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ADAPTIVE ANTENNA SHAPE CONTROL USING PIEZOELECTRIC ACTUATORS†

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Abstract—This paper presents improved techniques for the shape control of composite material plates using piezoelectric actuators. The application of this work is for the shape control of spacecraft antenna to correct surface errors introduced by manufacturing, in-orbit thermal distortion, and moisture. A finite element model has been developed for a composite plate with distributed piezoelectric actuators and sensors. To improve the accuracy of the prediction of plate deformation, a simple higher-order deformation theory is used. The electric potential is treated as a generalized coordinate, allowing it to vary over the element. The applied voltages to the actuators are optimized to minimize the error between the desired shape and the deformed shape. Based on these techniques, two computer programs were developed on finite element modeling and optimization. The analytical results demonstrate the use of piezo-electric actuators for the active shape control of spacecraft antennas.

1. INTRODUCTION

For a communications satellite designer, providing precision surface for antenna reflectors has been a challenging problem. The surface errors are introduced by manufacturing errors, thermal distortion in orbit, moisture, loose structural joints, material degradation, and creep. These reflectors are made of graphite–epoxy structures because of requirement for low thermal distortion. Significant time and cost are spent during fabrication, analyses, and ground tests to minimize and determine surface errors. Even with this effort, several current spacecraft antenna surfaces have higher than predicted surface errors, resulting in poor antenna performance. The problem is even more critical for support structures of optical payloads. Smart structures, with the ability to correct on-orbit surface errors, have great potential for precision structures. The application of smart structures technology will also result reduction in cost in analyses and ground tests. Smart structures can also provide antenna beam shaping.

In general, smart structures are elements of the system that are able to sense the state of the structure and change it as the system demands. There has been a number of recent studies [1] on the use of smart structures for vibration control, shape control, and noise reduction. While substantial research effort has been devoted to the use of smart structures for the active vibration suppression, considerably less attention has been focused on the use of smart structures for shape control. A number of materials are available which may be used as sensor or actuator elements of smart structures. These materials include piezoelectric polymers and ceramics, shape memory alloys, electric-rheological fluids and optical fibers.

Active vibration control and shape control using smart structures is an active area of research at Spacecraft Research and Design Center at the Naval Postgraduate School. This paper presents the recent results of shape control using piezoelectric actuators. Although piezoelectric are limited to the production of relatively small structural deformations, they may be more than adequate for certain applications, such as countering thermal distortion and manufacturing errors in precision spacecraft antennas.

The objective of this research is the shape control of a composite plate using piezoelectric actuators such that the mean-square error between the desired shape and the achieved shape is minimized. The problem is analyzed in three parts: (a) finite element model of laminated composite plate using simple high-deformation theory; (b) finite element model of laminate plate with piezoelectric actuators with actuator potential as a generalized coordinate; (c) optimization of actuator voltages to minimize mean square error between the desired shape and achieved shape.

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2. FINITE ELEMENT MODEL OF COMPOSITE PLATE

The finite element model is based on the following displacement field [2]

$$U(x,y,z) = U_0 + z\phi_{x_0} + z^2\zeta_{x0} + z^3\varsigma_{x0},$$

$$V(x,y,z) = V_0 + z\phi_{y0} + z^2\zeta_{y0} + z^3\zeta_{y0},$$

$$W(x,y,z) = W(x,y,0) = W_0, \quad (1)$$

where U_0, V_0, and W_0 are the displacement of a point on the reference surface with coordinates (x, y, z). ϕ_x and ϕ_y are the average rotation about y and x axes, respectively, of the normal to the mid-surface of the undeformed plate, and z is the distance of a point from the mid-plane along z axis. The remaining terms correspond to higher-order rotations and are determined by applying the conditions that the top and the bottom surface are free from transverse shear stresses.

A four-node, bilinear, isoparametric, rectangular element with seven degrees of freedom at each node is developed. The generalized displacement vector at a nodal point is given by

$$\{q\} = [U_0\ V_0\ \phi_{x0}\ \phi_{y0}\ w_0\ w_{,x0}\ w_{,y0}]^{\mathrm{T}}. \quad (2)$$

3. FINITE ELEMENT MODEL OF SMART COMPOSITE PLATE [3–7]

The linear piezoelectric constitutive equations coupling the elastic field and electric field are expressed as:

$$\{D\} = [e]^{\mathrm{T}}\{\varepsilon\} + [\varepsilon^s]\{E\}, \quad (3)$$

$$\{\sigma\} = [c]\{\varepsilon\} - [e]\{E\}, \quad (4)$$

where **D** is the electric displacement vector, **e** is the dielectric permittivity matrix, **ε** is the strain vector, $\boldsymbol{\varepsilon}^s$ is the dielectric matrix at constant mechanical strain, **E** is the electric field vector, **δ** is the stress vector, and **c** is elasticity matrix for a constant electric field.

Hamilton's principle is used to determine the equations of motion. The Lagrangian $\Im$ of a piezoelectric body is defined in terms of total kinetic energy λ and potential energy $\hbar$ (including strain and electrical energies);

$$\Im = \int_V (\lambda - \hbar)\mathrm{d}V, \quad (5)$$

where

$$\lambda = \frac{1}{2}\rho\{\dot{q}\}^{\mathrm{T}}\{\dot{q}\}, \quad (6)$$

and

$$\hbar = \frac{1}{2}\left[\{\varepsilon\}^{\mathrm{T}}\{\sigma\} - \{E\}^{\mathrm{T}}\{D\}\right]. \quad (7)$$

Thus, the Lagrangian is

$$\Im = \int_V \left[\frac{1}{2}\rho\{\dot{q}\}^{\mathrm{T}}\{\dot{q}\} - \frac{1}{2}(\{\varepsilon\}^{\mathrm{T}}\{\sigma\} - \{E\}^{\mathrm{T}}\{D\})\right]\mathrm{d}V, \quad (8)$$

where $\dot{q}$ is the velocity and V is the piezoelectric volume. The virtual work δW done by the external force and the applied surface charge density applied to the piezoelectric body is

$$\delta W = \int_{s_1} \{\delta q\}^{\mathrm{T}}\{P_s\}\mathrm{d}s_1 - \int_{s_2} \delta\phi\mu\mathrm{d}s_2, \quad (9)$$

where s_1 and s_2 are the surfaces at which the mechanical and electrical loads are applied, respectively, P_s is a surface load and ϕ is the electric potential.

The Hamilton's principle is

$$\int_{t_1}^{t_2} \delta(\Im + W)\mathrm{d}t = 0, \quad (10)$$

where t_1 to t_2 is the time interval and all variations must vanish at $t = t_1$ and $t = t_2$.

Since the thickness of the piezoelectric layers is very small, it is assumed that the electric potential function is zero at the interface with laminated substructure and varies linearly across the thickness of the sensor or actuator as follows:

$$\Phi^{\mathrm{L}}(x,y,z) = (z - h_{Lp})\phi_0^{\mathrm{L}}(x,y), \quad (11)$$

where h_{Lp} is the z coordinate of the lower interface and ϕ_0^{L} can be treated as the generalized coordinate like generalized displacement coordinates at the mid-plane of the actuator and sensor layers.

A finite element model of the composite plate with piezoceramic actuator/sensor, as shown in Fig. 1, is developed. The equation of motion of the system becomes as follows:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K_{qq}]\{q\} - [K_{q\Phi}]\{\bar{\Phi}\} - \{F\}, \quad (12)$$

$$[K_{\Phi q}]\{q\} + [K_{\Phi\Phi}]\{\bar{\Phi}\} = \{G\}. \quad (13)$$

In the above equation, **q** is nodal displacement vector as defined for composite plate and $\bar{\Phi}$ is electric generalized coordinate related to electrical potential at the nodal point, **M** is the mass matrix, **C** is the damping matrix, **K** are the stiffness and interaction matrices between actuator potential and displacement, F is the external mechanical force and G is charge excitation. It should be noted that in this formulation, electric potential is allowed to vary across the element.

4. SHAPE CONTROL AND OPTIMIZATION [8,9]

The problem is to determine optimum actuator voltages for given actuator locations to minimize

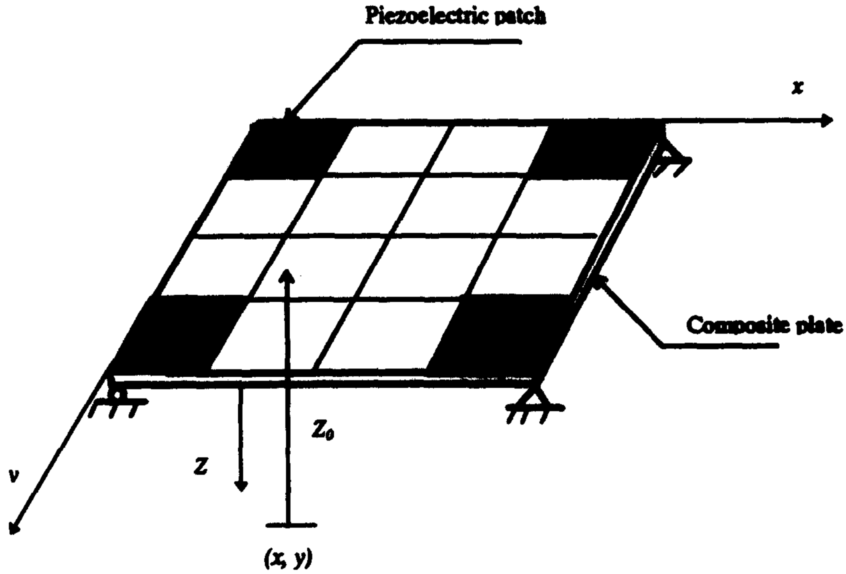

Fig. 1. Finite element model of composite plate with piezoelectric actuators and sensors.

the error between the desired shape and achieved shape. The analysis is based on the small deformation theory. The shape of the surface is defined by the z coordinate. The shape can be specified by a polynomial function in x and y as:

$$z_O = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2. \quad (14)$$

The desired shape is specified as

$$z_{\text{des}} = b_0 + b_1x + b_2y + b_3xy + b_4x^2 + b_5y^2, \quad (15)$$

where a and b are constants.

From the Fig. 2, as an example, A is a point on the original shape. Due to the actuator voltage, the deflection is δx_0 and δz_0. The error Δ is between the deformed point A' and desired point A'' with the same x and y coordinates. The error function for an element takes the form

$$\Lambda = \int_{\Omega_e} \bar{\Delta}^2 \mathrm{d}x\mathrm{d}y, \quad (16)$$

where Λ is error function for the element, Ω_e is the element domain, Δ represents the difference between the z coordinate on the reference surface of the desired shape and the z coordinate of a corresponding point on the actual surface. The error function for the surface is defined as the summation of the error functions of the elements. The objective is to make the error function as small as possible such that the error between the desired surface and the actual surface is minimized.

From Fig. 2, the distance Δ is defined as

$$\Delta = (z_0 + \delta z_0) - (z_{\text{des}} + \delta z_{\text{des}}).$$

As discussed in the previous sections, the deflection can be represented as a function of applied voltages. Therefore, the error function for one element in master element coordinate is written as:

$$\Lambda = \int_{-1}^{+1}\int_{-1}^{+1} \left[\left(z_0(\xi,\eta) - z_{\text{des}}(\xi,\eta)\right) + \eta 1(\xi,\eta)\tilde{w}(V) + \eta_2(\xi,\eta)\tilde{u}(V) + \eta_3(\xi,\eta)\tilde{v}(V)\right]^2 |\mathbf{J}| \mathrm{d}\xi \, \mathrm{d}\eta, \quad (18)$$

where ξ and η are the coordinates of the master element and $\mathbf{J}$ is determinant of the Jacobian matrix of transformation.

The structure objective function is defined as the summation of the error for each element such that

$$\bar{\Lambda} = \sum_{i=1}^{m} \Lambda, \quad (19)$$

subjected to constraint

$$\text{Lower limit} \leq V \leq \text{Upper limit}, \quad (20)$$

where m is the number of finite elements in the structure and V is actuator voltage.

MALAB optimization functions *f-min* and *f-mins* are used to find the optimum actuator voltage to minimize the error function. The function *f-min* is used for one constrained actuator voltage. The function *f-mins* is used for unconstrained multi-actuator voltages.

5. NUMERICAL ANALYSIS

Two MATLAB codes were developed based on these techniques. The first code COMPZ is a finite element code to solve a laminated plate with piezoelectric actuators and subjected to mechanical and electrical loads. The second MATLAB code OPTSHP determines change in the shape due to the application of mechanical loads and computes the

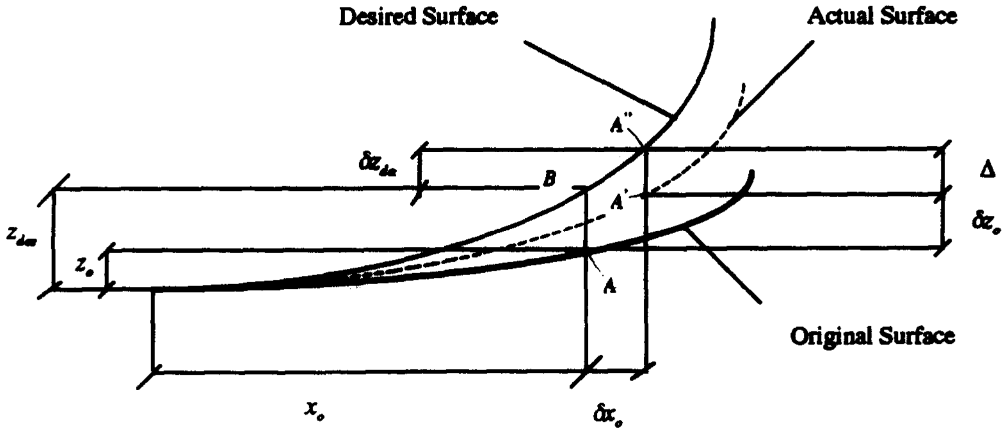

Fig. 2. Actual and desired shape.

optimum actuator voltages to minimize the objective function.

To validate the code COMPZ, the results from this code for composite plate with piezoelectric actuator were compared with the analytical results from Ref. [5] and finite elements results from Ref. [6]. The example used is a square plate consisting of a three-layered cross-ply laminated plate with thickness of 3 mm. Piezoelectric PVDF layers of 40 μm were used and it covered the whole surface.

The properties of the graphite–epoxy composite are as follows:

$E_{11} = 172.4$ GPa (25×10^6 psi)
$E_{22} = 6.9$ GPa (10^6 psi);
$G_{12} = G_{13} = 3.45$ GPa (0.5×10^6 psi);
$G_{23} = 1.38$ GPa (0.2×10^6);
$\nu_{12} = \nu_{13} = 0.25$.

The piezoelectric PVDF layer properties are as follows:

$e_{31} = 0.0460$ C/m^2;
$e_{32} = 0.0460$ C/m^2;
$e_{33} = 0.0000$ C/m^2.

Dielectricity

$\varepsilon^s_{11} = 0.1062 \times 10^{-9}$ F/m;
$\varepsilon^s_{22} = 0.1062 \times 10^{-9}$ F/m;
$\varepsilon^s_{33} = 0.1062 \times 10^{-9}$ F/m.

Poisson's ratio $\nu = 0.29$, mass density $\rho = 0.1800 \times 10^4$ kg/m^3, modulus of elasticity $E = 2 \times 10^4$ N/m^2.

The mechanical loading and electric potential distribution are described by:

$$q = q_0 \sin(\pi x/a) \sin(\pi y/b) \tag{21}$$

and

$$\phi(x,y,h_p) = V \sin(\pi x/a) \sin(\pi y/b). \tag{22}$$

The deflection is normalized as

$$\bar{w} = \frac{100E_T}{q_0 \lambda h} w,$$

where E_T is the transverse Young's modules of the graphite/epoxy layers, λ is the span to thickness ratio, h is the structure thickness.

Figure 3 shows the normalized center deflection versus length to thickness ratio and comparison with the analytical solution from Ref. [5] and finite element results from Ref. [6]. The results are in good agreement with the previous work.

To demonstrate the use of OPTSHP to determine optimum actuator voltages, a square 0.45×0.45 m plate with three layers (0°/90°/0°) is used and piezoelectric actuators are placed on the top surface. The length to thickness ratio is 50. The plate is divided into nine elements, as shown in Fig. 4. The properties of the graphite/epoxy and piezoelectric actuators are the same as in the previous example.

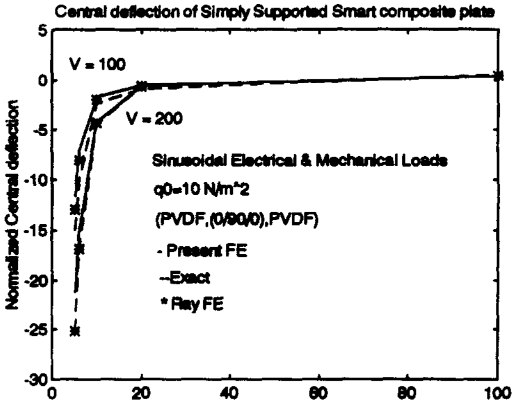

Fig. 3. Normalized center deflection versus length to thickness ratio.

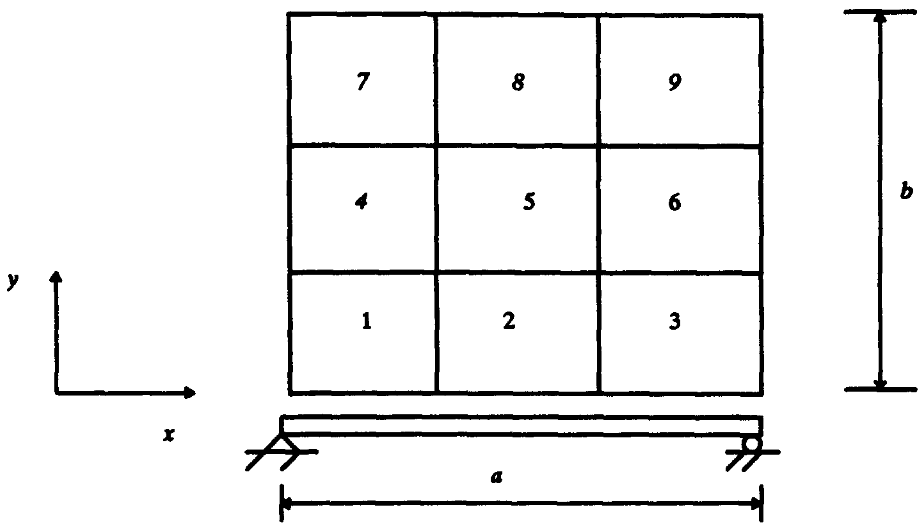

Fig. 4. Finite elements model of composite plate with piezoelectric actuator.

Table 1. Optimal applied voltage and error function at different actuator positions

Actuator position	Minimum applied voltage (V)	Minimum error function
Element 1	V = 78.226	f = 5.83495108e − 18
Element 2	V = 70.944	f = 4.78995246e − 18
Element 3	V = 99.999	f = 4.44647064e − 18
Element 4	V = 34.161	f = 5.88349351e − 18
Element 5	V = 37.346	f = 4.74690856e − 18
Element 6	V = 75.062	f = 4.03165923e − 18
Element 7	V = 78.611	f = 5.82972138e − 18
Element 8	V = 71.141	f = 4.78117803e − 18
Element 9	V = 99.999	f = 4.44063651e − 18

The original shape is chosen to be flat plate and the desired shape is selected as follows:

$$z_{des} = 1 \times 10^{-9}x^2 - 4.2 \times 10^{-10}xy + 6 \times 10^{-8}y^2. \tag{23}$$

Table 1 gives optimum actuator voltage and error function value as a function of location of the actuator. The program *f-min* is used to determine optimum actuator voltage to minimize the error between the desired surface and the deformed surface. The voltage has constraint such as $-100 < V < 100$. It is clear that the actuator voltage and error function are significantly dependent on the location of the actuator.

Table 2 gives the optimum voltages and error function when two actuators are used. In this case, the program *f-mins* is used to determine the optimum actuator voltages and the voltages are unconstrained. The results show that the location of actuators has significant influence on the optimum voltage and the error function.

6. CONCLUSIONS

A finite element model is developed to analyze a composite plate with piezoelectric actuators and sensors. A simple higher-order shear deformation theory is used to improve the prediction of plate deformation in comparison to linear theory. The actuator potential is used as a generalized coordinate to allow variation of the potential over the finite element. A MATLAB code COMPZ is developed based on these techniques and is validated by

Table 2. Optimal applied voltages and error functions for the case of pair of actuator used at a time

Actuators positions	Minimum appliedvoltages (V)	Minimum error function
Elements 1 and 7	V_1 = 41.612 V_7 = 47.265	f = 5.78007896e − 18
Elements 2 and 8	V_2 = 35.943 V_8 = 39.934	f = 4.74678213e − 18
Elements 3 and 9	V_3 = 85.710 V_9 = 90.547	f = 4.0583368e − 18
Elements 1 and 3	V_1 = − 26.368 V_3 = 175.055	f = 4.10114654e − 18
Elements 4 and 6	V_4 = − 8.231 V_6 = 79.660	f = 4.01544062e − 18
Elements 7 and 9	V_7 = 26.0895 V_9 = 175.193	f = 4.09288468e − 18
Elements 1 and 9	V_1 = − 17.007 V_9 = 170.053	f = 4.10970645e − 18
Elements 3 and 7	V_3 = 169.712 V_7 = − 16.138	f = 4.12087738e − 18

comparing the numerical results with analytical solutions and finite element solutions from other investigators.

A MATLAB code OPTSHP was developed to determine actuator voltages to minimize the error between the desired shape and the actual shape. The numerical results demonstrate the application of piezoceramic actuators for correcting antenna surface errors. Further work is required to develop systematic approach to determine optimum location of actuators to minimize shape error.

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