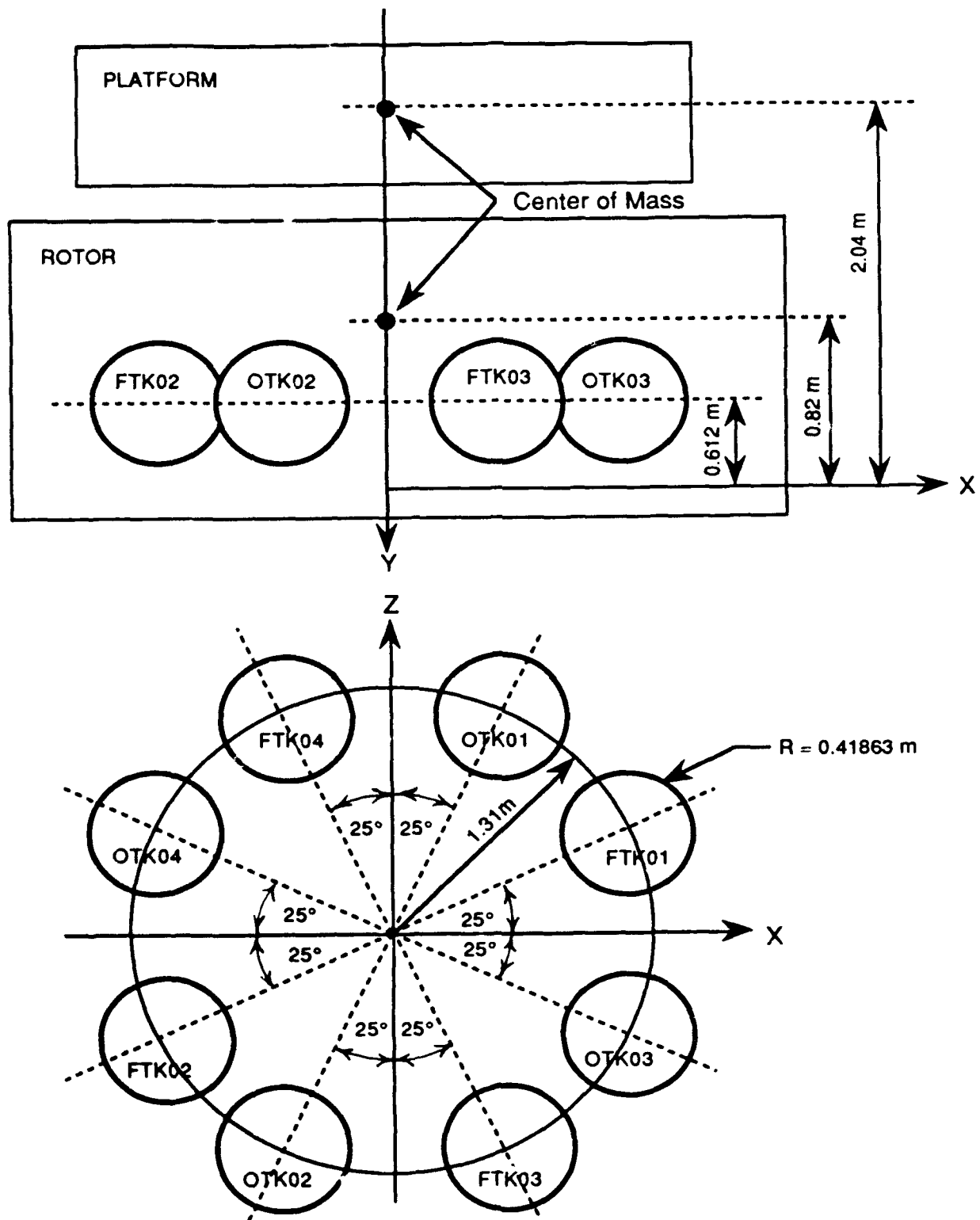


the system, several energy functions of the system, the velocity of the mass center of the system and the nutation angle. The energy functions available are the kinetic energy of the system, the energy dissipated through the viscous spherical joints, and the work done by the motor and the external forces. All the above quantities are produced as a function of time. It is the nutation angle as a function of time that is used a stability parameter throughout this study.

The attitude motion derived from Chung's model though, is decoupled from its orbital motion. Therefore, the above mentioned output parameters only represent the motion due to the generalized active and inertia forces. Chung has provided the equations to compute total motion, but these need to be solved for each specific orbit if complete information regarding the spacecraft attitude is desired.

#### **D. SATELLITE CONFIGURATION**

The satellite configuration used in the analysis is based on INTELSAT VI. This satellite has four fuel and four oxidizer tanks located on the rotor, symmetrically positioned around the axis of rotation. The placement of the rotor and platform as well as tank location and dimensions are depicted in Figure 3. The mass and dry inertia properties of the rotor and platform are listed in Table 1. These are the properties used in the unmodified simulation analysis. Fuel and oxidizer properties are listed in Table 2.



**Figure 3. Fuel Tank Arrangement for INTELSAT VI**

**TABLE 1. Mass and Inertia Properties of the Rotor and Platform  
(Dry)**

	Platform	Rotor
Mass	1058.8 kg	695.7 kg
$I_{xx}$	1587.1 kg m <sup>2</sup>	927.0 kg m <sup>2</sup>
$I_{yy}$	1518.3 kg m <sup>2</sup>	1166.2 kg m <sup>2</sup>
$I_{zz}$	1529.4 kg m <sup>2</sup>	973.7 kg m <sup>2</sup>
$I_{xy}$	0	0
$I_{xz}$	-44.4 kg m <sup>2</sup>	6.1 kg m <sup>2</sup>
$I_{yz}$	0	0

**TABLE 2. Liquid Fuel Characteristics**

	Fuel	Oxidizer
Viscosity	9.73 E-7 m <sup>2</sup> /s	2.92E-7 m <sup>2</sup> /s
Density	876.2 kg/m <sup>3</sup>	1448.3 kg/m <sup>3</sup>

### III. LIQUID SLOSH MODELS

The first two sections of the chapter discuss the two models of liquid slosh, the "rigid slug" model and Abramson's uniform gravity model. Section C presents Slafer's postflight analysis of the LEASAT instability problem. Slafer's comparison of the two models with actual flight data is also included.

#### A. RIGID SLUG MODEL

Chung's simulation models liquid propellant as two degree of freedom pendulums attached to a rigid, spinning rotor, with one pendulum for each propellant tank. This pendulum model (a "rigid slug") assumes that 100 percent of the fluid to be a distributed spherical mass, pivoting about the center of the tank. The pendulum mass is located at the fluid mass center. The location of this mass center determines the length of the pendulum arm. The frequency of the pendulum's oscillations is an input required by the simulation. Chung's model is depicted in Figure 4.

Chung's model of the pendulum's motion is very dependent on the following equation, which governs the angle  $q$ .

$$(I_E + m * l^2) \ddot{q} + C_E \dot{q} + (m * d * l * \Omega^2) q = 0$$

The variables in the above equation are either known or can be easily computed, except for the the inertia of the pendulum,  $I_E$ , which is a function of the natural frequency. The undamped natural frequency of the pendulum,  $\omega_n$ , is

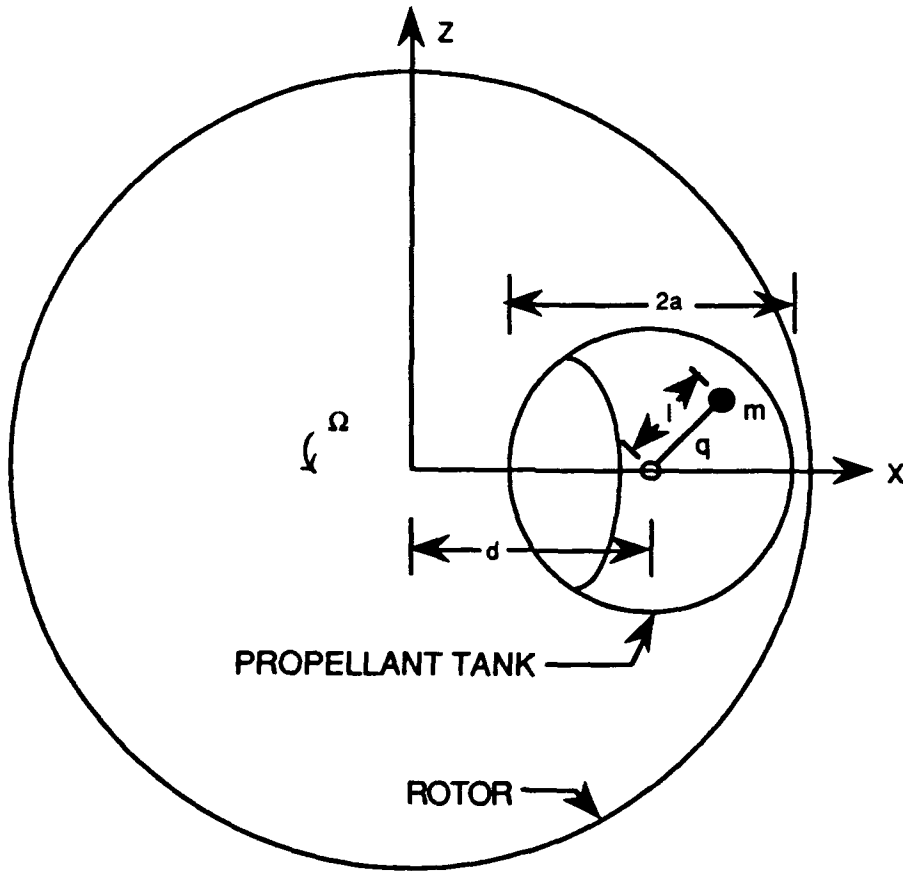
$$\omega_n = \Omega \sqrt{\frac{m * d * l}{I_E + m * l^2}}$$

where  $\Omega$  is the rotor rotation speed. Solving for the inertia term results in

$$I_E = m \left[ \left( \frac{\Omega}{\omega_n} \right)^2 * d * l - l^2 \right]$$

The inertia term and hence the accuracy of the results is determined by the user

given fundamental natural frequency. Chung's simulation provides a recommended frequency based on the analysis of Abramson. This value is used only to check the given value.



**Figure 4. Liquid Fuel Modelled as a Spherical Pendulum**

## **B. UNIFORM GRAVITY MODEL**

Abramson's model differs from the "rigid slug" model primarily in the fact that only a portion of the fuel is modelled as a spherical pendulum, and the rest as a point mass which is added to the dry properties of the rotor. Abramson's analytical model is depicted in Figure 5 with a schematic and in Figure 6 with the corresponding data curves. Figure 6 is taken from Reference 8. It is clear that

the fuel is separated into two different masses, one that is stationary and one that is free to move as a pendulum. Entering the curves with fill fraction, the values for pendulum mass and from that point mass, pendulum arm length, pendulum hinge point location, point mass location and liquid fuel slosh frequency can be extracted. All data extracted from the curves is normalized as a function of the specific spacecraft parameters.

**KEY:**

$m_{full} = \frac{4}{3}\rho\pi a^3$

$m_0 = m_{full} - m_1$

$m_1$  = pendulum mass

$\omega_0 = (g/a)^{0.5}$

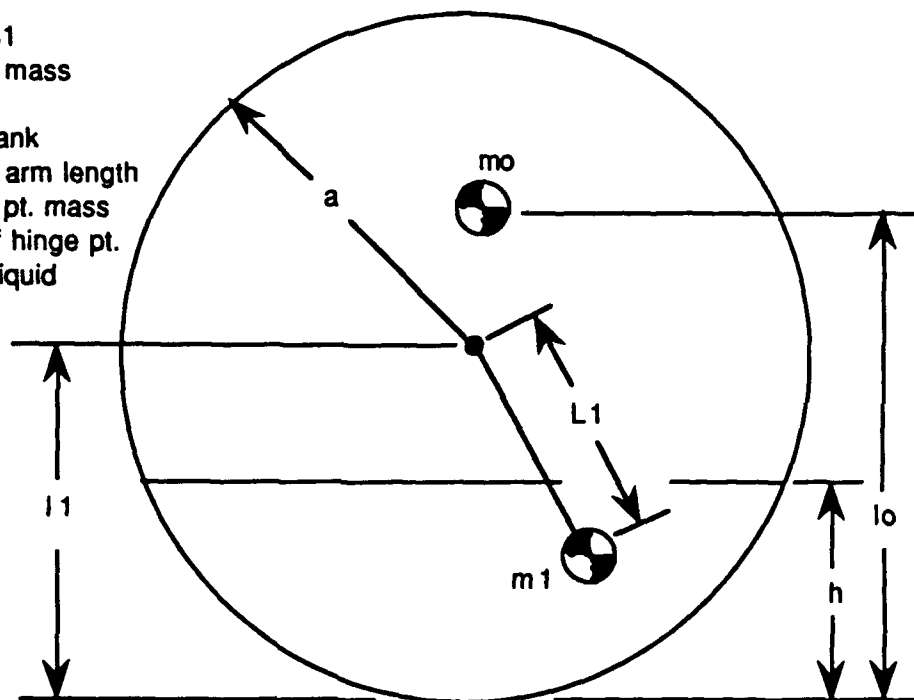
$a$  = radius of tank

$L_1$  = pendulum arm length

$l_0$  = location of pt. mass

$l_1$  = location of hinge pt.

$h$  = height of liquid



**Figure 5. Abramson's Analytical Model**

The fact that the pendulum's parameters vary as a function of fuel load differentiates this model from the "rigid slug" model. By incorporating the data

from Abramson's curves into Chung's simulation, the simulation is improved. The procedure how to do this is described in Chapter IV, Section C.

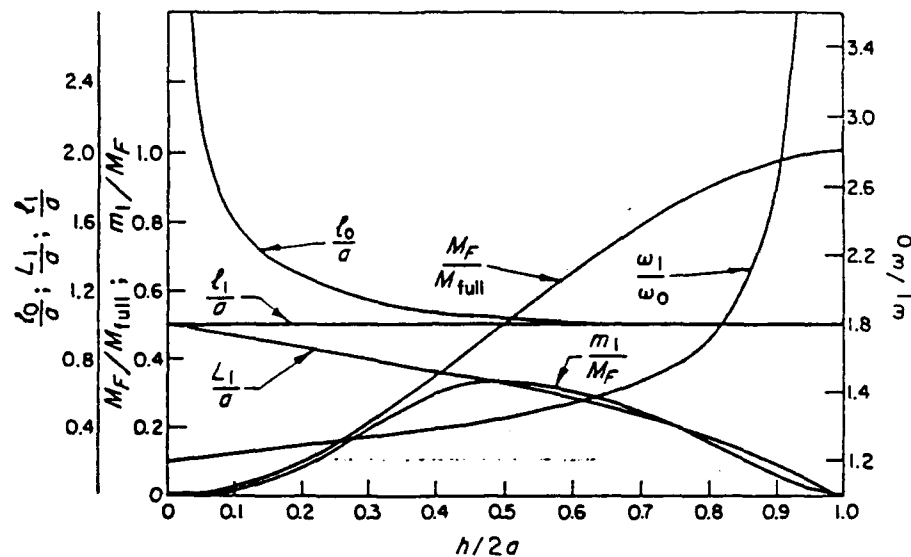
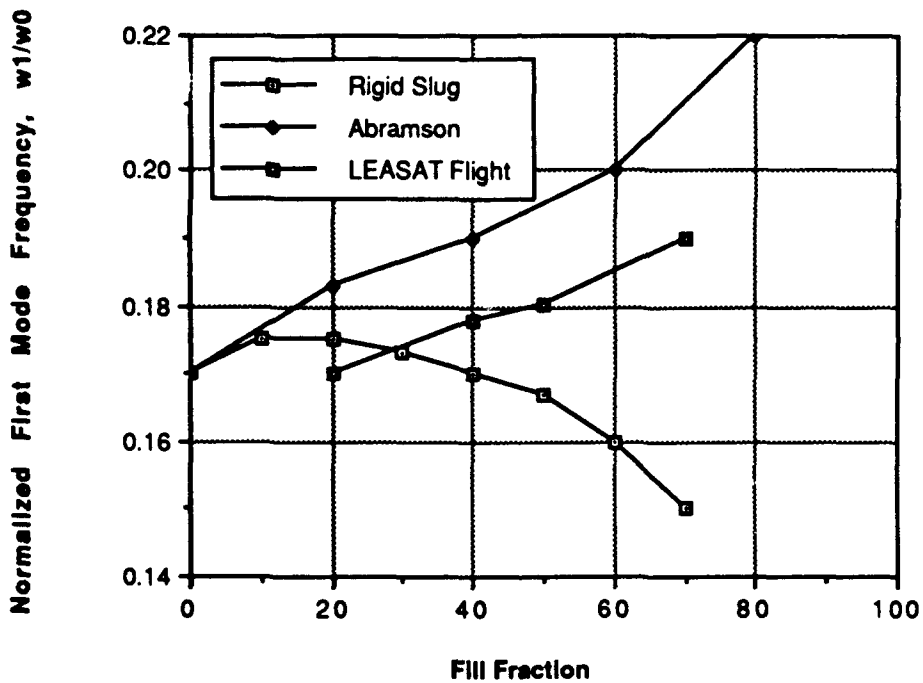


Figure 6. Abramson's Curves for Analytical Model [Ref. 8]

In Slafer's postflight tests, the rigid slug model could not duplicate the instability the LEASAT missions experienced. In fact, other shortcomings of the "rigid slug" model were also noted at this time.

Using a uniform gravity model similar to Abramson's however, although not predicting the exact results, Slafer did predict the mass and the frequency trends. Figure 7 shows the normalized first mode frequencies. In making the analogy to a mechanical pendulum, all modal frequencies of a pendulum should be considered. Slafer does not present the higher modes however, as the pendulum masses become very small for frequencies above the fundamental.

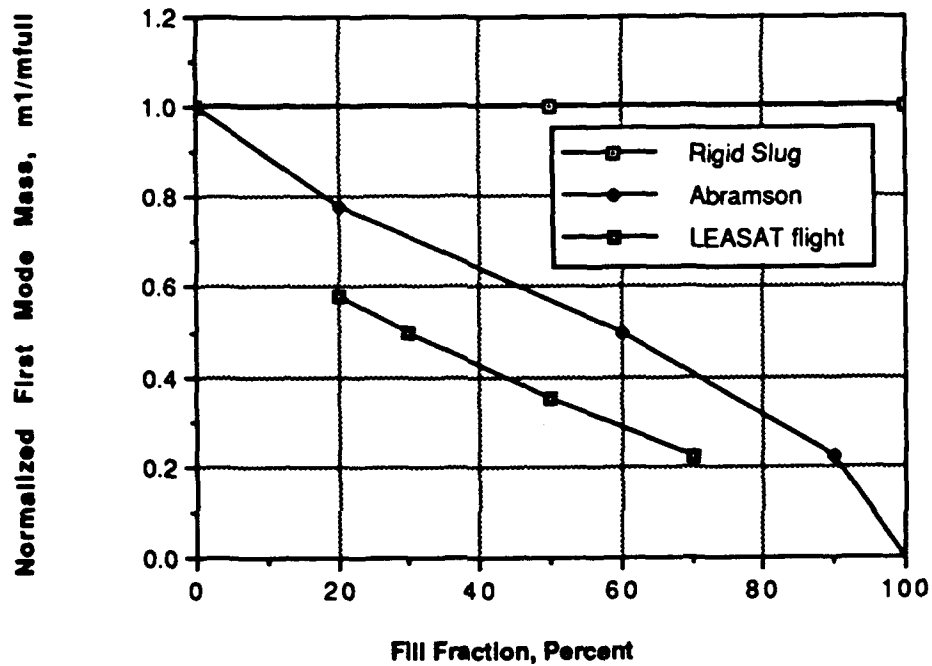


**Figure 7. Normalized First Mode Frequency Comparison**

The Abramson model predicted slightly higher first mode frequencies but the general trend matches the actual flight results very closely. The "rigid slug" model's accuracy for predicting first mode frequencies however, is in question. Only for selected fill fractions is the "rigid slug" model close to the actual flight



data. The value for  $\omega_0$  in this figure is  $(g/a)^{0.5}$  where  $g$  is the value for gravity and  $a$  is the fuel tank radius. The values for  $g$  are  $32.2 \text{ ft/sec}^2$  for the Abramson uniform gravity model and  $d*\Omega^2$  for the rigid slug model and the LEASAT flight data. Figure 8 shows the normalized first mode masses.



**Figure 8. Normalized First Mode Mass Comparison**

Throughout the entire range of fuel loading, the Abramson model's normalized mass ratio is slightly larger, but matches the trend of the LEASAT flight data very closely. The rigid slug model depicts that 100 percent of the fuel is modelled as a pendulum.

## **IV. PROCEDURE**

The procedures to generate the data for both the platform and the rotor asymmetry analysis are described in this chapter. The procedural steps to modify the simulation for the actual analysis are also described.

### **A. SYSTEMS PARAMETERS FOR ASYMMETRY ANALYSIS**

The main simulation program uses dry inertia properties of the spacecraft referenced to a specific point, and mass, positional and inertia data for each individual fuel tank. Chung authored an additional program named CONVRT that is used to calculate this data and convert it to a format usable by the main simulation program. CONVRT places this data into an output file named SYSPAR, which is used by INTLVI, the main simulation program. The primary inputs required by CONVRT are the following

1. Rotor rotation speed
2. Natural frequency of fuel
3. Fill fraction of the tanks
4. Mass of the rotor and the platform
5. Location of the center of mass of the rotor and the platform
6. Moments of inertia of the rotor and the platform

The output of CONVRT is placed in SYSPAR, to be used as an input file for INTLVI. The contents of SYSPAR include

1. Number and location of tanks
2. Mass and dry inertia properties of the rotor and platform
3. Mass and inertia properties of the fuel in each fuel tank
4. Spherical damping coefficients of the spherical pendulums

In addition, CONVRT places the following data into another output file, named TYPE.

1. Wet spacecraft inertia properties
2. Natural slosh frequency of the fuel
3. Damping ratios

Although the data in TYPE is not used by the main simulation program, it is used in this analysis. The wet spacecraft parameters for each of the fuel loads are used to compute new dry properties for the platform and the rotor. These wet spacecraft parameters are listed in Table 3.

**TABLE 3. Wet Spacecraft Parameters for Fuel Modelled as Spherical Pendulums**

	Fuel Load 15%	Fuel Load 20%	Fuel Load 26.2%	Fuel Load 50%	Fuel Load 75%
Mass	2183.1	2326.0	2503.1	3183.2	3897.5
$I_{xx}$	3950.6	4192.1	4469.5	5352.6	6029.5
$I_{yy}$	3778.3	4105.6	4491.2	5786.6	6851.8
$I_{zz}$	3939.6	4181.1	4458.5	5341.6	6018.5
$I_{xy}$	0.0	0.0	0.0	0.0	0.0
$I_{xz}$	38.3	38.3	38.3	38.3	38.3
$I_{yz}$	0.0	0.0	0.0	0.0	0.0

The actual rotor asymmetry analysis involves changing SYSPAR data prior to running the main simulation program, substituting in data which gives the desired inertia ratios. The specific data that is changed includes the dry transverse moments of inertia of the rotor and the platform, and the axial

moment of inertia of the rotor. The values used are generated with a spreadsheet program (SPRDSHT A) written for this purpose. SPRDSHT A generates new dry platform and rotor data for four inertia ratios and five fuel loads. The inertia ratios are 1.03, 1.05, 1.07 and 1.1. The five fuel loads to be examined for each inertia ratio are 15, 20, 26.2, 50 and 75 percent of the total spacecraft fuel capacity. These fuel loadings will be representative of the stability of a generic satellite from apogee kick motor firing to end of life. The procedure that SPRDSHT A uses to generate the dry inertia properties of the rotor and platform is as follows

1. Compute the average wet system transverse moment of inertia for each fuel load from the data obtained from output file TYPE. This data is listed in Table 3.

2. Multiply this average transverse moment of inertia by the desired inertia ratio. This is the new wet axial moment of inertia of the rotor,  $I_r$ .

3. Because the main simulation uses dry properties, the amount the dry rotor axial moment of inertia must be changed has to be computed. This is done by subtracting the old wet axial moment of inertia of the rotor ( $I_{zz}$  from Table 3 minus dry axial moment of inertia of the platform) from  $I_r$ . This quantity is  $\Delta I_r$ .

4. The new dry axial moment of inertia of the rotor  $I_{dr}$ , is computed by adding old dry axial moment of inertia and  $\Delta I_r$ .

5. So that the rule that the sum of two moments of inertia must be greater than the third is not violated, the dry transverse moments of inertia of the rotor must also be increased. To compute the amount of increase, the sum of the dry rotor transverse moments of inertia is subtracted from the new dry rotor axial moment of inertia,  $I_{dr}$ . The result is divided by two and added to the old dry rotor transverse moments of inertia. To achieve perfect symmetry, the smaller of the two moments is increased to equal the larger.

6. An additional adjustment is made in order that the wet spacecraft moments of inertia are not adversely effected by the previous step. The corresponding platform transverse moments of inertia are decreased by the amount the rotor transverse moments of inertia are increased.

7. These new dry platform and rotor moments of inertia are substituted into the file SYSPAR to be used by the main simulation program.

## **B. SIMULATION PROCEDURES FOR ASYMMETRY ANALYSIS**

The procedure for rotor asymmetry analysis involves changing file SYSPAR data and running the main simulation program. After each run of the main simulation, the nutation angle in the output file PLOT is examined to see if the nutation angle reduces to zero or if it increases. If the nutation angle decreases to zero, the system is considered passively stable. If the system is stable, rotor asymmetry is increased and the procedure repeated until instability occurs. The point at which this happens is the stability cutoff point. Once the stability cutoff point is reached, procedure is repeated for each inertia ratio and fuel load.

The percent asymmetry is defined as the difference of the two dry rotor transverse moments of inertia to the sum of the dry transverse moments of inertia multiplied by 100. Percent asymmetry is performed in the simulation by increasing  $I_{xx}$  of the dry rotor. The specific steps of the procedure follow. The procedure is also depicted graphically in a flowchart in Figure 9.

1. Modify CONVRT's input file with fuel load, fundamental liquid slosh frequency and rotation speed of the rotor. The rotor speed is assumed constant throughout the study at 30 RPM.

2. Execute CONVRT.

3. Modify CONVRT's output file SYSPAR prior to running the main simulation program with data from SPRDSHT A. Specifically, dry moments of inertia are modified.

4. Execute the main simulation program INTLVI.

5. Check the output file PLOT to determine if nutation angle is decreasing to zero. If the system appears to be marginally stable, increase the damping coefficient exponents and run the simulation again. It was often necessary to increase the damping coefficients by a factor of 1000 to determine definitively, the nutation angle response over time.

6. If the system is stable, increment the dry rotor asymmetry and run the simulation again. Once the stability cutoff point is located, change SYSPAR data with data for the next inertia ratio

7. Go to Step 4 until inertia ratio is equal to 1.1.

8. After all the stability cutoff points for that fuel load are found, CONVRT's input file is changed with the next fuel load and the entire procedure repeated.

The platform asymmetry analysis was done identically to the rotor asymmetry analysis with few exceptions . The most important being that not only was  $I_{xx}$  of the dry platform increased , but  $I_{zz}$  of the dry platform reduced, while still satisfying the percent asymmetry equation. The motivation for this deviation from the rotor asymmetry analysis was to allow a direct comparison of the results with the work of Myers in Reference 7.

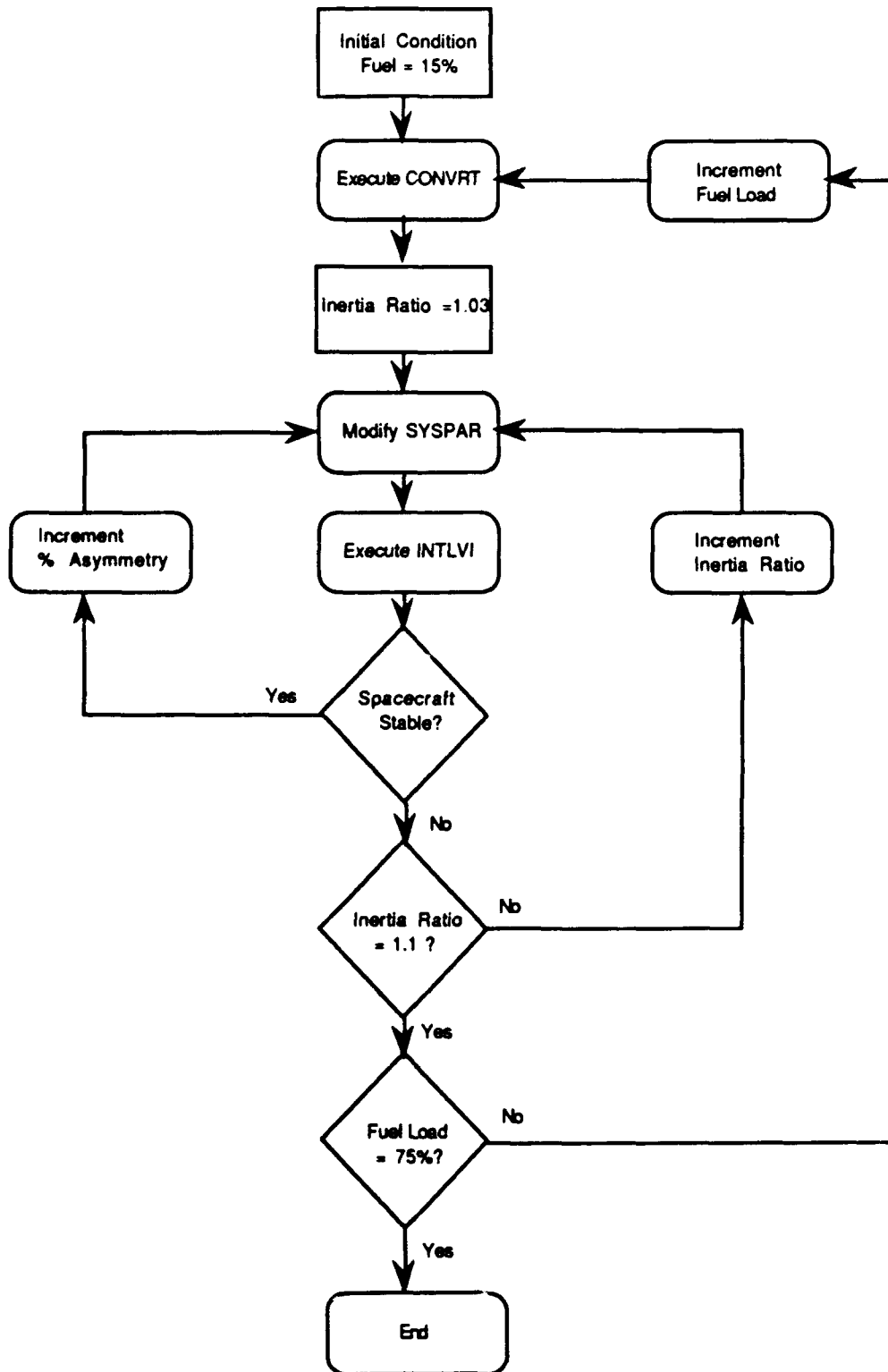


Figure 9. Flowchart for Rotor Asymmetry Analysis Procedure

### **C. SIMULATION PROCEDURES FOR MODEL ENHANCEMENT**

The modification to Chung's simulation incorporates Abramson's model for liquid slosh. Abramson's model changes the following input data for the main simulation program.

1. Pendulum length
2. Center of mass of rotor location
3. Mass of dry rotor
4. Dry inertia properties
5. Slosh frequency
6. Liquid percentage

The first step is to use Abramson's curves to compute what percentage of the liquid mass is to be modeled as spherical pendulums and what percentage is to be added to the dry rotor. Another spreadsheet (SPRDSHT B) was written to compute the amount of liquid that is sloshing as well as add the remaining fuel as masses to the dry rotor using the parallel axis theorem. The fuel that was added to the dry rotor was modelled as spheres equivalent in size to the mass of fuel being added. SPRDSHT B also calculates liquid slosh frequency and pendulum length using Abramson's curves.

The initial conditions of CONVRT are modified with the new dry rotor mass, new location of the center of mass of the rotor, the pendulum arm length, and dry moments of inertia from SPRDSHT B. CONVRT is then executed with Abramson's recommended fuel slosh frequency and fuel percentage. The objective of this part of the procedure is to obtain the wet spacecraft moments of inertia. As mentioned previously, these are generated by CONVRT and can be found in CONVRT's other output file, TYPE. The wet spacecraft moments of inertia are then used in SPRDSHT A to obtain new dry spacecraft properties for



the various fuel loads and inertia ratios. These wet spacecraft properties for the modified Chung program are found in Table 4.

**TABLE 4. Wet Spacecraft Parameters for Fuel Modelled as Spherical Pendulums (Modified Simulation)**

	Fuel Load 15%	Fuel Load 20%	Fuel Load 26.2%	Fuel Load 50%	Fuel Load 75%
Mass	2190.8	2334.1	2511.8	3194.0	3885.0
$I_{xx}$	4006.8	4251.5	4539.9	5758.6	7268.4
$I_{yy}$	3878.5	4215.8	4638.6	5791.6	6851.2
$I_{zz}$	3995.9	4241.8	4528.9	5742.7	7244.7
$I_{xy}$	0.0	0.0	0.0	0.0	0.0
$I_{xz}$	38.3	38.3	38.3	38.3	38.3
$I_{yz}$	0.0	0.0	0.0	0.0	0.0

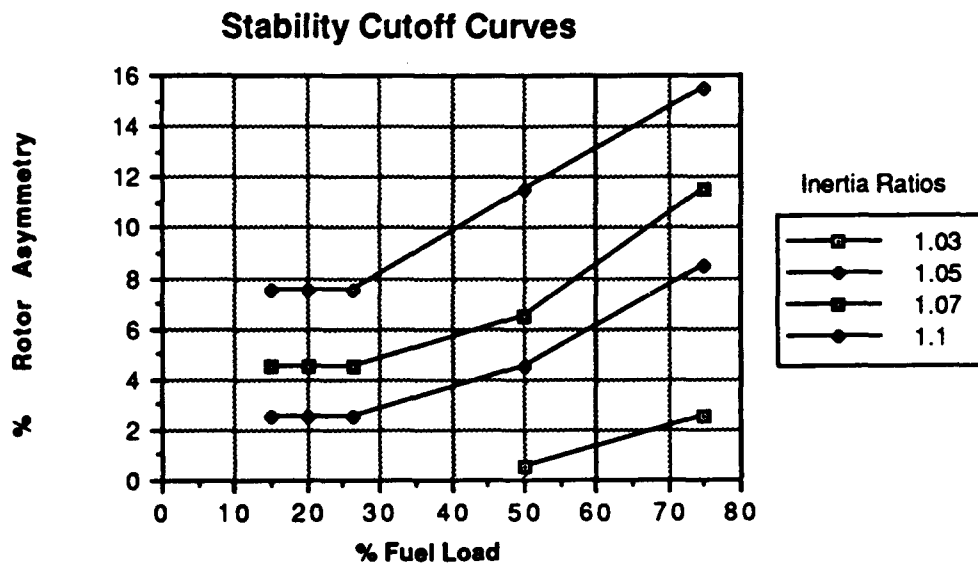
As with the unmodified simulation, the output of SPRDSHT A is then used to modify SYSPAR data prior to running the main simulation program. The procedure to do this is identical as that for the unmodified model.

## V. RESULTS AND ANALYSIS

This chapter will present plots of the rotor asymmetry analysis for both the modified and unmodified simulations. The boundary between stability and instability will be plotted as a function of fuel load and rotor asymmetry. The minimum inertia ratio for both rotor and platform asymmetry as a function of fuel load will also be plotted.

### A. SIMULATION RESULTS FOR ROTOR ASYMMETRY

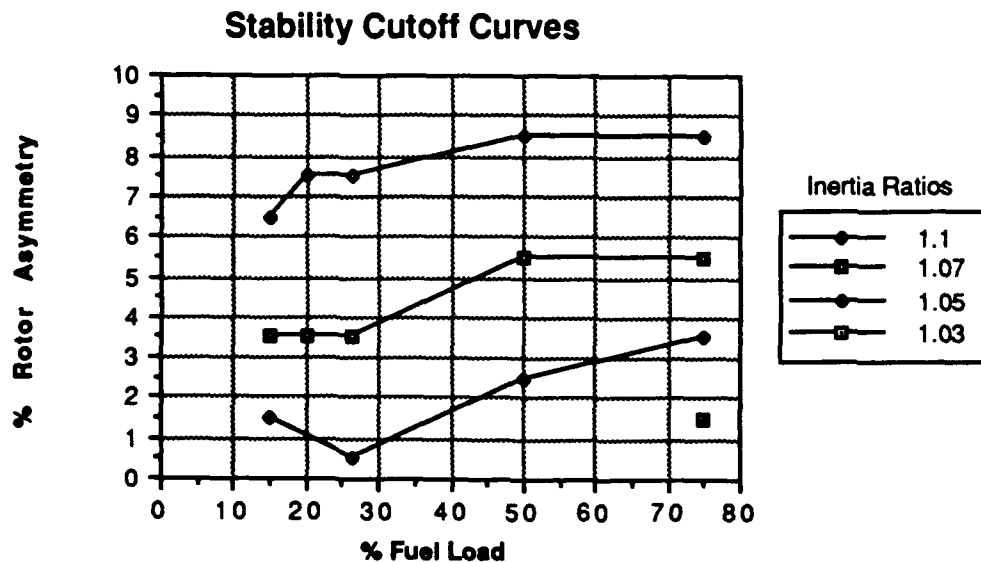
The results for rotor asymmetry using Chung's "rigid slug" model are depicted in Figure 10.



**Figure 10. Stability Cutoff Points for Inertia Ratios 1.03, 1.05, 1.07 and 1.1.**

The points are plotted midway between the last stable percent asymmetry and the first unstable percent asymmetry. For an inertia ratio of 1.03, fuel loads of 15, 20 and 26.2 percent are unstable in their symmetric rotor configuration.

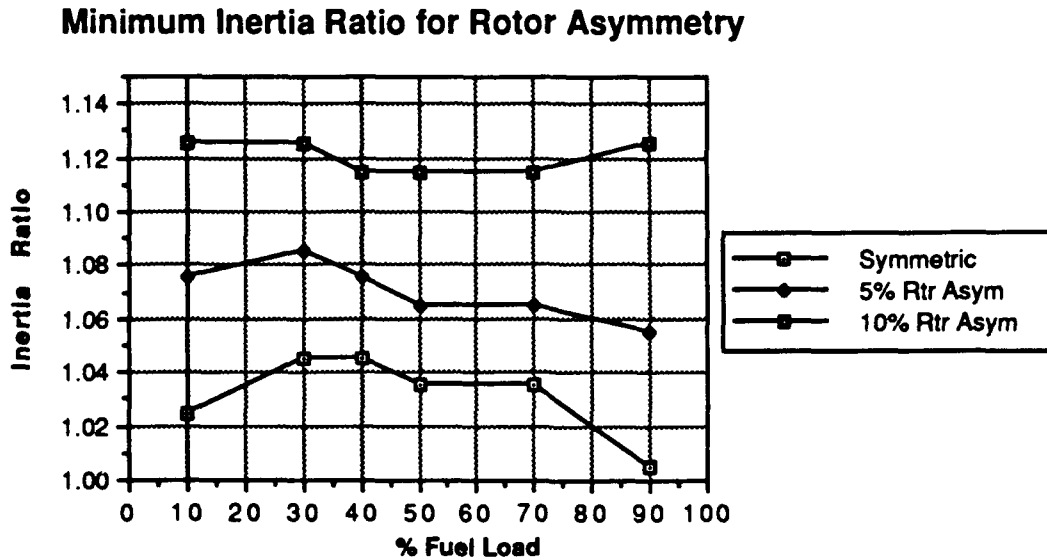
The results for rotor asymmetry using the modified version of Chung's simulation are depicted in Figure 11. Again, the points are plotted midway between the last stable percent asymmetry and the first unstable percent asymmetry.



**Figure 11. Stability Cutoff Points for Inertia Ratios 1.03, 1.05, 1.07 and 1.1. (Modified Simulation)**

For an inertia ratio of 1.03, all fuel loads except 75 percent were unstable in the symmetrical rotor configuration. For an inertia ratio of 1.05, the 20 percent fuel load was unstable in its symmetric configuration. The line connecting the 15 and 26.2 percent fuel load points, shows only the trend and does not include any intermediate points. The other two inertia ratios show a much more gradual increase in allowable percent rotor asymmetry as fuel load increases, than the unmodified Chung simulation.

Figure 12 depicts the minimum inertia ratio for zero, five and ten percent asymmetry. As expected, significant increases in inertia ratio are required as the rotor percent asymmetry is increased.



**Figure 12. Minimum Inertia Ratio for Stability (Rotor Asymmetry)**

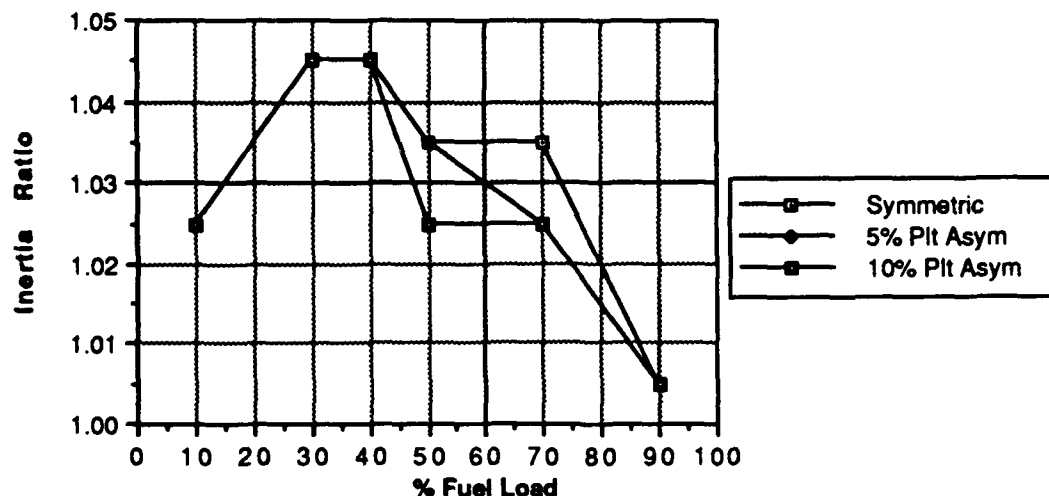
The results also indicate that the maximum inertia ratio required is between 20-40 percent fuel load. It was expected that the maximum inertia ratio would be required at 50 percent fuel load because at this fuel load, liquid fuel surface area is a maximum.

## **B. SIMULATION RESULTS FOR PLATFORM ASYMMETRY**

The results for the platform asymmetry analysis are depicted in Figure 13. The stability curve for zero percent asymmetry is identical to the one depicted in the rotor asymmetry plot. The curves for five and ten percent platform

asymmetry however, show that for fuel loads of 50 and 70 percent, stability increases as the percent platform asymmetry increases.

**Minimum Inertia Ratio For Platform Asymmetry**



**Figure 13. Minimum Inertia Ratio for Stability (Platform Asymmetry)**

The five and ten percent asymmetry curves are generated by keeping the rotor symmetric and varying the dry platform asymmetry. As with the minimum inertia ratio for the rotor asymmetry plot, fuel loads of 10, 30, 40, 50, 70 and 90 were run.

### C. ANALYSIS

For the rotor asymmetry analysis, both the modified and unmodified simulations were run for five fuel loads and four inertia ratios. Data points are plotted halfway between the last stable asymmetry percent and the first unstable asymmetry percent. The data points that are not plotted were unstable in their symmetric configuration.

The results of Chung's original "rigid slug" model depict relatively smooth, increasing curves. The curves show that as the percentage of the fuel in the tanks increases, so does the allowable percent rotor asymmetry. The curves also show that as inertia ratio increases, the allowable percent asymmetry increases. This result is in agreement with Likins' energy sink results, that is, inertia ratio must be greater than one for passive stability. It is reasonable to assume that as inertia ratio becomes greater than one, the spacecraft can tolerate a higher degree of rotor asymmetry.

The curves of the modified simulation appear somewhat irregular and for lower inertia ratios, broken. These results need to be assessed for each inertia ratio. For inertia ratios of 1.1 and 1.07, the modified simulation comes into very close agreement with Slafer's finding that propellant damping (with unbaffled tanks) is very small. The curves are almost linear and is what would be expected based on actual flight results. The curve is somewhat irregular however, for an inertia ratio of 1.05. A more thorough discussion of the modification procedures is required to explain this. First off, for low fuel loads, Chung's and Abramson's models are nearly identical in terms of the mass size of the pendulum. However, the length of the pendulum arms and the fundamental frequencies are quite different. This difference can be seen in Table 5.

In substituting several Abramson frequencies and pendulum arm lengths, CONVRT generated negative axial and transverse moments of inertia for the spherical pendulums. This of course, is physically unrealizable. In order to generate positive values for these inertias, it was decided to shorten the length of the pendulum arms rather than to decrease the frequency of oscillation. If CONVRT generated negative moments of inertias, the pendulum arm lengths were reduce by five percent until positive moments were generated. The actual pendulum arm lengths used in the study can be found in Table 6.

**TABLE 5. Frequencies and Pendulum Arm Lengths of Models**

Fuel Load (%)	Fundamental Frequency (rad/sec)		Length of Pendulum arm (meters)	
	Chung	Abramson	Chung	Abramson
15.0	6.4843	6.0619	.285	.351
20.0	6.4660	6.2236	.263	.336
26.2	6.4867	6.4771	.238	.322
50.0	6.7998	7.2175	.157	.282
75.0	7.9852	7.6959	.081	.218

**TABLE 6. Recommended and Actual Pendulum Arm Lengths**

Fuel Load (%)	Length of Pendulum Arm (meters)		Percent Change
	Abramson	Actually used	
15.0	.351	.351	0
20.0	.336	.319	5
26.2	.322	.306	5
50.0	.282	.240	15
75.0	.218	.218	0

The irregularity in the stability cutoff curves for an inertia ratios 1.05 can be explained in the change required to Abramson's model for low fuel loads. For an inertia ratio of 1.03, only a singular point was obtained at 75 percent fuel load with no general trend. This discrepancy is related either to the fact that frequency and arm lengths for Abramson's model produced physically unrealizable results, or because 1.03 may be close to the actual minimum inertia ratio required for stability.

The most obvious difference between the modified and unmodified simulations though, is that the unmodified version often predicts stability when the modified simulation does not. This is especially true at the lower fuel loads. The unmodified simulation obviously assumes a higher dampening effect caused by liquid slosh. The Abramson modified simulation appears more realistic based on Slafer's finding that liquid slosh contributes less than 0.05% of the total damping.

The platform asymmetry results show that as platform asymmetry increases, so does stability for certain fuel loads. At fuel loads of 50 and 70 percent, the minimum inertia ratio decreases for five and ten percent asymmetry by 0.01. The platform asymmetry apparently acts as a damping mechanism for the system. Also of note is the fact with an inertia ratio of 1.03, a symmetric satellite with 50 percent fuel is unstable. Increasing the platform asymmetry to ten percent however, makes the minimum inertia ratio less than 1.03. This increase in platform asymmetry made an inherently unstable symmetric spacecraft, stable.



## **VI. SUMMARY AND CONCLUSIONS**

This chapter presents a synopsis of the results of both the modified and unmodified computer simulations from Chapter V. Conclusions drawn from the study are presented in Section B.

### **A. SUMMARY**

The results of this analysis agree with Slafer in showing that the "rigid slug" method of modelling liquid fuel is inadequate. Chung's "rigid slug" model assumed the whole liquid is moving which is incorrect. Using this simulation in its current form then, may result in determining incorrect stability boundaries for various spacecraft configurations. The simulation can be improved using Abramson's model however, resulting in improved prediction of attitude stability conditions of dual-spin spacecraft.

### **B. CONCLUSIONS**

Slafer's experience with LEASAT satellites is that the "rigid slug" model gave incorrect predictions of attitude stability and that Abramson's model followed closely with LEASAT flight data. The modified version of Chung's simulation will give improved predictions of the attitude of asymmetric dual-spin spacecraft with large liquid fractions. However, there is still a need to further improve the liquid slosh model. Liquid slosh modelling should include modelling the point masses as their actual shape, rather than symmetrical spheres for more accurate dry rotor inertia calculations. This would yield more accurate new dry rotor moments of inertia. In addition, fuel slosh frequency and pendulum arm length predictions need improvement. Using Abramson's frequencies and pendulum arm lengths in Chung's simulation, resulted in

negative pendulum moments of inertia for some cases. Future work in this area should determine the exact relationship between liquid slosh frequency and pendulum arm length.

Future work should also more closely examine the effect of increasing the damping coefficient exponents by 1000. The coefficients had to be increased to determine definitively the trend of the nutation angle within 50 seconds of simulation time. Factors less than 1000 required simulation times of 200 to 300 seconds to determine the trend, times which would necessitate an unreasonable amount of CPU time for each simulation run.

Lastly, Likins' energy sink equation states that it does not matter in what body the asymmetry is physically located, so long as the average transverse moments of inertia do not change. The results here show otherwise. Increases in platform asymmetry are stabilizing, while increases in rotor asymmetry are destabilizing. The energy sink equation, as Chung states in Reference 2, should be used with extreme caution for dual-spin stabilized spacecraft with large fraction liquid loads

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