Adaptive Control of Uncertain Hamiltonian Multi-Input Multi-Output Systems: With Application to Spacecraft Control

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***Abstract*—A novel adaptive tracking control law for nonlinear Hamiltonian multi-input–multi-output (MIMO) systems with uncertain parameters in the actuator modeling as well as the inertia and/or the Coriolis and centrifugal terms is developed. The physical properties of the Hamiltonian systems are effectively used in the control design and the stability analysis. The number of the parameter estimates is significantly lowered as compared to the conventional adaptive control methods which are based on the state-space form. The developed control scheme is applied for attitude control of a spacecraft with both the inertia and the actuator uncertainties, and numerical examples show that the controller successfully deals with the unknown inertia/actuator parameters.**

***Index Terms*—Actuator uncertainty, adaptive control, Hamiltonian system, multiple-input–multiple-output (MIMO) system control, smooth projection algorithm, spacecraft attitude control.**

I. Introduction

ADAPTIVE control of multi-input–multi-output (MIMO) dynamic systems with uncertain parameters has been extensively studied in the literature. Especially, dynamic systems whose equations of motion are described in the general form of second order differential equations have received significant attention in the literature. Expressing dynamics of systems in the generic form, rather than the state-space form, has several advantages. The equation can be easily derived by applying Lagrange's equation, and its form is so general that it can represent various kinds of dynamic systems, such as a multilink robot manipulator[1], [2] and a spacecraft[3]–[5], etc. In addition, there are physical properties, such as energy conservation, which are extremely useful in designing advanced control schemes [1].

Most (if not all) of the previous research, however, deals only with uncertainties in the inertia, centripetal/Corilois, and gravitational terms, assuming that an exact model of the actuators is available. This assumption is rarely satisfied in practice because the actuator parameters may also have uncertainties due to installation error, aging and wearing out of the mechanical and electrical parts, etc.

Adaptive control with actuator uncertainty does not seem to have received much attention in the literature, even though this uncertainty may result in significant degeneration of controller performance. Ge [6] has derived an adaptive control law for multilink manipulator systems with uncertainties in the control input term, but the uncertainty must be in the input scalings, and thus the uncertainty matrix must be diagonal when represented in multiplicative form. Chang [7] has provided an adaptive, robust tracking control algorithm for nonlinear MIMO systems which is based on the "smooth projection algorithm," which has also been used in [8] and [9] for adaptive control of SISO systems. More recently, using Chang's method [7], Yoon and Tsiotras [10] have also provided an adaptive control scheme which is applied to spacecraft attitude tracking with uncertain misalignments/inertia of the actuator flywheels. However, these previous results [7], [10] are based on purely mathematical approaches and do not exploit the useful physical properties of the Hamiltonian systems (In this paper, Hamiltonian systems are referred to as conservative systems). More significantly, they considered MIMO systems represented the state-space form, and thus they need "over-parameterization" (more details are in Section II).

In this paper, an adaptive tracking control algorithm for dynamic systems with parameter uncertainties in the dynamic modeling as well as the actuator modeling is developed. This paper is organized as follows. Section II introduces the general form of second order differential equations and the mathematical expression of the uncertain matrices. In Section III, adaptive control algorithm is derived based on the smooth projection algorithm. The proposed method is then applied for a spacecraft attitude control problem with inertia/actuator uncertainties in Section IV. Finally, Section V provides numerical examples with the spacecraft to validate the proposed law, and Section VI concludes this paper.

II. Problem Formulation

We consider a MIMO nonlinear Hamiltonian system represented by the second-order differential equation

$$H(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = D(\mathbf{q})\mathbf{u} \tag{1}$$

where $\mathbf{q} \in \mathbb{R}^n$ is the generalized coordinates vector, $H \in \mathbb{R}^{n\times n}$ is the (symmetric positive definite) inertia matrix, $C\dot{\mathbf{q}}$ is a nonlinear vector of Coriolis and centripetal forces, and $\mathbf{g} \in \mathbb{R}^n$ is the gravity vector. The generalized force is generated by a control input vector $\mathbf{u} \in \mathbb{R}^m$ and the actuator matrix (or the control influence matrix) $D \in \mathbb{R}^{n\times m}$. For full tracking control of $\mathbf{q}$, it is generally required that $n \le m$ and D has full row rank.

Suppose now that the system matrices have *constant or slowly-varying uncertain parameters* so that they can be assumed to be constant. Let us express these matrices as

$$H(\mathbf{q},\Theta_s) = H^n(\mathbf{q}) + H^{\Delta}(\mathbf{q},\Theta_s) \tag{2a}$$

$$C(\mathbf{q},\dot{\mathbf{q}},\Theta_s) = C^n(\mathbf{q},\dot{\mathbf{q}}) + C^{\Delta}(\mathbf{q},\dot{\mathbf{q}},\Theta_s) \tag{2b}$$

$$\mathbf{g}(\mathbf{q},\Theta_s) = \mathbf{g}^n(\mathbf{q}) + \mathbf{g}^{\Delta}(\mathbf{q},\Theta_s) \tag{2c}$$

$$D(\mathbf{q},\Theta_a) = D^n(\mathbf{q}) + D^{\Delta}(\mathbf{q},\Theta_a) \tag{2d}$$

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where the matrices with a superscript of "n" are with their known nominal values, $\Theta_a \in \mathbb{R}^p$ is a vector of unknown bounded constant uncertainties in the actuator matrix D, and $\Theta_s \in \mathbb{R}^q$ is in the other system matrices/vector, H, C, and $\mathbf{g}$. We also assume that, by proper definition of the unknown parameters Θ_s and Θ_a, the uncertain matrices $H^\Delta(\mathbf{q},\Theta_s), C^\Delta(\mathbf{q},\dot{\mathbf{q}},\Theta_s), \mathbf{g}^\Delta(\mathbf{q},\Theta_s)$ and $D^\Delta(\mathbf{q},\Theta_a)$ are *linearly* dependent on Θ_s, and Θ_a, respectively.

The problem we study in this paper is formulated as follows.

1) Adaptive Tracking Control of Uncertain MIMO System: Consider the MIMO Hamiltonian System (1) with system matrices with uncertainties as expressed in (2). For given reference trajectory $\mathbf{q}_d(t)$, $\dot{\mathbf{q}}_d(t)$ and $\ddot{\mathbf{q}}_d(t)$ which are bounded, design a control algorithm which makes the tracking error $\tilde{\mathbf{q}} \triangleq \mathbf{q} - \mathbf{q}_d$ globally asymptotically stable at $\tilde{\mathbf{q}} = 0$.

Slotine's adaptive controller[1] can handle only the special case with $D^\Delta = 0$. On the other hand, Chang's method [7] needs to convert the equation of motion (1) into a state-space form, that is

$$\begin{bmatrix}\dot{\mathbf{q}}\\ \ddot{\mathbf{q}}\end{bmatrix} = \begin{bmatrix}0_{n\times n} & \mathbf{I}_n\\ 0_{n\times n} & 0_{n\times n}\end{bmatrix}\begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix} + \begin{bmatrix}0_{n\times n}\\ \mathbf{I}_n\end{bmatrix}\left(-H^{-1}C\dot{\mathbf{q}} - H^{-1}\mathbf{g} + H^{-1}D\mathbf{u}\right) \quad (3)$$

in which the uncertainties in the inertia matrix H are multiplied with the uncertainties in the other terms. Since Chang's method also uses the linear dependency of uncertainties, it needs to estimate the combination of uncertain parameters in H and those in the others ("over-parameterization"). Therefore, the number of parameter estimates would significantly increase when H has multiple uncertain parameters. In the following section, we derive a novel algorithm which remedies these drawbacks, that is an adaptive tracking controller for a case of $D^\Delta \neq 0$ without over-parameterization.

III. Adaptive Control Law

The first part of the derivation of the adaptive control law follows the standard design procedure for Hamiltonian systems in [1] and [4]. Let us define a measure of tracking error $\mathbf{s} \triangleq \dot{\tilde{\mathbf{q}}} + \Lambda\tilde{\mathbf{q}} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_r$ and the reference velocity $\dot{\mathbf{q}}_r \triangleq \dot{\mathbf{q}}_d - \Lambda\tilde{\mathbf{q}}$, where the matrix $-\Lambda$ is assumed to be Hurwitz. Let $\hat{\Theta}_*$ be the parameter estimate vector and let $\tilde{\Theta}_* \triangleq \hat{\Theta}_* - \Theta_*$ be a parameter estimate error vector, when $*$ is a or s.

As suggested by Slotine *et al.* [1] from a physical insight that $\dot{\mathbf{q}}^T H\dot{\mathbf{q}}$ is the system's kinetic energy, the following Lyapunov function candidate is defined as:

$$V(t) = \frac{1}{2}\left[\mathbf{s}^T H\mathbf{s} + \tilde{\Theta}_a^T\Gamma_a^{-1}\tilde{\Theta}_a + \tilde{\Theta}_s^T\Gamma_s^{-1}\tilde{\Theta}_s\right] \quad (4)$$

where Γ_a, Γ_s are positive definite weighting matrices. Differentiating $V(t)$ with respect to time and using the skew-symmetry property of the matrix $\dot{H} - 2C$ to replace the term $(1/2)\mathbf{s}^T\dot{H}\mathbf{s}$ with $\mathbf{s}^T C\mathbf{s}$, one can have the following expression:

$$\begin{aligned}\dot{V} &= \mathbf{s}^T H\dot{\mathbf{s}} + \frac{1}{2}\mathbf{s}^T\dot{H}\mathbf{s} + \tilde{\Theta}_a^T\Gamma_a^{-1}\dot{\hat{\Theta}}_a + \tilde{\Theta}_s^T\Gamma_s^{-1}\dot{\hat{\Theta}}_s\\ &= \mathbf{s}^T\left(H\dot{\mathbf{s}} + C\mathbf{s}\right) + \tilde{\Theta}_a^T\Gamma_a^{-1}\dot{\hat{\Theta}}_a + \tilde{\Theta}_s^T\Gamma_s^{-1}\dot{\hat{\Theta}}_s\\ &= \mathbf{s}^T(D\mathbf{u} - H\ddot{\mathbf{q}}_r - C\dot{\mathbf{q}}_r - \mathbf{g}) + \tilde{\Theta}_a^T\Gamma_a^{-1}\dot{\hat{\Theta}}_a + \tilde{\Theta}_s^T\Gamma_s^{-1}\dot{\hat{\Theta}}_s. \quad (5)\end{aligned}$$

When the actuator matrix D is exactly known (i.e., $D^\Delta = 0$) and has full row rank (i.e., $\mathrm{rank}(D) = \mathrm{n}$), one can easily design an adaptive control law using the methods proposed in the previous work [1]. However, since D is assumed to contain unknown parameters as well as H, C and $\mathbf{g}$, a novel control law is proposed as follows:

$$(D^n + \hat{D}^\Delta)\mathbf{u} = \Big((H^n + \hat{H}^\Delta)\ddot{\mathbf{q}}_r + (C^n + \hat{C}^\Delta)\dot{\mathbf{q}}_r + (\mathbf{g}^n + \hat{\mathbf{g}}^\Delta) - K_d\mathbf{s}\Big) \quad (6)$$

where the matrices/vector with a 'hat' symbol are constructed using the parameter estimates $\hat{\Theta}_a$ and $\hat{\Theta}_s$ instead of the (unknown) actual parameters. K_d is a gain matrix which is a positive definite. When the matrix $D^n + \hat{D}^\Delta$ is assumed to have full row rank, the (weighted) minimum norm solution is given by

$$\mathbf{u} = (D^n + \hat{D}^\Delta)^\dagger\Big((H^n + \hat{H}^\Delta)\ddot{\mathbf{q}}_r + (C^n + \hat{C}^\Delta)\dot{\mathbf{q}}_r + (\mathbf{g}^n + \hat{\mathbf{g}}^\Delta) - K_d\mathbf{s}\Big) \quad (7)$$

where $(\cdot)^\dagger$ denotes the (weighted) pseudo-inverse of a matrix [5], [10].

The control law (7) leads to

$$\begin{aligned}\dot{V} &= \mathbf{s}^T[\{(D^n + \hat{D}^\Delta) - \tilde{D}^\Delta\}\mathbf{u} - (H^n + H^\Delta)\ddot{\mathbf{q}}_r\\ &\quad - (C^n + C^\Delta)\dot{\mathbf{q}}_r - (\mathbf{g}^n + \mathbf{g}^\Delta)] + \tilde{\Theta}_a^T\Gamma_a^{-1}\dot{\hat{\Theta}}_a + \tilde{\Theta}_s^T\Gamma_s^{-1}\dot{\hat{\Theta}}_s\\ &= \mathbf{s}^T[(H^n + \hat{H}^\Delta)\ddot{\mathbf{q}}_r + (C^n + \hat{C}^\Delta)\dot{\mathbf{q}}_r + (\mathbf{g}^n + \hat{\mathbf{g}}^\Delta)\\ &\quad - K_d\mathbf{s} - \tilde{D}^\Delta\mathbf{u} - (H^n + H^\Delta)\ddot{\mathbf{q}}_r - (C^n + C^\Delta)\dot{\mathbf{q}}_r\\ &\quad - (\mathbf{g}^n + \mathbf{g}^\Delta)] + \tilde{\Theta}_a^T\Gamma_a^{-1}\dot{\hat{\Theta}}_a + \tilde{\Theta}_s^T\Gamma_s^{-1}\dot{\hat{\Theta}}_s\\ &= \mathbf{s}^T\left[\tilde{H}^\Delta\ddot{\mathbf{q}}_r + \tilde{C}^\Delta\dot{\mathbf{q}}_r + \tilde{\mathbf{g}}^\Delta - K_d\mathbf{s} - \tilde{D}^\Delta\mathbf{u}\right]\\ &\quad + \tilde{\Theta}_a^T\Gamma_a^{-1}\dot{\hat{\Theta}}_a + \tilde{\Theta}_s^T\Gamma_s^{-1}\dot{\hat{\Theta}}_s\\ &= -\mathbf{s}^T K_d\mathbf{s} + \mathbf{s}^T(\tilde{H}^\Delta\ddot{\mathbf{q}}_r + \tilde{C}^\Delta\dot{\mathbf{q}}_r + \tilde{\mathbf{g}}^\Delta)\\ &\quad - \mathbf{s}^T\tilde{D}^\Delta\mathbf{u} + \tilde{\Theta}_a^T\Gamma_a^{-1}\dot{\hat{\Theta}}_a + \tilde{\Theta}_s^T\Gamma_s^{-1}\dot{\hat{\Theta}}_s. \end{aligned} \quad (8)$$

Since the uncertainty matrices/vector are assumed to depend linearly on Θ's, one can define *known* regressor matrices (in fact, row vectors) $Y_s = Y_s(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r,\mathbf{s}) \in \mathbb{R}^{1\times q}$ and $Y_a = Y_a(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r,\mathbf{s},\mathbf{u}) \in \mathbb{R}^{1\times p}$, such that

$$\mathbf{s}^T(H^\Delta\ddot{\mathbf{q}}_r + C^\Delta\dot{\mathbf{q}}_r + \mathbf{g}^\Delta) = Y_s\Theta_s \quad (9)$$

and

$$-\mathbf{s}^T D^\Delta\mathbf{u} = Y_a\Theta_a. \quad (10)$$

Notice that the definition of the regressor matrix Y_s in (9) is slightly different from that in the previous works by Slotine [1], [3], in which the regressor matrix was defined as $(H^\Delta\ddot{\mathbf{q}}_r + C^\Delta\dot{\mathbf{q}}_r + \mathbf{g}^\Delta) = Y_s\Theta_s$. By including $\mathbf{s}$ in the regressor's definition, the size of the regressor Y_s becomes smaller than that of Slotine's works where the regressor has a size of $n \times q$.

The time derivative of V then becomes

$$\dot{V} = -\mathbf{s}^T K_d\mathbf{s} + Y_s\tilde{\Theta}_s + Y_a\tilde{\Theta}_a + \tilde{\Theta}_s^T\Gamma_s^{-1}\dot{\hat{\Theta}}_s + \tilde{\Theta}_a^T\Gamma_a^{-1}\dot{\hat{\Theta}}_a \quad (11)$$

and taking the adaptation laws of the parameter estimates to be

$$\dot{\hat{\Theta}}_s = -\Gamma_s Y_s^T \tag{12}$$

and

$$\dot{\hat{\Theta}}_a = -\Gamma_a Y_a^T \tag{13}$$

then yields $\dot{V}(t) = -\mathbf{s}^T K_d \mathbf{s} \le 0$. Using standard arguments in [1], [10] which use Barbalat's lemma, one can easily show that $\dot{V} \to 0$ and thus $\mathbf{s} \to 0$ and $\tilde{\mathbf{q}} \to 0$ as $t \to \infty$.

A. Smooth Projection Algorithm

We previously used an assumption that the matrix $D^n + \hat{D}^\Delta$ has full row rank in deriving the adaptive control laws (7), (12), and (13). In general, for the full tracking control, the nominal matrix D^n in general has full row rank. However, a drift of the parameter estimates $\hat{\Theta}_a$, governed by an update law (13), can result in $D^n + \hat{D}^\Delta$ losing rank. We will refer to this situation as a "singularity" of the steering law due to the adaptation. This singularity hinders the use of the derived control laws, so they need to be modified.

If the nominal matrix D^n has full row rank, and the true value of the parameter uncertainty Θ_a is bounded by a sufficiently small number, and the parameter estimate $\hat{\Theta}_a$ is also kept small, then the matrix $D^n + \hat{D}^\Delta$ will also have full row rank. To this end, we define the following two convex sets:

$$\Omega_{\Theta_a} \triangleq \{\Theta_a \in \mathbb{R}^p |\, \|\Theta_a\|^2 < \beta\} \tag{14}$$

$$\hat{\Omega}_{\Theta_a} \triangleq \{\hat{\Theta}_a \in \mathbb{R}^p |\, \|\hat{\Theta}_a\|^2 < \beta + \delta\} \tag{15}$$

where $\beta > 0$ and $\delta > 0$ are known constants. Notice that $\Omega_{\Theta_a} \subset \hat{\Omega}_{\Theta_a}$. We make the following three assumptions.

- **Assumption 1.** The nominal value D^n has full row rank of n.
- **Assumption 2.** The actual value Θ_a belongs to the set Ω_{Θ_a}.
- **Assumption 3.** If $\hat{\Theta}_a \in \hat{\Omega}_{\Theta_a}$, then $D^n + \hat{D}^\Delta$ is non-singular.

These assumptions allow us to modify the adaptation law (13) by using the "smooth projection algorithm" as follows:[1]

$$\dot{\hat{\Theta}}_a = \mathrm{Proj}(\hat{\Theta}_a, \Phi_a) \tag{16}$$

where

$$\Phi_a \triangleq -\Gamma_a Y_a^T \tag{17}$$

[1]The adaptation law is, in fact, only Lipschitz continuous, not continuously differentiable. The use of the term "smooth" is a slight misnomer in this context, but we use it here in accordance to prior usage in the literature. It should be noted that a new parameter projection operator which is $\mathcal{C}^n$ has been recently introduced in [11].

and shown in (18) at the bottom of the page. This adaptation law is identical to (13) in cases (i) and (ii), and switches smoothly to a new expression in case (iii). The projection operator $\mathrm{Proj}(\hat{\Theta}_a, \Phi_a)$ is locally Lipschitz in $(\hat{\Theta}_a, \Phi_a)$, thus the system has a unique solution defined for some time interval $[0, T)$, $T > 0$.

Proposition 1: Under Assumptions 1–3, the control law (7) along with the adaptation laws (12) and (16) yields

$$\dot{V} \le -\mathbf{s}^T K_d \mathbf{s} \le 0 \tag{19}$$

and

$$\hat{\Theta}_a(t=0) \in \Omega_{\Theta_a} \Rightarrow \hat{\Theta}_a(t) \in \hat{\Omega}_{\Theta_a}, \quad \forall t \ge 0. \tag{20}$$

Proof: The proof is straightforward and therefore omitted here. It is similar with the proof in the author's previous work [10]. ■

From Proposition 1, one can conclude that, using the feedback control law (7) and the adaptation laws (12) and (16), $\tilde{\mathbf{q}} \to 0$ as $t \to \infty$ and $(D^n + \hat{D}^\Delta)$ will not lose rank, if we choose the initial parameter guess $\hat{\Theta}_a(0)$ inside the set Ω_{Θ_a}. For instance, we may take $\hat{\Theta}_a(0) = 0$.

It is also worth mentioning that the proposed adaptation law (16) has the additional benefit of keeping the parameter estimates from "bursting," which may happen when the persistency of excitation condition does not hold [12].

IV. Application to Spacecraft Attitude Control

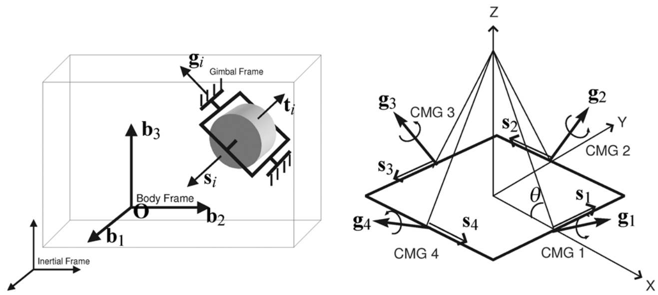
A. Equations of Motion

In this section, applying the proposed adaptive control scheme, we design an adaptive attitude tracking control law for a spacecraft which has uncertainties both in its inertia and actuator modeling. (Since the system remains Hamiltonian in the presence of uncertainties, non-adaptive passivity-based methods [13], [14] may be applied to control the spacecraft attitude, but they are *regulating* control laws which can be used only when $\mathbf{q}_d(t)$ is constant.)

A cluster of variable-speed control moment gyros (VSCMGs) with N flywheels is used for the torque actuator. While a conventional control moment gyro (CMG) keeps its flywheel spinning at a constant rate, a VSCMG—as its name implies—is essentially a single-gimbal CMG with the flywheel allowed to have variable speed. (See [15] and [16] for more details and applications of VSCMGs.)

Fig. 1(a) shows a spacecraft with the ith VSCMG, where $\mathbf{g}_i$ is (body-fixed) gimbal axis, $\mathbf{s}_i$ is spin axis, and $\mathbf{t}_i \triangleq \mathbf{g}_i \times \mathbf{s}_i$ is transverse torque axis. The equations of motion of a spacecraft with VSCMGs are complicated as shown in the aforementioned

$$\mathrm{Proj}(\hat{\Theta}_a, \Phi_a) = \begin{cases} \Phi_a, & \text{if (i) } \|\hat{\Theta}_a\|^2 < \beta \text{, or (ii) } \|\hat{\Theta}_a\|^2 \ge \beta \text{ and } \Phi_a^T\, \hat{\Theta}_a \le 0, \\ \left(\Phi_a - \frac{(\|\hat{\Theta}_a\|^2 - \beta)\Phi_a^T \hat{\Theta}_a}{\delta \|\hat{\Theta}_a\|^2} \hat{\Theta}_a\right), & \text{if (iii) } \|\hat{\Theta}_a\|^2 \ge \beta \text{ and } \Phi_a^T\, \hat{\Theta}_a > 0. \end{cases} \tag{18}$$



(a) Spacecraft body and the i-th VSCMG frames (b) The Pyramid Configuration

Fig. 1. Spacecraft and VSCMGs.

references, but under assumptions which are standard in the literature [10], they can be simplified as follows:

$$J\dot{\boldsymbol{\omega}} - [\mathbf{h}^{\times}]\boldsymbol{\omega} + Q\mathbf{u} = 0 \tag{21}$$

and

$$\mathbf{h} = J\boldsymbol{\omega} + A_s I_w \boldsymbol{\Omega} \tag{22}$$

where $Q = [Q_{cmg}, Q_{rw}] \in \mathbb{R}^{3\times 2N}$ and where $Q_{cmg}(\boldsymbol{\gamma}, \boldsymbol{\Omega}) = A_t I_w \boldsymbol{\Omega}^d \in \mathbb{R}^{3\times N}$, $Q_{rw}(\boldsymbol{\gamma}) = A_s I_w \in \mathbb{R}^{3\times N}$, and the control input of this system is $\mathbf{u} = [u_1, \ldots, u_{2N}]^T = [\dot{\boldsymbol{\gamma}}^T, \dot{\boldsymbol{\Omega}}^T]^T \in \mathbb{R}^{2N}$. J is the total moment of inertia of the spacecraft which is assumed to be constant,[2] $\boldsymbol{\omega}$ is the body rate vector of the spacecraft, $\mathbf{h}$ is the total angular momentum of the spacecraft, $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_N]^T \in \mathbb{R}^N$ and $\boldsymbol{\Omega} = [\Omega_1, \ldots, \Omega_N]^T \in \mathbb{R}^N$ are vectors of gimbal angles and flywheel spinning speeds, respectively, and I_w is a diagonal matrix with the inertias of VSCMGs flywheels. The skew-symmetric matrix $[\mathbf{v}^{\times}]$, for $\mathbf{v} \in \mathbb{R}^3$, represents the cross product operation. The matrices $A_* \in \mathbb{R}^{3\times N}$ have as columns the gimbal ($\mathbf{g}_i$), spin ($\mathbf{s}_i$), and transverse ($\mathbf{t}_i$) directional unit vectors expressed in the body-frame, where $*$ is g, s or t. These matrices depend on the gimbal angles as follows:

$$A_{\mathrm{g}} = A_{\mathrm{g}0} \tag{23}$$

$$A_s = A_{s0}[\cos\boldsymbol{\gamma}]^d + A_{t0}[\sin\boldsymbol{\gamma}]^d \tag{24}$$

$$A_t = A_{t0}[\cos\boldsymbol{\gamma}]^d - A_{s0}[\sin\boldsymbol{\gamma}]^d \tag{25}$$

where the A_{*0}'s denote the values of A_* at $\boldsymbol{\gamma} = 0$. The symbol $\mathbf{x}^d$ denotes the diagonal matrix with elements the components of the vector $\mathbf{x}$, and $\cos\boldsymbol{\gamma} \triangleq [\cos\gamma_1, \ldots, \cos\gamma_N]^T$ and $\sin\boldsymbol{\gamma} \triangleq [\sin\gamma_1, \ldots, \sin\gamma_N]^T$.

The modified Rodrigues parameters (MRPs) [17] are chosen to describe the attitude kinematics of the spacecraft.[3] The kinematics in terms of the MRPs is given by

$$\dot{\boldsymbol{\sigma}} = G(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{26}$$

where

$$G(\boldsymbol{\sigma}) = \frac{1}{2}\Big(\mathbf{I}_3 + [\boldsymbol{\sigma}^{\times}] + \boldsymbol{\sigma}\boldsymbol{\sigma}^T - \Big[\frac{1}{2}(1 + \boldsymbol{\sigma}^T\boldsymbol{\sigma})\Big]\mathbf{I}_3\Big) \tag{27}$$

[2]In fact, J is a function of $\boldsymbol{\gamma}$ but the dependence is weak in general.

[3]We hasten to point out that the use of the MRPs to describe the kinematics is done without loss of generality. Any other suitable kinematic description could have been used with the conclusions of the paper remaining essentially the same.

and $\mathbf{I}_r$ is the $r \times r$ identity matrix.

As suggested in [1] and [5], we combine the kinetic (21) and the kinematic (26) into one second-order system as follows:

$$H(\boldsymbol{\sigma})\ddot{\boldsymbol{\sigma}} + C(\boldsymbol{\sigma}, \dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} = D(\boldsymbol{\sigma})\mathbf{u} \tag{28}$$

where

$$H(\boldsymbol{\sigma}) = G^{-T}JG^{-1} \tag{29}$$

$$C(\boldsymbol{\sigma}, \dot{\boldsymbol{\sigma}}) = -G^{-T}JG^{-1}\dot{G}G^{-1} - G^{-T}[(R(\boldsymbol{\sigma})\mathbf{h}^I)^{\times}]G^{-1} \tag{30}$$

$$D(\boldsymbol{\sigma}) = -G^{-T}Q. \tag{31}$$

$R(\boldsymbol{\sigma})$ is a rotational matrix from the inertial frame to the body frame, and $\mathbf{h}^I$ is the total angular momentum of a spacecraft expressed in the inertial frame which is conserved to be constant if there is no external torque applied to the spacecraft. Therefore, $\mathbf{h} = R(\boldsymbol{\sigma})\mathbf{h}^I$.

Notice that the equation of motion (28) has the form of (1) with the gravitational term $\mathbf{g} = 0$. Moreover, it can be easily shown that the matrix $\dot{H} - 2C$ is skew-symmetric[1].

B. Adaptive Attitude Tracking Control

Suppose that there are uncertainties in D as well as J. We assume that the exact values of the initial axis directions of VSCMGs actuator at $\boldsymbol{\gamma} = 0$ are unknown. This can happen when the VSCMGs are installed with small misalignments and/or the measure of gimbal angles has constant unknown bias. In addition, $\mathbf{h}^I$ is also unknown constant not only because of uncertain J but also because of uncertain A_s.

For most cases the effect of axes uncertainties on the overall system performance may be not significant. However, when flywheels spin at high speeds as in a case of integrated power and attitude control system (IPACS) [5], [18], [19], even small misalignments of the flywheel axes can be detrimental.

The uncertain parameters in H and C can be defined as follows:

$$\Theta_s = [\Delta j_{11}, \Delta j_{22}, \Delta j_{33}, \Delta j_{12}, \Delta j_{13}, \Delta j_{23}, \Delta h_1, \Delta h_2, \Delta h_3]^T \in \mathbb{R}^9 \tag{32}$$

where Δj's are the elements of $J^{\Delta} \triangleq J - J^n$ and J^n is the nominal value of the actual inertia matrix J. Similarly, Δh's are elements of $\mathbf{h}^{I\Delta} \triangleq \mathbf{h}^I - \mathbf{h}^{In}$. The uncertain parameters in D are defined as

$$\Theta_a = [\Theta_{t,1}^T, \ldots, \Theta_{t,N}^T, \Theta_{s,1}^T, \ldots, \Theta_{s,N}^T]^T \in \mathbb{R}^{6N} \tag{33}$$

where

$$\Theta_{t,i} \triangleq \mathbf{t}_{i,0} - \mathbf{t}_{i,0}^n, \quad \Theta_{s,i} \triangleq \mathbf{s}_{i,0} - \mathbf{s}_{i,0}^n, \quad i = 1, \ldots, N. \tag{34}$$

and $\mathbf{t}_{i,0}$ and $\mathbf{s}_{i,0}$ are actual value of $\mathbf{t}_i$ and $\mathbf{s}_i$ at $\gamma_i = 0$, respectively, and $\mathbf{t}_{i,0}^n$ and $\mathbf{s}_{i,0}^n$ are their nominal values. The total number of parameter estimates is then $6N + 9$. If one uses the methods in the previous works [7], [10], then the number of the estimates will be as much as $6 \times (6N + 3)$.

Exact mathematical expressions of the regressor vectors Y_s defined in (9) can be easily obtained using symbolic math packages, and can be constructed from the measurements of $\boldsymbol{\sigma}$, $\dot{\boldsymbol{\sigma}}$, and the desired trajectories $\boldsymbol{\sigma}_d, \dot{\boldsymbol{\sigma}}_d, \ddot{\boldsymbol{\sigma}}_d$. The regressor vector Y_a defined in (10) can be obtained in the same way, but it is also possible to derive its mathematical expression by manipulating the matrices as follows:

$$Y_a = \mathbf{s}^T G^{-T}[(M\mathbf{u})_1\mathbf{I}_3, \ldots, (M\mathbf{u})_{2N}\mathbf{I}_3] \tag{35}$$

where

$$M \triangleq \begin{bmatrix} [\cos\boldsymbol{\gamma}]^d & [\sin\boldsymbol{\gamma}]^d \\ -[\sin\boldsymbol{\gamma}]^d & [\cos\boldsymbol{\gamma}]^d \end{bmatrix} \begin{bmatrix} I_w\Omega^d & 0 \\ 0 & I_w \end{bmatrix} \tag{36}$$

and $(\mathbf{x})_i$ is the ith element of a vector $\mathbf{x}$. Then using the developed control scheme in Section III, one can design an adaptive attitude tracking control law for the spacecraft.

V. Numerical Examples

Numerical examples for a satellite with a VSCMGs cluster are provided in this section to test the proposed adaptive control algorithm. In addition, the null motion technique presented in [19] is also applied along with the proposed control law in order to avoid the geometric singularity of VSCMGs. Since the control input $\mathbf{u}$ in (7) is replaced by $\mathbf{u}+\mathbf{u}_{\text{null}}$, where the null motion $\mathbf{u}_{\text{null}}$ satisfies $(D^n + \hat{D}^\Delta)\mathbf{u}_{\text{null}} = 0$, the stability analysis of the adaptive control law still holds. A standard four-VSCMG pyramid configuration ($N = 4$) is utilized as shown in Fig. 1(b).

When the skew angle θ in Fig. 1(b) is chosen so that $\cos\theta = 1/\sqrt{3}$, the nominal values of the axis directions at $\boldsymbol{\gamma} = [0, 0, 0, 0]^T$ are

$$A_{s0}^n = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{37}$$

$$A_{t0}^n = \begin{bmatrix} -0.5774 & 0 & 0.5774 & 0 \\ 0 & -0.5774 & 0 & 0.5774 \\ 0.8165 & 0.8165 & 0.8165 & 0.8165 \end{bmatrix}. \tag{38}$$

The (unknown) actual axis directions at $\boldsymbol{\gamma} = 0$ used in the present example are assumed as

$$A_{s0} = \begin{bmatrix} -0.0072 & -0.9999 & -0.0072 & 0.9999 \\ 0.9999 & 0.0143 & -0.9999 & 0.0057 \\ -0.0071 & 0.0071 & -0.0142 & -0.0117 \end{bmatrix} \tag{39}$$

$$A_{t0} = \begin{bmatrix} -0.5657 & -0.0024 & 0.5831 & 0.0061 \\ 0.0018 & -0.5715 & -0.0157 & 0.5868 \\ 0.8246 & 0.8206 & 0.8123 & 0.8097 \end{bmatrix} \tag{40}$$

which are obtained by rotating each nominal gimbal frame $(\mathbf{g}_i, \mathbf{s}_i, \mathbf{t}_i)$ with 1 degree about arbitrary direction. With these values, $||\Theta_a||^2 \simeq 0.0015$. The nominal value of the spacecraft inertia matrix is

$$J^n = \begin{bmatrix} 15000 & 3000 & -1000 \\ 3000 & 6500 & 2000 \\ -1000 & 2000 & 12000 \end{bmatrix} \text{ kg m}^2 \tag{41}$$

and the (unknown) actual inertia matrix is

$$J = \begin{bmatrix} 18000 & 2400 & -1200 \\ 2400 & 5200 & 1600 \\ -1200 & 1600 & 14400 \end{bmatrix} \text{ kg m}^2 \tag{42}$$

which is obtained by adding/subtracting 20% of the nominal values. The (unknown) actual value of Θ_s defined in (32) is then

$$\Theta_s \simeq [3000, -1300, 2400, -600, -200, -400, -89.77, 141.43, -162.77]^T.$$

The remained parameters used for the simulations are set as follows: the initial attitude is aligned with the inertia frame at rest ($\boldsymbol{\sigma}(0) = \boldsymbol{\omega}(0) = \dot{\boldsymbol{\omega}}(0) = [0, 0, 0]^T$), the initial gimbal angle is also zero at rest ($\boldsymbol{\gamma} = \dot{\boldsymbol{\gamma}} = [0, 0, 0, 0]^T$), the moment of inertia of the flywheels are $I_w = \text{diag}\{2.0, 2.0, 2.0, 2.0\}\ \text{kg m}^2$, the initial wheel spin speed is $\boldsymbol{\Omega}(0) = 10^4 \times [2.5, 3.5, 3.5, 3.0]^T$ rpm, and the controller parameters are chosen as $K_d = 10^3\mathbf{I}_3$, $\Lambda = \mathbf{I}_3$, $\beta = 0.01$, and $\delta = 0.01$. (Notice that Assumption 2 is satisfied with the given value of β.)

Notice that the initial wheel speeds of the VSCMGs are set to 25 000 $\sim$ 35 000 r/min, which are an order of magnitude larger than the speed of conventional CMGs, since the flywheels of VSCMGs used for IPACS in general need to spin very fast so that they are competitive against traditional chemical batteries. According to [20], even a higher speed than these vales is implementable, at least in a laboratory.

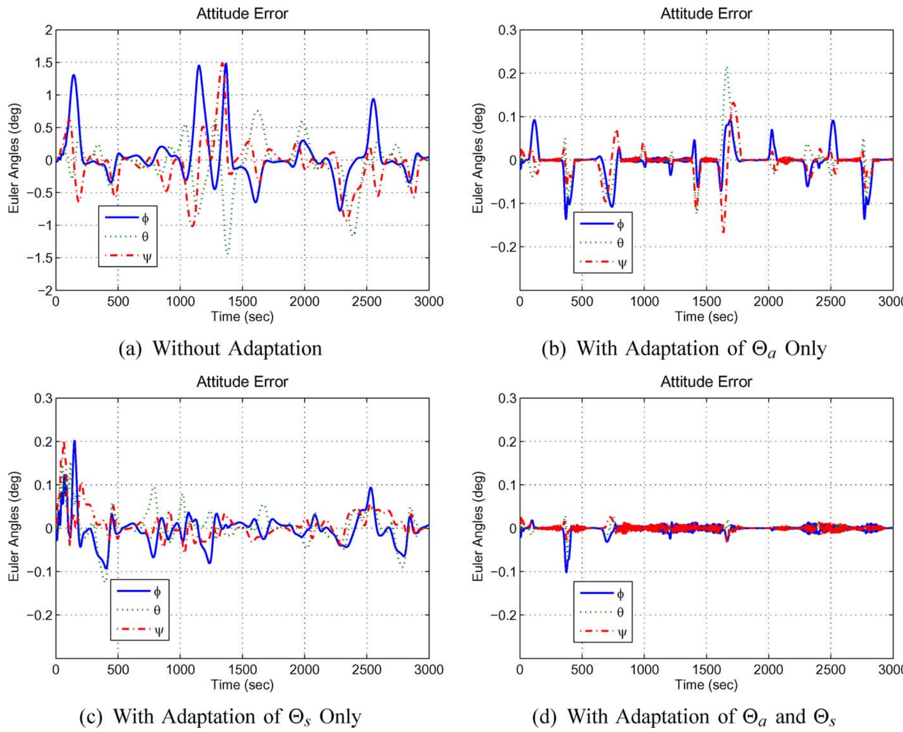
The reference trajectory is chosen so that the initial reference attitude is aligned with the body frame which is also aligned with the inertial frame, and the angular velocity of the reference attitude is chosen as $\omega_d(t) = 0.04(\sin 2\pi t/400, \sin 2\pi t/300, \sin 2\pi t/200)^T$ rad/s.

First, in order to show the effect of the misalignment of the axes of the VSCMG cluster and the uncertain inertia matrix, a simulation without adaptation was performed. (This nonadaptive control law can be regarded as a combination of a feedforward term and a simple proportional-derivative (PD) control term [1].) Fig. 2(a) shows the attitude tracking error (expressed with "3–2-1" Euler angles)[4] under control law with the adaptation gains Γ_a and Γ_s set to zero matrices. Since the flywheel speeds are very fast, there is large attitude tracking error without adaptation.

Next, another simulation was run with adaptation of the actuator uncertainty Θ_a only. The adaptation gains are set to $\Gamma_a = 10\,\mathbf{I}_{6N}$ and $\Gamma_s = 0$, and the resulting attitude error is shown in Fig. 2(b). There is significant performance improvement with the adaptation of Θ_a only, but tracking error with a magnitude of about 0.2 degree remains. On the other hand, Fig. 2(c) shows the attitude tracking error with adaptation of Θ_s only. The adaptation gains are $\Gamma_a = 0$ and $\Gamma_s = \text{diag}(10^7\,\mathbf{I}_6, 10^5\,\mathbf{I}_3)$. In fact, the control law in this scenario is almost identical with Slotine's method [1]. The attitude error is significantly attenuated using the adaptive controller, but there are again residual tracking errors with a magnitude of about 0.1 degree.

Finally, a simulation is performed with adaptation of both Γ_a and Γ_s, that is $\Gamma_a = 10\,\mathbf{I}_{6N}$ and $\Gamma_s = \text{diag}(10^7\,\mathbf{I}_6, 10^5\,\mathbf{I}_3)$. Fig. 2(d) shows the tracking performance is significantly improved upon Slotine's control law. Fig. 3(a) shows the time history of $||\hat{\Theta}_a||^2$. It is confirmed that $||\hat{\Theta}_a||^2$ does not drift

[4]The use of Euler angles in the figures in done solely for the convenience of the reader who may not be familiar with the MRPs.



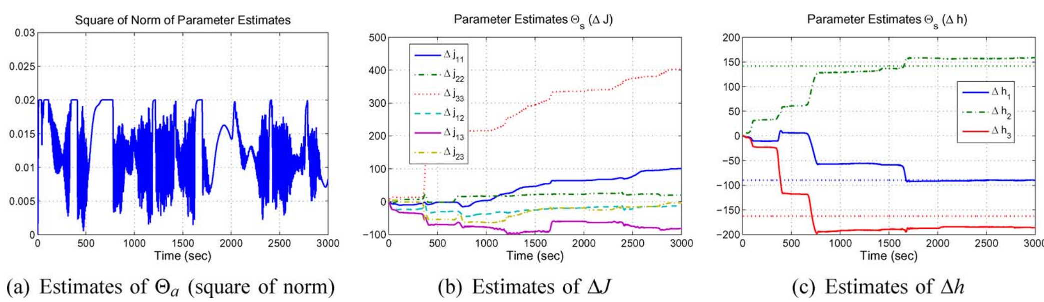
(a) Without Adaptation

(b) With Adaptation of Θ_a Only

(c) With Adaptation of Θ_s Only

(d) With Adaptation of Θ_a and Θ_s

Fig. 2. Attitude tracking errors.



(a) Estimates of Θ_a (square of norm)

(b) Estimates of ΔJ

(c) Estimates of Δh

Fig. 3. Parameter estimates.

more than $\beta + \delta = 0.02$ owing to the smooth projection algorithm. As a result, the steering law (7) remains well-defined. Fig. 3(b) and (c) show the time history of the parameter estimate $\hat{\Theta}_s$. In Fig. 3(c) the bold horizontal dotted lines denote the actual values of the components of $\mathbf{h}^{I\Delta}$. Some of the estimates approach their actual values, but they do not exactly converge to these values due to the lack of persistency of excitation. Note that, in general—and for a linear system—convergent estimation of m parameters requires at least $m/2$ sinusoids in the reference signal. For the nonlinear case such simple relation may not be valid [1]. In this example, the number of the parameters to be estimated is $6N + 9$, while the reference signal has only three sinusoids, thus the persistency condition is not satisfied. However, the tracking error will eventually converge to zero nevertheless.

VI. Conclusion

In this paper, we proposed an adaptive tracking control law for a nonlinear Hamiltonian MIMO dynamic system. The proposed control scheme has several significant improvements over the previous works in the literature. First, the proposed method can deal with uncertainties in the actuator terms. Second, the proposed method exploits the physical properties of the Hamiltonian systems and so the designed law is more compact than those in previous mathematics-based methods. Finally, it does not need over-parameterization to deal with uncertainties both in the inertia and the actuator modeling at the same time, while the previous methods do.

The developed adaptive algorithm is shown to significantly improve the tracking performance in the application to the spacecraft attitude control, but it still has room for improvement. For instance, while only three parameters are generally needed to express a misalignment of axis frame for one VSCMG, a total of 6 parameters are used in this method. Development of methods to reduce the number of estimated parameters would be extremely beneficial. In addition, robustness of the proposed control law against unknown external disturbances is also suggested for future study.

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