JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 21, No. 3, May–June 1998

Lyapunov Controller for Cooperative Space Manipulators

Gary E. Yale*
U.S. Air Force Academy, Colorado Springs, Colorado 80840
and
Brij N. Agrawal†
U.S. Naval Postgraduate School, Monterey, California 93940

The cooperative control of multiple manipulators attached to the same base as they reposition a common payload is discussed. The theory is easily applied to inertially based problems, as well as space-based free-floating platforms. The system equations of motion are developed, as well as a Lyapunov-based controller that ensures stability. The closed chain aspect of the problem reduces the system's degrees of freedom resulting in more actuators than degrees of freedom. This actuator redundancy is used to minimize a weighted norm of the actuator torques. A polynomial reference trajectory describes the path the payload will follow. The disturbance torque transmitted to the spacecraft centerbody by the motion of the manipulators is reduced by altering the order of the reference trajectory polynomial and its coefficients. Results from a two-dimensional, dual-arm configuration are included. Compared to the Lyapunov point controller alone, the addition of a fifth-order polynomial reference trajectory leads to superior performance in terms of actuator torque magnitudes, spacecraft centerbody attitude control, and payload repositioning accuracy and time. An eighth-order polynomial reference trajectory results in only small improvement over the fifth-order case.

Background

SPACE-based robotics platforms experience conditions unlike those of their terrestrial counterparts. With respect to the dynamics of the systems, the most notable difference is the absence of a fixed base on which to locate the manipulators. The consequence of this difference is that motion of the space-based manipulator transmits forces and moments to its mounting base resulting in translation and rotation of the base itself.[1,2] Generally, this motion is unwanted because the attitude control subsystem of the vehicle must compensate. One can estimate the spacecraft attitude disturbance caused by manipulator motion and use that information to command reaction wheels on the main body.[3–5] As an alternative, one could try to minimize the attitude disturbance the manipulators transmit to the main body. For a spacecraft with a single manipulator with redundant kinematics, the excess degrees of freedom can be used to minimize reactions transmitted to the main body.[6] Teleoperating a space manipulator to reduce satellite attitude disturbances has also been studied.[7] If the manipulator is sufficiently redundant, the attitude disturbances may be eliminated altogether.[8]

Using space manipulators to stabilize tethered satellite systems has also been proposed.[9] For spacecraft with multiple manipulators, cooperative control takes on more than one meaning. In one case, one manipulator repositions an object while a second manipulator, which is not grasping the object, moves to provide counterbalancing torques on the main body thereby reducing the spacecraft attitude disturbance.[10] A more traditional concept of cooperative control of multiple manipulators assumes the manipulators are each in contact with the payload. One control strategy developed for a fixed-base system controls the payload position and its internal forces using a Lyapunov controller or an adaptive controller.[11] A space-based version uses object impedance control to position the payload and control its internal forces.[12] In this paper, cooperative control means multiple manipulators grasping a common object moving in harmony to reposition the object. When more than one manipulator grasps an object, the actuator redundancy created by the closed chain dynamics permits tradeoffs to be made regarding how the actuators are used. Through appropriate selection of weighting factors, the user has great flexibility in choosing to what degree each actuator is involved in repositioning the payload.

The following development of an analytical model is based on a multiple-manipulator space robotics system. The manipulators already have a firm grasp of the payload. The initial conditions for the system are known although there may be some error in these values. Desired final conditions are also known. The equations of motion are derived from Lagrange's equations. This results in a set of second-order, nonlinear, coupled, differential equations. The initial and final boundary conditions for the payload are connected by means of a reference trajectory. Based on the payload reference trajectory, actuator torques are computed by means of inverse kinematics. The actuator torques are modified using a Lyapunov-derived controller. The controller compares the reference trajectories with the actual trajectories. The reference trajectories are selected by means of an optimization algorithm to reduce the attitude disturbance on the main spacecraft.

Equations of Motion

Development of the analytical model is predicated on establishing the variables and coordinate systems that will describe the system. The most general case is for a spacecraft with n manipulators involved in controlling the positioning of a common payload. The centerbody, manipulator links, and payload are rigid bodies. A semi-inertial axis system is located somewhere on the centerbody. The origin of this coordinate system remains fixed to the spacecraft. However, this coordinate frame maintains an inertial orientation. The centerbody attitude is referenced to this coordinate frame. Each manipulator link has its own set of body axes. The axes for each link are attached at the point of rotation nearest the centerbody. The x axis for each link points along the longitudinal axis of the link. The angles that describe link orientation are joint angles with two subscripts. The first subscript indicates which manipulator the link belongs to. The second subscript indicates the particular link of that manipulator. The links are numbered outward from the centerbody. The payload position and orientation is referenced back to the coordinate frame on the centerbody. The dual two-link manipulator case is shown in Fig 1. To eliminate gravity, this two-dimensional model is in the horizontal plane. The z axis is perpendicular to the plane of the motion. The generalized coordinates are

$$q = [\theta_0 \quad \theta_{L1} \quad \theta_{L2} \quad \theta_{R1} \quad \theta_{R2} \quad \theta_P \quad X_P \quad Y_P]^T \tag{1}$$

Received Feb. 20, 1995; revision received Aug. 1, 1997; accepted for publication Oct. 1, 1997.

*Assistant Professor, Department of Astronautics. Member AIAA.

†Professor, Department of Aeronautics and Astronautics. Associate Fellow AIAA.

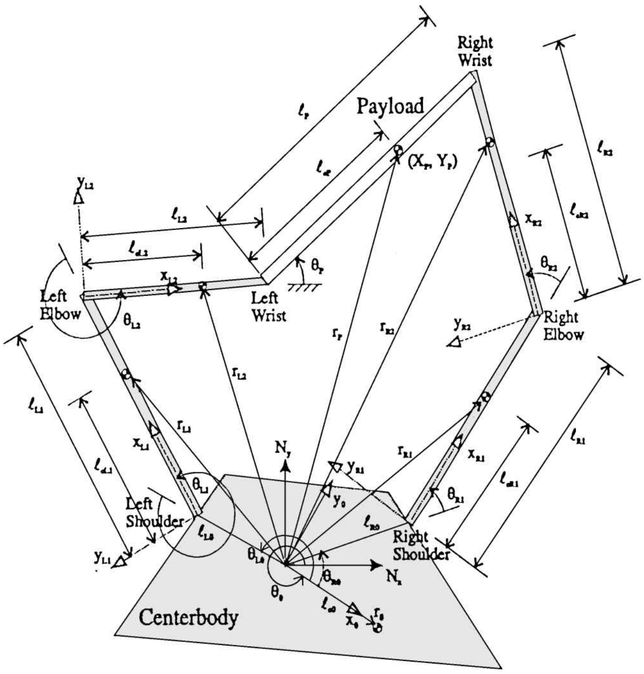

Fig. 1 Dual two-link manipulator case: —▶, semi-inertial axes, and —▷, body axes.

They include centerbody attitude, left and right arm joint angles, payload attitude, and payload center of mass Cartesian coordinates. Like the centerbody angle, the payload angle is referenced to an inertial coordinate frame. The mounting location for the left and right shoulders are given by the two constant angles θ_{L0} and θ_{R0}. The distances from the semi-inertial coordinate frame to the shoulders are l_{L0} and l_{R0}. Link lengths are designated as l_{Li} and l_{Ri}. Distances to link centers of mass contain the letter c. The control actuators for this system consist of a reaction wheel mounted on the centerbody and joint motors at the shoulder, elbow, and wrist of each manipulator. The resulting control input vector is

$$\boldsymbol{u} = [u_{wh} \quad u_{LS} \quad u_{LE} \quad u_{LW} \quad u_{RS} \quad u_{RE} \quad u_{RW}]^T \tag{2}$$

The first element is the reaction wheel torque. The next three elements are the shoulder, elbow, and wrist torques for the left manipulator. The final three elements are for the right manipulator.

The equations of motion for this system are developed using Lagrange's equations for a dynamic system with holonomic constraints:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right) - \frac{\partial L}{\partial \boldsymbol{q}} = \boldsymbol{Q} + A^T \boldsymbol{\lambda} \tag{3}$$

subject to the constraint equations $A\dot{\boldsymbol{q}} + A_0 = 0$, where $L = T - V$, T is kinetic energy, V is potential energy, $\boldsymbol{q}$ are the generalized coordinates, $\dot{\boldsymbol{q}}$ are the generalized velocities, $\boldsymbol{Q}$ are the applied nonconservative forces, and $A^T\boldsymbol{\lambda}$ are the constraint forces. The constraints are imposed by the geometry of the system.

Because of the closed chain nature of the system, the choice of generalized coordinates in Eq. (1) is not a minimum coordinate formulation. Consequently, the constraint forces [last term in Eq. (3)] will be nonzero.

Beginning with Lagrange's equation, the equations of motion can be rearranged into the alternate form

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + G(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \frac{\partial V}{\partial \boldsymbol{q}} = \boldsymbol{Q} + A^T\boldsymbol{\lambda} \tag{4}$$

where M is the inertia matrix. The two-dimensional system of Fig. 1 is confined to the horizontal plane. This prevents gravity from having any effect and causes $\partial V/\partial \boldsymbol{q} = 0$, which reduces Eq. (4) to

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + G(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{Q} + A^T\boldsymbol{\lambda} \tag{5}$$

where M is a function of the generalized coordinates and can be found by expressing the kinetic energy in the form

$$T = \tfrac{1}{2}\dot{\boldsymbol{q}}^T[M(\boldsymbol{q})]\dot{\boldsymbol{q}} \tag{6}$$

The G matrix contains all of the centripetal and Coriolis terms. It is most easily found using the following equations:

$$G(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} \dot{\boldsymbol{q}}^T C^{(1)}\dot{\boldsymbol{q}} \\ \dot{\boldsymbol{q}}^T C^{(2)}\dot{\boldsymbol{q}} \\ \cdots \\ \dot{\boldsymbol{q}}^T C^{(8)}\dot{\boldsymbol{q}} \end{bmatrix} \tag{7}$$

$$C^{(i)}_{jk} = \frac{1}{2}\left(\frac{\partial M_{ij}}{\partial q_k} + \frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{jk}}{\partial q_i}\right) \tag{8}$$

where $C^{(i)}_{jk}$ is the jkth element in the ith C matrix.

The nonconservative forces $\boldsymbol{Q}$ may be expressed as the product of a control influence matrix and the input vector ($\boldsymbol{Q} = B\boldsymbol{u}$). For the configuration of Fig. 1, the control influence matrix is

$$B = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

The constraints matrix A is derived by writing the system constraints in the Pfaffian form $A\dot{\boldsymbol{q}} + A_0 = 0$. The system constraints are those equations that describe the closed chain geometry of the system. Explicit terms for the constraints matrix are developed in the Appendix.

After substituting the matrix form of the generalized forces into the equations of motion [Eq. (5)], one has

$$M\ddot{\boldsymbol{q}} + G = B\boldsymbol{u} + A^T\boldsymbol{\lambda} \tag{10}$$

Because the M, G, B, and A matrices have already been found, the only remaining unknowns in Eq. (10) are the generalized accelerations, the actuator torques, and the Lagrange multipliers. By using the equations of motion and the Pfaffian form of the constraints, one can eliminate the Lagrange multipliers. The time derivative of the constraint equations ($A\dot{\boldsymbol{q}} + A_0 = 0$) is

$$A\ddot{\boldsymbol{q}} + \dot{A}\dot{\boldsymbol{q}} = 0 \tag{11}$$

Solving Eq. (10) for $\ddot{\boldsymbol{q}}$ and substituting the result into Eq. (11) permits one to find an expression for the Lagrange multipliers

$$\boldsymbol{\lambda} = (AM^{-1}A^T)^{-1}(AM^{-1}(G - B\boldsymbol{u}) - \dot{A}\dot{\boldsymbol{q}}) \tag{12}$$

Equation (12) can be substituted back into the equations of motion [Eq. (10)] leaving the generalized accelerations and the actuator torques as the only unknowns. As discussed in the next section, torques are found by means of inverse kinematics. Once the torques are known, the equations of motion can be integrated to find the generalized coordinates as functions of time.

Inverse Kinematics

If the motion of the system is to follow a prescribed trajectory, then the generalized accelerations at any point on that reference trajectory are known. Using reference trajectory displacements, velocities, and accelerations in the reference trajectory equivalent of the equations of motion [(Eq. 10)] and of the Lagrange multipliers [Eq. (12)] allow one to solve for the actuator torques needed to produce the reference accelerations. These equations are

$$M_{\mathrm{ref}}\ddot{\boldsymbol{q}}_{\mathrm{ref}} + G_{\mathrm{ref}} = B\boldsymbol{u}_{\mathrm{ref}} + A^T_{\mathrm{ref}}\boldsymbol{\lambda}_{\mathrm{ref}} \tag{13}$$

$$\boldsymbol{\lambda}_{\mathrm{ref}} = \left(A_{\mathrm{ref}}M^{-1}_{\mathrm{ref}}A^T_{\mathrm{ref}}\right)^{-1}\left(A_{\mathrm{ref}}M^{-1}_{\mathrm{ref}}(G_{\mathrm{ref}} - B\boldsymbol{u}_{\mathrm{ref}}) - \dot{A}_{\mathrm{ref}}\dot{\boldsymbol{q}}_{\mathrm{ref}}\right) \tag{14}$$

After substituting Eq. (14) into Eq. (13), the terms can be rearranged to produce equations of motion in the form

$$\tilde{M}\ddot{\boldsymbol{q}}_{\text{ref}} + \tilde{G} = \tilde{B}\boldsymbol{u}_{\text{ref}} \tag{15}$$

where

$$\tilde{M} = M_{\text{ref}}$$

$$\tilde{G} = G_{\text{ref}} - A_{\text{ref}}^T\left(A_{\text{ref}}M_{\text{ref}}^{-1}A_{\text{ref}}^T\right)^{-1}\left(A_{\text{ref}}M_{\text{ref}}^{-1}G_{\text{ref}} - \dot{A}_{\text{ref}}\dot{\boldsymbol{q}}_{\text{ref}}\right)$$

$$\tilde{B} = \left(I - A_{\text{ref}}^T\left(A_{\text{ref}}M_{\text{ref}}^{-1}A_{\text{ref}}^T\right)^{-1}A_{\text{ref}}M_{\text{ref}}^{-1}\right)B$$

In this study, the total number of actuators is more than the system degrees of freedom. This situation is caused by the geometric constraints of multiple manipulators handling a common object producing an excess of actuators as compared to degrees of freedom. As a result, there are an infinity of solutions for the reference torques. One method to select a specific solution is to establish and minimize a cost function. An obvious cost function is a weighted norm of the actuator torques

$$J = \tfrac{1}{2}\boldsymbol{u}_{\text{ref}}^T W_u \boldsymbol{u}_{\text{ref}} \tag{16}$$

where W_u is a user defined weighting matrix. The problem now becomes one of minimizing the cost function [Eq. (16)] subject to the constraint that the reference equations of motion are satisfied [Eq. (15)]. Augmenting the cost function with the constraint by means of another Lagrange multiplier γ leads to

$$J = \tfrac{1}{2}\boldsymbol{u}_{\text{ref}}^T W_u \boldsymbol{u}_{\text{ref}} + \gamma^T(\tilde{B}\boldsymbol{u}_{\text{ref}} - \tilde{M}\ddot{\boldsymbol{q}}_{\text{ref}} - \tilde{G}) \tag{17}$$

The minimum of the augmented cost function is found by taking the gradient of Eq. (17) with respect to the reference torques and with respect to the Lagrange multiplier. Each of the gradients is set to zero as follows:

$$\nabla_{\boldsymbol{u}_{\text{ref}}} J = 0 = W_u \boldsymbol{u}_{\text{ref}} + \tilde{B}^T\gamma \tag{18}$$

$$\nabla_{\gamma} J = 0 = \tilde{B}\boldsymbol{u}_{\text{ref}} - \tilde{M}\ddot{\boldsymbol{q}}_{\text{ref}} - \tilde{G} \tag{19}$$

Equations (18) and (19) are two equations in two unknowns $(\gamma, \boldsymbol{u}_{\text{ref}})$. To eliminate γ, solve Eq. (18) for $\boldsymbol{u}_{\text{ref}}$ and substitute the result into Eq. (19). Solve this equation for γ and substitute back into Eq. (18). Then solve for $\boldsymbol{u}_{\text{ref}}$ to get an expression for the reference actuator torques:

$$\boldsymbol{u}_{\text{ref}} = W_u^{-1}\tilde{B}^T\left(\tilde{B}W_u^{-1}\tilde{B}^T\right)^{-1}(\tilde{M}\ddot{\boldsymbol{q}}_{\text{ref}} + \tilde{G}) \tag{20}$$

These values for reference actuator torques minimize the augmented cost function [Eq. (17)] at each instant in time.

Lyapunov Controller

To develop a controller with guaranteed stability for this highly nonlinear system, one could choose a Lyapunov approach. If one substitutes Eq. (12) into Eq. (10) and solves for $\ddot{\boldsymbol{q}}$, the result can be expressed as

$$\ddot{\boldsymbol{q}} = C_1\boldsymbol{u} + C_2\dot{\boldsymbol{q}} + C_3 \tag{21}$$

where

$$C_1 = M^{-1}\{I - A^T(AM^{-1}A^T)^{-1}AM^{-1}\}B$$

$$C_2 = -M^{-1}A^T(AM^{-1}A^T)^{-1}\dot{A}$$

$$C_3 = M^{-1}\{A^T(AM^{-1}A^T)^{-1}AM^{-1} - I\}G$$

Similarly, the reference maneuver accelerations can be expressed as

$$\ddot{\boldsymbol{q}}_{\text{ref}} = C_{1\text{ref}}\boldsymbol{u}_{\text{ref}} + C_{2\text{ref}}\dot{\boldsymbol{q}}_{\text{ref}} + C_{3\text{ref}} \tag{22}$$

where the ref subscripts on the C matrices indicate that reference maneuver values need to be used in their calculation. Let error quantities between the actual variables and their reference maneuver counterparts be defined by

$$\delta\boldsymbol{q} = \boldsymbol{q} - \boldsymbol{q}_{\text{ref}}, \qquad \delta\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_{\text{ref}}, \qquad \delta\ddot{\boldsymbol{q}} = \ddot{\boldsymbol{q}} - \ddot{\boldsymbol{q}}_{\text{ref}} \tag{23}$$

Now define an arbitrary error Lyapunov function as

$$U = 0.5(\delta\dot{\boldsymbol{q}}\cdot\delta\dot{\boldsymbol{q}}) + f(\delta\boldsymbol{q}) \tag{24}$$

where $f(\delta\boldsymbol{q}) \geq 0$. Differentiating Eq. (24) results in

$$\dot{U} = \delta\dot{\boldsymbol{q}}\cdot\delta\ddot{\boldsymbol{q}} + \sum_i \frac{\partial f}{\partial(\delta q_i)}\delta\dot{q}_i \tag{25}$$

Let

$$\boldsymbol{F} = \begin{bmatrix} \dfrac{\partial f}{\partial(\delta q_1)} & \dfrac{\partial f}{\partial(\delta q_2)} & \cdots & \dfrac{\partial f}{\partial(\delta q_7)} \end{bmatrix}^T \tag{26}$$

Then Eq. (25) can be rewritten as

$$\dot{U} = \delta\dot{\boldsymbol{q}}\cdot(\delta\ddot{\boldsymbol{q}} + \boldsymbol{F}) \tag{27}$$

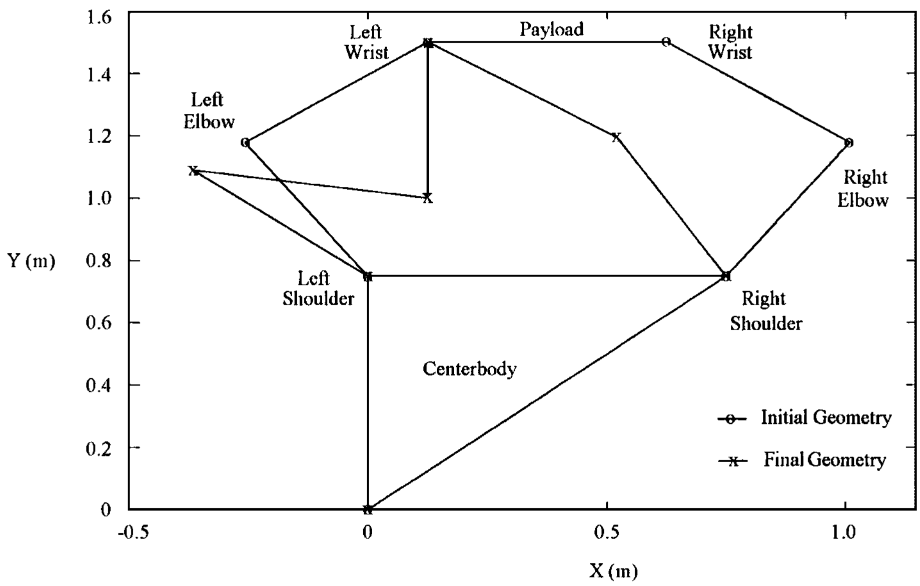

Fig. 2 Desired repositioning maneuver.

Substituting Eqs. (21) and (22) into Eq. (23) and then Eq. (23) into Eq. (27) produces

$$\dot{U} = \delta\dot{\boldsymbol{q}} \cdot [(C_1\boldsymbol{u} - C_{1\mathrm{ref}}\boldsymbol{u}_{\mathrm{ref}}) + (C_2\dot{\boldsymbol{q}} - C_{2\mathrm{ref}}\dot{\boldsymbol{q}}_{\mathrm{ref}}) + (C_3 - C_{3\mathrm{ref}}) + \boldsymbol{F}] \tag{28}$$

If one lets the quantity inside the brackets of Eq. (28) equal $-K_v\delta\dot{\boldsymbol{q}}$ where K_v is a positive definite matrix, then one is guaranteed that $\dot{U} \leq 0$ and, therefore, the system will be stable in the Lyapunov sense. K_v is assumed to be a diagonal matrix with generalized coordinate vector velocity gains on the main diagonal and zeros elsewhere. Solving Eq. (28) for command torques $\boldsymbol{u}$ leads to

$$\boldsymbol{u} = C_1^{\dagger}[-K_v\delta\dot{\boldsymbol{q}} + C_{1\mathrm{ref}}\boldsymbol{u}_{\mathrm{ref}} - (C_2\dot{\boldsymbol{q}} - C_{2\mathrm{ref}}\dot{\boldsymbol{q}}_{\mathrm{ref}}) - (C_3 - C_{3\mathrm{ref}}) - \boldsymbol{F}] \tag{29}$$

Equation (29) finds the torques that should be used rather than the reference torques. C_1 is an 8×7 matrix so that $C_1^{\dagger}$ is its pseudoinverse. All that remains is to choose a function for $f(\delta\boldsymbol{q})$ that satisfies $f(\delta\boldsymbol{q}) \geq 0$. One can chose $f(\delta\boldsymbol{q}) = 0.5\delta\boldsymbol{q}^T K_P \delta\boldsymbol{q}$, where K_P has the same diagonal form as K_v. This makes the error Lyapunov function analogous to mechanical energy.

Reference Trajectories

The reference trajectories describe the nominal path that the system follows in moving from the initial conditions to the desired final conditions. One need only specify reference trajectories for as many

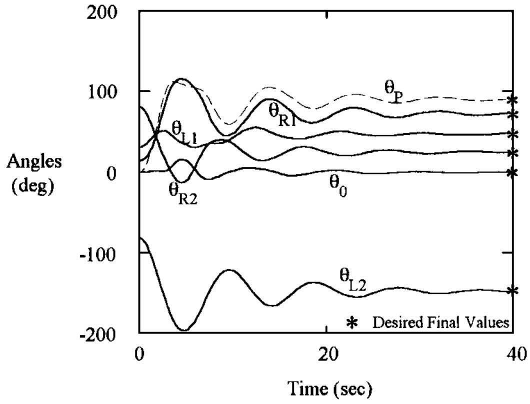

Fig. 3 Lyapunov point controller angles.

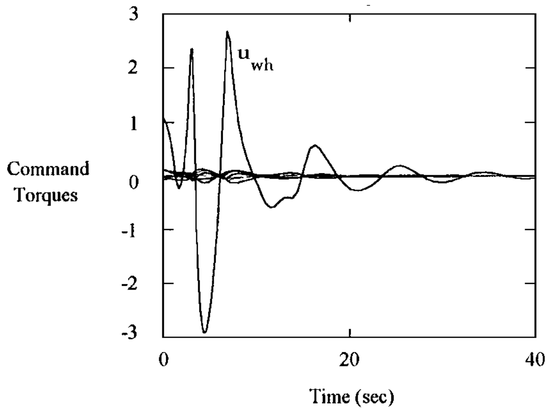

Fig. 4 Lyapunov point controller command torques.

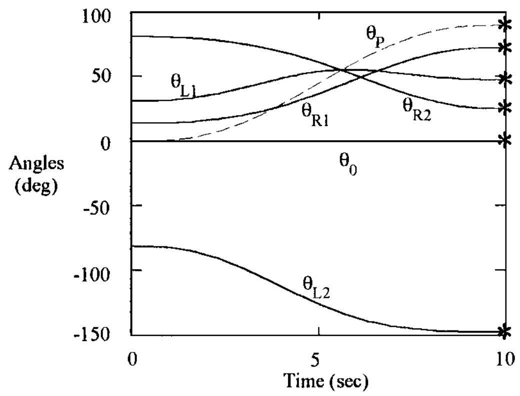

Fig. 6 Nominal fifth-order angles.

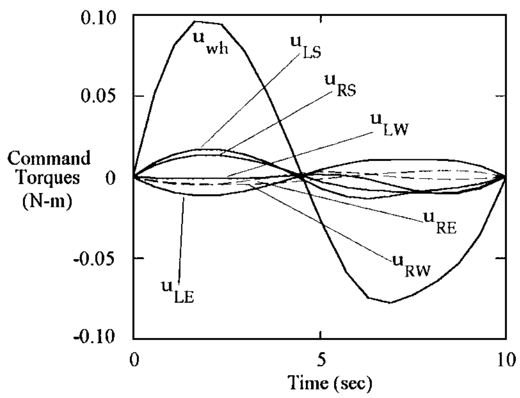

Fig. 7 Nominal fifth-order command torques.

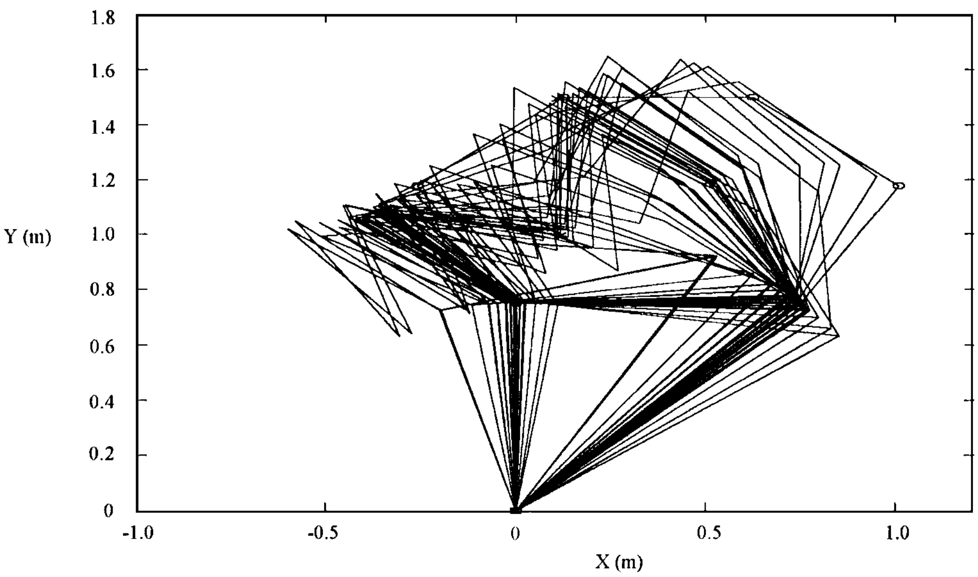

Fig. 5 Lyapunov point controller time-lapse stick figure.

generalized coordinates as there are degrees of freedom. The choice of which generalized coordinates to specify is entirely arbitrary. A reasonable choice is any set that includes the payload coordinates and centerbody attitude because the user will probably be especially interested in these coordinates. Any path that connects the associated endpoints can be a reference trajectory. Recall, however, that the usefulness of the reference trajectories is to permit calculation of the generalized coordinates positions, velocities, and accelerations for use in the inverse kinematics calculations. To help ensure that the payload does not experience any unnecessary jerk, one might further constrain the path such that the velocities and accelerations are zero at the endpoints and continuous in between. Therefore, a convenient form for the reference trajectory is as a polynomial function of time. The user decides the maneuver duration in advance. The minimum-order polynomial that satisfies the preceding boundary conditions is

$$f(\tau) = 6\tau^5 - 15\tau^4 + 10\tau^3 \tag{30}$$

where τ is the normalized time, $\tau = (t - t_0)/(t_f - t_0)$, and t_0 and t_f are maneuver start and stop times.

The following equations illustrate how this fifth-order polynomial reference trajectory would apply to the payload attitude generalized coordinate:

$$\Delta\theta_P = \theta_P(t_f) - \theta_P(t_0) \tag{31}$$

$$\theta_{P_{\text{ref}}}(t) = \theta_P(t_0) + (6\tau^5 - 15\tau^4 + 10\tau^3)(\Delta\theta_P) \tag{32}$$

$$\dot{\theta}_{P_{\text{ref}}}(t) = (30\tau^4 - 60\tau^3 + 30\tau^2)(\Delta\theta_P)\left(\frac{1}{t_f - t_0}\right) \tag{33}$$

$$\ddot{\theta}_{P_{\text{ref}}}(t) = (120\tau^3 - 180\tau^2 + 60\tau)(\Delta\theta_P)\left(\frac{1}{(t_f - t_0)^2}\right) \tag{34}$$

Higher-order polynomials can increase the complexity of the path but offer the advantage that an infinity of polynomial coefficients satisfy the position, velocity, and acceleration boundary conditions. This affords an opportunity to select the coefficients based on another optimization function. Because a reaction wheel on the centerbody will be required to maintain spacecraft attitude, the reaction wheel torque history is a prime candidate for optimization. Possible cost functions include the integral of the absolute value of reaction wheel torque or the maximum reaction wheel torque given by

$$J = \int_{t_0}^{t_f} |u_{\text{wheel}}|\, \mathrm{d}t \qquad \text{or} \qquad J = \max(|u_{\text{wheel}}|) \tag{35}$$

Results

The system used to generate these results is a dual two-link manipulator configuration similar to Fig. 1. The system properties used for the simulations are listed in Table 1.

The stick figure representation of Fig. 2 depicts the initial and final conditions of the desired maneuver (payload will rotate 90 deg and its right endpoint will finish where the left endpoint started). Four cases are presented to illustrate the system dynamics and the effect of using a reference trajectory. In all but one case, the boundary conditions of the payload are the same. All seven actuators are weighted equally in the torque calculations [Eqs. (16–20)].

In the first simulation, the repositioning is done entirely by the Lyapunov controller without the benefit of a reference trajectory. Figure 3 presents the angular displacement history. The payload Cartesian coordinate profiles (X_P and Y_P) are not shown but are very similar in appearance to the payload attitude profile θ_P. The asterisks on the right side of the plot indicate the desired final angles. Although the system is approaching the desired final geometry, it has not completely settled down even after 40 s. Position errors are still present, as well as nonzero velocities. Also, the reaction wheel torque is quite high during the maneuver (Fig. 4). The oscillatory nature of the system is evident in the angular position and velocity

Table 1 System properties

Length, m		Mass, kg		Center of mass, m		Moment of inertia, kg-m^2		Shoulder location, deg	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
l_{L0}	0.75	m_0	5	lc_0	0	I_0	5	θ_{L0}	90
l_{L1}	0.5	m_{L1}	1	lc_{L1}	0.25	I_{L1}	0.02083	θ_{R0}	45
l_{L2}	0.5	m_{L2}	1	lc_{L2}	0.25	I_{L2}	0.02083	——	——
l_{R0}	0.5	m_{R1}	1	lc_{R1}	0.25	I_{R1}	0.02083	——	——
l_{R1}	0.5	m_{R2}	1	lc_{R2}	0.25	I_{R2}	0.02083	——	——
l_{R2}	0.5	m_P	1	lc_P	0.25	I_P	0.02083	——	——
l_P	$0.75\sqrt{2}$	——	——	——	——	——	——	——	——

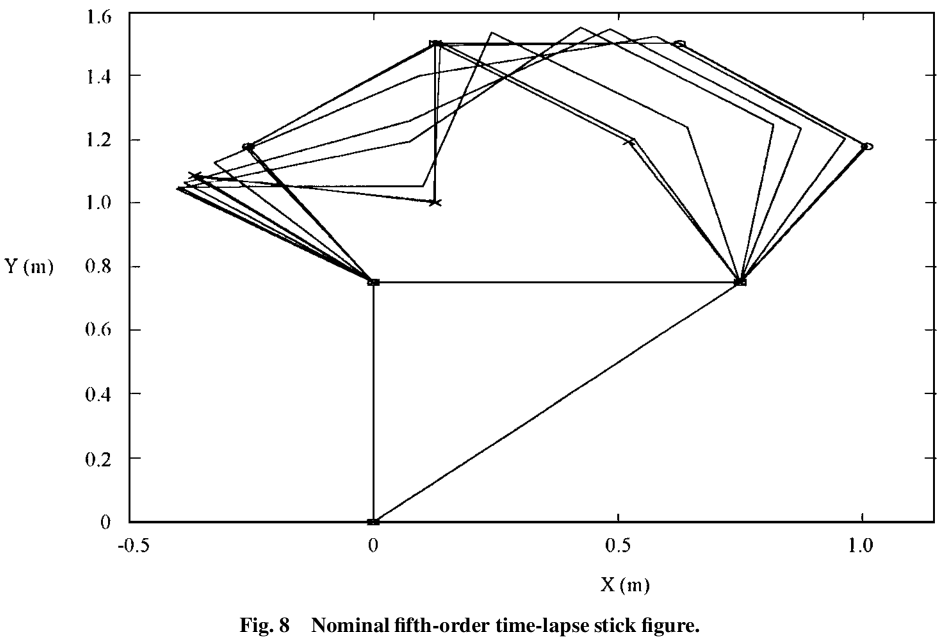

Fig. 8 Nominal fifth-order time-lapse stick figure.

plots. This behavior is also evident in Fig. 5, which shows a time-lapse representation of the system geometry at several instances during the maneuver. This controller also does a poor job of maintaining the centerbody attitude. This is clearly evident in Figs. 3 and 5. The attitude error peaks at about 16 deg.

The second simulation uses a fifth-order polynomial reference trajectory [Eq. (30)] applied to the payload generalized coordinates. Commanded control torques are calculated based on Eq. (29). This equation considers the errors with a reference trajectory, as well as reference torques produced from minimizing a weighted norm of the actuator torques associated with the reference trajectory. The maneuver time was selected to be 10 s. As is evident in Fig. 6, the system successfully moves from initial conditions to desired final conditions. The command torques (Fig. 7) are an order of magnitude smaller than in the earlier case. More importantly, the centerbody attitude is maintained throughout the maneuver. Figure 8 shows the time-lapse depiction of the maneuver.

The third simulation is a variation on the second one. Everything is the same except for the initial conditions. The controller is told that the initial conditions are the same, but the true initial conditions are such that the payload is tilted 10 deg. This case tests the stability of the controller and illustrates that perfect information is not required in order to obtain good results. As can be seen in Figs. 9–11, the simulation exhibits damped oscillatory behavior but not as severe as the Lyapunov point controller.

The fourth simulation is the same as the nominal fifth-order case except for the use of an eighth-order reference trajectory polynomial. The polynomial was picked to minimize the integral of the absolute value of the reaction wheel torque [Eq. (35)]. The resulting polynomial is

$$f(\tau) = 0.0794\tau^8 + 0.6410\tau^7 + 0.0278\tau^6 + 1.2764\tau^5 - 8.5973\tau^4 + 7.5727\tau^3 \qquad (36)$$

The trajectories that result from this polynomial are very similar to the fifth-order reference trajectories. As one might expect, the performance is also very similar.

Comparing the values produced by integrating the absolute value of the reaction wheel torque [Eq. (35)] for the four simulations provides a means to distinguish between the cases. A second metric is the absolute value of the maximum reaction wheel torque [also Eq. (35)]. Another obvious choice is to bound the centerbody attitude error during each simulation. The results are summarized in Table 2. Clearly the point controller is the worst controller based on all three metrics. The difference between the nominal fifth- and eighth-order tracking controllers is only slight.

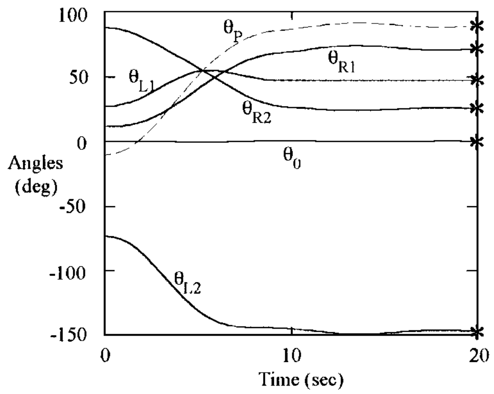

Fig. 9 Perturbed fifth-order angles.

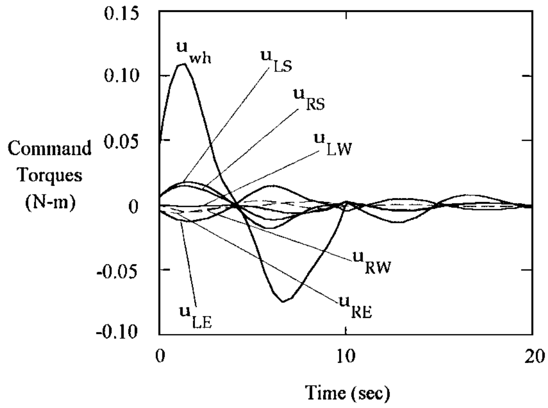

Fig. 10 Perturbed fifth-order command torques.

Table 2 Comparisons of simulations

Controllers	$\int \lvert u_{\text{wheel}} \rvert \, dt$	$\lvert u_{\max} \rvert$	Centerbody attitude error, deg
Lyapunov point controller	17.3841	2.9365	16.2261
Tracking controller			
Nominal fifth order	0.5746	0.0961	0.0000
Perturbed fifth order	0.5748	0.1092	0.3565
Nominal eighth order	0.5705	0.0885	0.0000

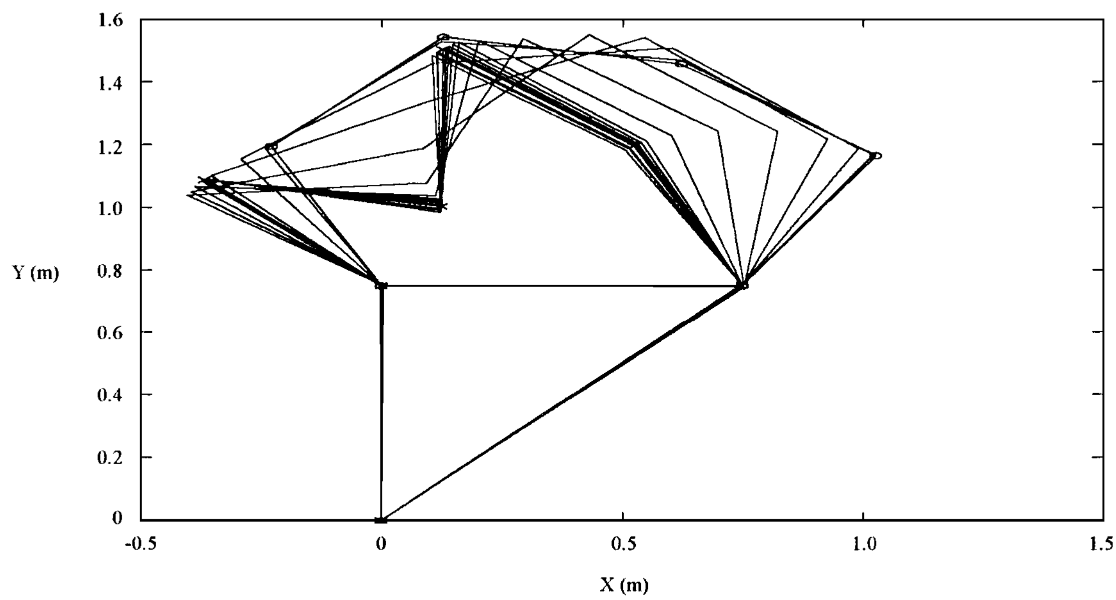

Fig. 11 Perturbed fifth-order time-lapse stick figure.

Conclusions

The problem of repositioning a payload that is grasped by multiple manipulators mounted on the same free-floating base is addressed. The closed chain nature of the problem allows for an infinite set of joint actuator torques to accomplish the maneuver. A technique is presented whereby a weighted norm approach selects a torque profile to use. Use of polynomial reference trajectory significantly improves the performance of the system. As the order of the polynomial increases, the redundancy of the coefficients can be used to select values that lead to reduced centerbody attitude disturbance. The biggest improvement is from including the polynomial trajectory in the first place. The minimal improvement achieved by increasing the order of the polynomial probably does not warrant the additional computational expense.

Appendix: Dual Two-Link Manipulator Matrix Terms

For the dual two-link manipulator case shown in Fig. 1, the inertia matrix is given by

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & 0 & 0 & 0 \\ M_{21} & M_{22} & M_{23} & 0 & 0 & 0 & 0 & 0 \\ M_{31} & M_{32} & M_{33} & 0 & 0 & 0 & 0 & 0 \\ M_{41} & 0 & 0 & M_{44} & M_{45} & 0 & 0 & 0 \\ M_{51} & 0 & 0 & M_{54} & M_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_P & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_P & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_P \end{bmatrix} \tag{A1}$$

where

$$M_{55} = I_{R2} + m_{R2} lc_{R2}^2$$

$$M_{45} = M_{54} = M_{55} + m_{R2} l_{R1} lc_{R2} \cos\theta_{R2}$$

$$M_{23} = M_{32} = M_{33} + m_{L2} l_{L1} lc_{L2} \cos\theta_{L2}$$

$$M_{15} = M_{51} = M_{45} + m_{R2} l_{R0} lc_{R2} \cos(\theta_{R1} + \theta_{R2})$$

$$M_{44} = M_{45} + I_{R1} + m_{R2} l_{R1} lc_{R2} \cos\theta_{R2} + m_{R1} lc_{R1}^2 + m_{R2} l_{R1}^2$$

$$M_{14} = M_{41} = M_{44} + l_{R0}(m_{R1} lc_{R1} + m_{R2} l_{R1})\cos\theta_{R1} + m_{R2} l_{R0} lc_{R2} \cos(\theta_{R1} + \theta_{R2})$$

$$M_{33} = I_{L2} + m_{L2} lc_{L2}^2$$

$$M_{22} = M_{23} + I_{L1} + m_{L2} l_{L1} lc_{L2} \cos\theta_{L2} + m_{L1} lc_{L1}^2 + m_{L2} l_{L1}^2$$

$$M_{13} = M_{31} = M_{23} + m_{L2} l_{L0} lc_{L2} \cos(\theta_{L1} + \theta_{L2})$$

$$M_{12} = M_{21} = M_{22} + l_{L0}(m_{L1} lc_{L1} + m_{L2} l_{L1})\cos\theta_{L1} + m_{L2} l_{L0} lc_{L2} \cos(\theta_{L1} + \theta_{L2})$$

$$\begin{aligned} M_{11} = {} & I_0 + m_0 lc_0^2 + 2m_{R2} l_{R0} lc_{R2} \cos(\theta_{R1} + \theta_{R2}) \\ & + \big((m_{R1} + m_{R2}) l_{R0}^2 + 2 l_{R0}(m_{R1} lc_{R1} + m_{R2} l_{R1})\cos\theta_{R1}\big) \\ & + (m_{L1} + m_{L2}) l_{L0}^2 + 2 l_{L0}(m_{L1} lc_{L1} + m_{L2} l_{L1})\cos\theta_{L1} \\ & + 2m_{L2} l_{L0} lc_{L2} \cos(\theta_{L1} + \theta_{L2}) + M_{22} + M_{44} \end{aligned}$$

Because the generalized coordinates for the payload are referenced to the centerbody coordinate frame, the inertia matrix is decoupled between the payload and the rest of the system. Coupling does exist between the spacecraft centerbody and each of the manipulators.

To develop the constraints matrix A, the dual two-link manipulator system described uses eight generalized coordinates. Because this system has only four degrees of freedom, an additional four equations are needed to describe the constraints. These equations come from geometric relationships describing the payload center of mass Cartesian coordinates in terms of the left and right arm generalized coordinates:

$$X_P = l_{L0}\cos(\theta_0 + \theta_{L0}) + l_{L1}\cos(\theta_0 + \theta_{L0} + \theta_{L1}) + l_{L2}\cos(\theta_0 + \theta_{L0} + \theta_{L1} + \theta_{L2}) + lc_P\cos\theta_P \tag{A2}$$

$$Y_P = l_{L0}\sin(\theta_0 + \theta_{L0}) + l_{L1}\sin(\theta_0 + \theta_{L0} + \theta_{L1}) + l_{L2}\sin(\theta_0 + \theta_{L0} + \theta_{L1} + \theta_{L2}) + lc_P\sin\theta_P \tag{A3}$$

$$X_P = l_{R0}\cos(\theta_0 + \theta_{R0}) + l_{R1}\cos(\theta_0 + \theta_{R0} + \theta_{R1}) + l_{R2}\cos(\theta_0 + \theta_{R0} + \theta_{R1} + \theta_{R2}) - (l_P - lc_P)\cos\theta_P \tag{A4}$$

$$Y_P = l_{R0}\sin(\theta_0 + \theta_{R0}) + l_{R1}\sin(\theta_0 + \theta_{R0} + \theta_{R1}) + l_{R2}\sin(\theta_0 + \theta_{R0} + \theta_{R1} + \theta_{R2}) - (l_P - lc_P)\sin\theta_P \tag{A5}$$

To get the Pfaffian form, differentiate Eqs. (A2–A5) and rearrange terms. The following equations express the result. The constant term $\boldsymbol{A}_0$, is a zero vector:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & A_{16} & -1 & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 & A_{26} & 0 & -1 \\ A_{31} & 0 & 0 & A_{34} & A_{35} & A_{36} & -1 & 0 \\ A_{41} & 0 & 0 & A_{44} & A_{45} & A_{46} & 0 & -1 \end{bmatrix} \begin{bmatrix} \dot\theta_0 \\ \dot\theta_{L1} \\ \dot\theta_{L2} \\ \dot\theta_{R1} \\ \dot\theta_{R2} \\ \dot\theta_P \\ \dot X_P \\ \dot Y_P \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{A6}$$

where

$$A_{16} = -lc_P\sin\theta_P, \qquad A_{26} = lc_P\cos\theta_P$$

$$A_{36} = (l_P - lc_P)\sin\theta_P, \qquad A_{46} = -(l_P - lc_P)\cos\theta_P$$

$$A_{45} = l_{R2}\cos(\theta_0 + \theta_{R0} + \theta_{R1} + \theta_{R2})$$

$$A_{44} = A_{45} + l_{R1}\cos(\theta_0 + \theta_{R0} + \theta_{R1})$$

$$A_{41} = A_{44} + l_{R0}\cos(\theta_0 + \theta_{R0})$$

$$A_{35} = -l_{R2}\sin(\theta_0 + \theta_{R0} + \theta_{R1} + \theta_{R2})$$

$$A_{34} = A_{35} - l_{R1}\sin(\theta_0 + \theta_{R0} + \theta_{R1})$$

$$A_{31} = A_{34} - l_{R0}\sin(\theta_0 + \theta_{R0})$$

$$A_{23} = l_{L2}\cos(\theta_0 + \theta_{L0} + \theta_{L1} + \theta_{L2})$$

$$A_{22} = A_{23} + l_{L1}\cos(\theta_0 + \theta_{L0} + \theta_{L1})$$

$$A_{21} = A_{22} + l_{L0}\cos(\theta_0 + \theta_{L0})$$

$$A_{13} = -l_{L2}\sin(\theta_0 + \theta_{L0} + \theta_{L1} + \theta_{L2})$$

$$A_{12} = A_{13} - l_{L1}\sin(\theta_0 + \theta_{L0} + \theta_{L1})$$

$$A_{11} = A_{12} - l_{L0}\sin(\theta_0 + \theta_{L0})$$

References

[1]Lindberg, R. E., Longman, R. W., and Zedd, M. F., "Kinematic and Dynamic Properties of an Elbow Manipulator Mounted on a Satellite," *Journal of the Astronautical Sciences*, Vol. 38, No. 4, 1990, pp. 397–421.

[2]Alexander, H. L., and Cannon, R. H., Jr., "An Extended Operational-Space Control Algorithm for Satellite Manipulators," *Journal of the Astronautical Sciences*, Vol. 38, No. 4, 1990, pp. 473–486.

[3]Oda, M., "On the Dynamics and Control of ETS-7 Satellite and Its Robot Arm," *IROS '94: Proceedings of the IEEE/RSJ/GI International Conference on Intelligent Robots and Systems*, Inst. of Electrical and Electronics Engineers, Piscataway, NJ, pp. 1586–1593.

[4]Longman, R. W., Lindberg, R. E., and Zedd, M. F., "Satellite–Mounted Robot Manipulators-New Kinematics and Reaction Moment Compensation," *International Journal of Robotics Research*, Vol. 6, No. 3, 1987, pp. 87–103.

[5]Yoshida, K., "Practical Coordination Control Between Satellite Attitude and Manipulator Reaction Dynamics Based on Computed Momentum Concept," *IROS '94: Proceedings of the IEEE/RSJ/G1 International Conference on Intelligent Robots and Systems*, Inst. of Electrical and Electronics Engineers, Piscataway, NJ, pp. 1578–1585.

[6]Chung, C. L., Desa, S., and deSilva, C. W., *Base Reaction Optimization of Redundant Manipulators for Space Applications*, Carnegie Mellon Univ., Pittsburgh, PA, 1988.

[7]Sato, Y., Hirata, M., Nagashima, F., Maruyama, T., and Uchiyama, T., "Reducing Attitude Disturbances While Teleoperating a Space Manipulator," *Proceedings IEEE International Conference on Robotics and Automation*, IEEE Computer Society Press, Los Alamitos, CA, 1993, pp. 516–523.

[8]Vafa, Z., "Space Manipulator Motions with No Satellite Attitude Disturbances," *Proceedings of 1990 IEEE International Conference on Robotics and Automation*, IEEE Computer Society Press, Los Alamitos, CA, 1990, pp. 1770–1775.

[9]Sadigh, M. J., and Misra, A. K., "Stabilizing Tethered Satellite Systems Using Space Manipulators," *IROS '94: Proceedings of the IEEE/RSJ/G1 International Conference on Intelligent Robots and Systems*, Inst. of Electrical and Electronics Engineers, Piscataway, NJ, pp. 1546–1553.

[10]Yoshida, K., Kurazume, R., and Umetani, Y., "Torque Optimization Control in Space Robots with a Redundant Arm," *Proceedings of IROS '91*, Inst. of Electrical and Electronics Engineers, New York, 1991, pp. 1647–1652.

[11]Ahmad, S., and Zribi, M., "Lyapunov Based Control Design for Multiple Robots Handling a Common Object," *Mechanics and Control: Proceedings of 4th Workshop on Mechanics and Control*, edited by J. M. Skowronski, H. Flashner, and R. S. Guttaul, Springer–Verlag, Berlin, 1992, pp. 1–17.

[12]Schneider, S. A., and Cannon, R. H., Jr., "Object Impedance Control for Cooperative Manipulation: Theory and Experimental Results," *Proceedings of 1989 IEEE International Conference on Robotics and Automation*, IEEE Computer Society Press, Washington, DC, 1989, pp. 1076–1083.