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Dynamic Characteristics of Liquid Motion in Partially Filled Tanks of a Spinning Spacecraft

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This paper presents a boundary-layer model to predict dynamic characteristics of liquid motion in partially filled tanks of a spinning spacecraft. The solution is obtained by solving three boundary-value problems: an inviscid fluid problem, a boundary-layer problem, and a viscous correction problem. The boundary-layer solution is obtained analytically, and the solutions to inviscid and viscous correction problems are obtained by using finite element methods. The model has been used to predict liquid natural frequencies, mode shapes, damping ratios, and nutation time constants for a spinning spacecraft. The results show that liquid motion in general will contain significant circulatory motion due to Coriolis forces except in the first azimuth and first elevation modes. Therefore, only these two modes can be represented accurately by equivalent pendulum models. The analytical results predict a sharp drop in nutation time constants for certain spacecraft inertia ratios and tank fill fractions. This phenomenon was also present during on-orbit liquid slosh tests and ground air-bearing tests.

I. Introduction

A RECENT trend in geosynchronous spacecraft design is to use liquid apogee motors, which results in liquid constituting almost half of the spacecraft mass during transfer orbit. In these spacecraft, liquid motion significantly influences the spacecraft attitude stability and control. LEASAT, a geosynchronous spacecraft with liquid apogee motor, launched in September 1984, experienced attitude control motion instability[1] during the pre-apogee injection phase, immediately following the activation of despin control. The instability was found to be the result of interaction between liquid lateral sloshing modes and the attitude control. This experience demonstrated that the analysis of dynamic interaction between liquid slosh motion and attitude control is critical in the attitude control design of these spacecraft. To perform this analysis, accurate determination of liquid dynamic characteristics, such as natural frequencies, mode shapes, damping, and modal masses becomes important. Accurate prediction of liquid dynamic characteristics is, however, a difficult problem because of the complexity of the hydrodynamical equations of motion.

Several investigators have analyzed the fluid motion in rotating containers. Greenspan[2] analyzed the transient motion during spin up of an arbitrarily shaped container filled with viscous imcompressible fluid. Stewartson[3] developed a stability criterion for a spinning top containing fluid. This stability criterion was corrected by Wedemeyer[4] by considering fluid viscosity. Nayfeh and Meirovitch[5] analyzed a spinning rigid body with a spherical cavity partially filled with liquid. Viscous effects are considered only for a boundary layer near the wetted surface. Hendricks and Morton[6] analyzed the stability of a rotor partially filled with a viscous incompressible fluid. Stergiopoulous and Aldridge[7] studied inertial waves in a partially filled cylindrical cavity during spin up. Pfeiffer[8] introduced the concept of homogeneous vorticity to the problem of partially filled containers. El-Raheb and Wagner[9] developed a finite element model based on a homogeneous vorticity assumption. These investigations, however, have not resulted in the development of accurate models to predict liquid dynamic characteristics in a spinning spacecraft.

Because of lack of availability of accurate models to predict liquid dynamic characteristics in a spinning spacecraft, simplified models, such as rigid pendulum models, have been used by the space industry. These models have been found to be significantly inaccurate, resulting in some cases in unstable attitude control designs.[1] This has provided impetus to the space industry to develop improved analytical models to predict liquid dynamic characteristics in a spinning spacecraft.

Recently, a finite element model has been developed, under INTELSAT sponsorship by MBB, to predict liquid natural frequencies, mode shapes, damping ratios, nutational frequency, and nutation time constant for spinning spacecraft with partially filled tanks. This model has been extensively used to study the effects of liquid motion on the attitude dynamics and control of INTELSAT VI, a dual-spin spacecraft with a liquid apogee motor. This paper presents the boundary-layer model, an analytical prediction of liquid dynamic characteristics and nutation time constants using this model, and a comparison of theoretical and experimental results.

II. Equations of Motion

An analytical model is developed for the spacecraft configuration shown in Fig. 1. The tank is offset from the spin axis and is partially filled with liquid. The tank can be of rotational symmetry but its contour may be of arbitrary shape. The symmetry axis may be inclined with respect to the spin axis. The coordinate system XYZ is fixed in the spinning spacecraft with origin at the center of mass of the spacecraft, including liquid, and the Z axis along the spin axis in steady state. The coordinate system $x''y''z''$ is fixed in the tank with origin at the center of the tank and the z'' axis along the tank symmetry axis.

General equations of motion for the liquid are represented by Navier-Stokes equations as follows:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + 2\Omega \times u + \dot{\Omega} \times R + \Omega \times (\Omega \times R) = -\frac{1}{\rho}\nabla p + f + \nu\nabla^2 u \tag{1}$$

where u is the relative velocity of the liquid with respect to the tank, R the position vector of liquid particle, Ω the angular velocity, ρ the liquid density, p the liquid pressure, f the body force per unit mass, and ν the kinematic viscosity.

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Assuming the liquid to be incompressible, we may write the continuity equation as

$$\nabla \cdot u = 0 \tag{2}$$

There are two sets of boundary conditions. The first condition which enforces no flow through the tank wall is

$$u \cdot n = 0 \tag{3}$$

where n is a unit vector normal to the tank wall. The second set of boundary conditions enforces constant pressure and surface equation on the free surface as

$$p = \text{const} \tag{4}$$

$$\frac{\mathrm{d}F}{\mathrm{d}t} = 0 \tag{5}$$

where F is the free surface equation. The steady-state equilibrium surface is a paraboloid of revolution. It should be noted that for a spinning spacecraft, centrifugal force dominates and the surface tension effects are small and are neglected in this formulation.

The equation of motion of a spinning spacecraft can be written as

$$I \cdot \dot{\Omega} + \Omega \times I \cdot \Omega = T \tag{6}$$

where I is the inertia dyadic of the spacecraft with empty tanks and T is the torque exerted by the liquid on the spacecraft and is given by

$$T = \int_{S_w} R \times np \,\mathrm{d}s + \rho\nu \int_{S_w} R \times \left\{(n \cdot \nabla)\left[n \times (n \times u)\right]\right\} \mathrm{d}s \tag{7}$$

The integrals are taken over the wetted surface S_w, where the first integral is the torque due to the liquid pressure and the second integral is the torque due to shear stress on the wall of the tank.

III. Analytical Model

The solution of general equations of motion of liquid, Eqs. (1) and (2), with associated boundary conditions, Eqs. (3–5), is very complex. Therefore, several simplified analytical models have been used to study the dynamic behavior of liquids in a spinning spacecraft. A simple model, commonly used by the space industry, is the rigid pendulum model, where the liquid propellant is treated as a distributed mass pivoting about the center of the tank with the total liquid mass located at the mass center of the liquid. This model has been found to be very inaccurate in the prediction of liquid natural frequencies, resulting in some cases in unstable attitude control design.[1] Abramson's model[10] is based on an ideal fluid executing an irrotational motion. The centrifugal force is represented by an equivalent constant gravitational force and the Coriolis effects are neglected.

In the homogeneous vortex model,[9] the simplifying assumptions are that the liquid vorticity is independent of the spatial coordinates and only time dependent, and the Coriolis acceleration is retained inside the liquid but is neglected in the free surface boundary condition to obtain an integral. In the boundary-layer model developed by Pohl,[11] three boundary-value problems are solved: an inviscid fluid problem, a viscous boundary-layer problem, and a viscous correction problem. For large Reynolds numbers, $Re > \mathcal{O}(10^5)$, the boundary-layer approximation is appropriate. For INTELSAT VI parameters, the Reynolds number is greater than 10^6. During the INTELSAT VI study,[12] all of the analytical models discussed here were compared and the conclusion reached was that the boundary-layer model gives the most accurate prediction of liquid dynamic characteristics. The boundary-layer model is briefly discussed in the following section.

IV. Boundary-Layer Model

The equation of motion of the liquid, Eq. (1), is first nondimensionalized. For nondimensionalization, a reference velocity $U = a \cdot \omega_n$ is defined, where a denotes a reference length of the tank and ω_n is the nutation frequency of the rigid spacecraft. The following dimensionless quantities are introduced.

$$\begin{aligned} &\bar{t} = t\omega_n, && \bar{\Omega} = \bar{\bar{\Omega}}/\omega_n \\ &\bar{r} = r/a, && \bar{R}_0 = R_0/a \\ &\bar{R} = R/a, && \bar{u}e^{\lambda t} = u/U \\ &\bar{p}e^{\lambda t} = p/(\rho U^2), && \bar{\omega}e^{\lambda t} = \omega/\omega_n \\ &\bar{g}_0 = g/(a \cdot \omega_n^2), && Re = \omega_n^2 a^2/\nu \end{aligned} \tag{8}$$

Here Re denotes Reynolds number and λ is the eigenvalue to be determined. The angular velocity is written as

$$\Omega = \bar{\bar{\Omega}}k + \omega \tag{9}$$

where $\bar{\bar{\Omega}}$ is steady-state angular velocity. Introducing the nondimensional parameters into Eq. (1) and neglecting body force f, it can be written as

$$\begin{aligned} &\lambda\bar{u} + 2\bar{\Omega}k \times \bar{u} + \bar{\omega} \times r\lambda + \bar{\Omega}\left[\bar{\omega} \times (k \times \bar{r}) + k \times (\bar{\omega} \times \bar{r})\right] \\ &\quad = -\nabla P + Re^{-1}\nabla^2\bar{u} \end{aligned} \tag{10}$$

where

$$\begin{aligned} P = \bar{p} &+ \left\{\lambda\bar{\omega} \times \bar{R}_0 + \bar{\Omega}\left[\bar{\omega} \times (k \times \bar{R}_0) + k \times (\bar{\omega} \times \bar{R}_0)\right]\right\}\bar{r} \\ &- \frac{\bar{\Omega}^2}{2}(k \times \bar{R})(k \times \bar{R}) \end{aligned} \tag{11}$$

The spacecraft equation (6) becomes

$$\lambda\bar{I} \cdot \bar{\omega} + \bar{\Omega}\left[k \times \bar{I} \cdot \bar{\omega} + \bar{\omega} \times \bar{I} \cdot k\right] = \bar{T} \tag{12}$$

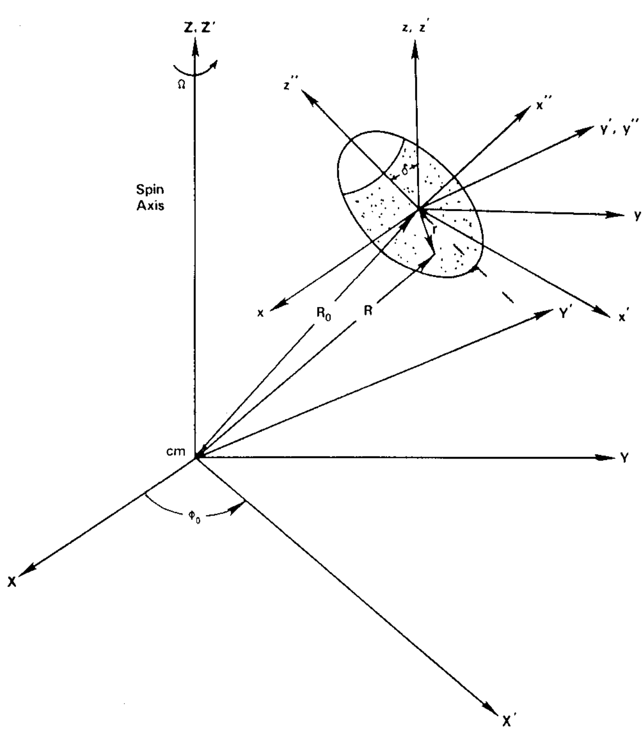

Fig. 1 Coordinate system.

where

$$\bar{I} = I/\rho a^5$$

$$\bar{T} = T/(\rho a^5 \omega_n^2)$$

$$= \int_{S_w} \bar{R} \times n\bar{p}\, \mathrm{d}s + \frac{1}{Re} \int_{S_w} \bar{R} \times \left\{ (n \cdot \nabla)\left[n \times (n \times \bar{u}) \right] \right\} \mathrm{d}s$$

For a multitank configuration, the contributions from the tanks are summed up.

The effects of viscosity are included in the liquid motion through a boundary-layer analysis. Using the procedure proposed by Hendricks and Morton,[6] the unknown parameters are expanded in the power of $Re^{-1/2}$ as follows:

$$\begin{aligned} \omega &= \omega_0 + Re^{-1/2}\omega_1 \\ u &= u_0 + \tilde{u}_0 + Re^{-1/2}(u_1 + \tilde{u}_1) \\ P &= P_0 + \tilde{P}_0 + Re^{-1/2}(P_1 + \tilde{P}_1) \\ \lambda &= \lambda_0 + Re^{-1/2}\lambda_1 \end{aligned} \tag{13}$$

where quantities with a tilde refer to boundary-layer variables. Substituting these perturbation expressions into Eq. (10) and equating powers of $Re^{-1/2}$ yields three boundary-value problems corresponding to an inviscid problem, a boundary-layer problem, and the correction to the inviscid solution. By solving these boundary-layer problems, the complete solution is obtained.

The unknown variables for the inviscid solution are the pressure function P_0, the velocity u_0, angular velocity ω_0, and the eigenvalue λ_0. Neglecting the viscous terms in Eq. (10), we may solve for the velocity components in terms of pressure. Substituting the velocity component, the continuity equation becomes

$$\frac{\partial^2 P_0}{\partial x^2} + \frac{\partial^2 P_0}{\partial y^2} + \left(\frac{\lambda_0^2 + 4\bar{\Omega}^2}{\lambda_0^2} \right) \frac{\partial^2 P_0}{\partial z^2} = 0 \tag{14}$$

The boundary conditions are also expressed in terms of the pressure. From Eq. (14) and the boundary conditions, the pressure and the eigenvalues of the liquid motion are determined by using a finite element method. The boundary-layer equations are written with respect to the wetted surface polar normal coordinate. The unknown variables $\tilde{P}_0$, $\tilde{u}_0$, $\tilde{P}_1$, and $\tilde{u}_1$ are determined analytically as described in Ref. 11.

The equations for the viscous correction are obtained by comparing all terms of order $Re^{-1/2}$. The equations for the unknown viscous correction parameters P_1, u_1, and λ_1 are

$$\frac{\partial^2 P_1}{\partial x^2} + \frac{\partial^2 P_1}{\partial y^2} + \frac{\lambda_0^2 + 4\bar{\Omega}^2}{\lambda_0^2} \frac{\partial^2 P_1}{\partial z^2} = 8 \frac{\lambda_1 \bar{\Omega}^2}{\lambda_0^3} \frac{\partial^2 P_0}{\partial z^2} \tag{15}$$

The finite element method is used to solve Eq. (15) with appropriate boundary conditions to determine viscous correction to the liquid natural frequencies and mode shapes.

V. Numerical Solution

The finite element computer program based on the boundary-layer model calculates liquid natural frequencies, mode shapes, damping ratios, torque exerted by the liquid on the spacecraft, energy dissipation rates, spacecraft nutation frequency, and nutation time constant. This model has been used extensively on the INTELSAT VI program[12,13] to calculate liquid natural frequencies, mode shapes, and spacecraft nutation time constants.

Liquid Natural Frequencies and Mode Shapes

The finite element model used in the analysis is shown in Fig. 2. It consists of 81 node points. The inviscid solution gives eigenvalues representing liquid natural frequencies and complex eigenvectors, representing modes shapes in terms of three velocity components at each node point. The complex velocity components are represented in terms of amplitudes and phase angles. The liquid motion can be considered as a combination of two types of natural oscillations: sloshing waves and inertial waves. The sloshing waves are characterized by free surface liquid oscillation. Mathematically, slosh motion can be analyzed on the basis of an ideal fluid executing irrotational motion. These modes are characterized by velocity components either in phase or out of phase. Inertial waves represent a dynamic interaction between Coriolis forces and pressure forces. Inertial waves are circulatory in the liquid interior and they may or may not have any apparent motion at the free surface. Inertial wave frequencies are less than twice the spin rate.[3] These waves are characterized by velocity components at a node having 90-deg phase difference, creating a circulatory motion. In general, a mode will have a combination of slosh and inertial wave motion. The viscous correction solution calculates damping for the modes.

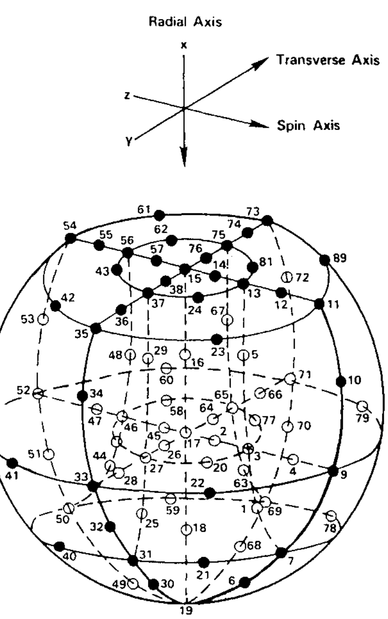

Fig. 2 Finite element model.

The calculated natural frequency ratios, ratios of natural frequencies to the spin rate, as a function of fraction fill for INTELSAT VI parameters are given in Table 1. It should be noted that only the modes near the frequency ratio = 2 are calculated. There will be additional modes present below and above the calculated modes. The parameters are tank radius = 0.42 m, radial distance of the tank center from the spin axis = 1.31 m, and spin rate = 30 rpm. The modes are divided into three groups: inertial, slosh, and higher modes containing both inertial and slosh components. In the slosh modes, two modes are present: azimuth and elevation. In the azimuth mode, the liquid motion is in a plane normal to the spin axis with a major velocity component in the transverse direction. The azimuth mode produces a reaction torque along the spin axis and therefore can cause interaction with the despin control. In the elevation mode, the major velocity component is in the axial direction. The elevation mode produces a reaction torque normal to the spin axis and can interact with the nutation control.

Figures 3 and 4 show typical mode shapes of first azimuth and first elevation modes, respectively. To simplify the plot, the velocity is plotted only for the node points along the x axis and on the free surface parallel to the z axis. In the azimuth mode, the liquid motion is in the xy plane with the major

Table 1 Liquid natural frequency ratios

Fraction fill, %	Inertial modes			Slosh modes First azimuth	Slosh modes First elevation	Higher modes		
95	1.996	2.0	2.009	2.876	3.04	4.114	4.42	4.81
90	1.992	1.997	2.002	2.57	2.784	3.778	3.998	4.255
80	1.992	1.996	2.0	2.314	2.556	3.366	3.694	3.792
70	1.991	1.996	2.0	2.118	2.434	3.287	3.552	3.580
60	1.992	1.996	2.0	2.109	2.351	3.167	3.396	3.53
50	1.992	1.996	2.0	2.056	2.287	3.07	3.278	3.507
40	1.989	1.996	2.0	2.023	2.233	3.008	3.17	3.512
30	1.989	1.995	2.0	2.007	2.187	2.95	3.09	3.536
20	1.979	1.993	1.997[a]	2.003[a]	2.142	2.90	3.01	3.57
10	1.993		1.997[a]	2.005[a]	2.09	2.849	2.92	3.6

[a]Difficult to distinguish between inertial and first azimuth mode.

Table 2 Analytical results for elevation mode

Fill fraction, %	Frequency ratio	Damping ratio
10	2.08	0.0088
20	2.12	0.0064
30	2.16	0.0025
50	2.26	0.0040
80	2.52	0.0023
90	2.74	0.0019

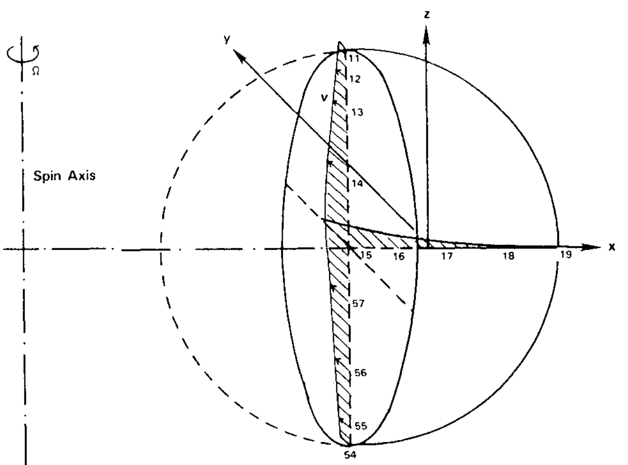

Fig. 3 First azimuth slosh mode.

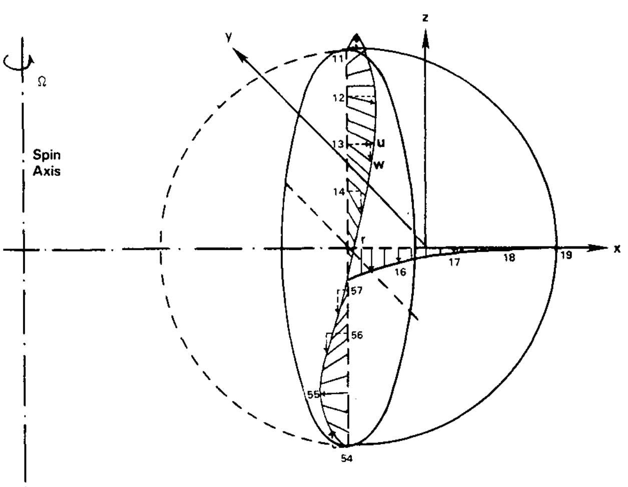

Fig. 4 First elevation slosh mode.

velocity along the y axis. In the elevation mode, the liquid motion is in the xz plane, with the major component in the axial z direction. The inertial and higher modes are difficult to plot due to their circulatory motion characteristic.

Scaled Model Tests

Under the INTELSAT VI program, Hughes Aircraft Company performed scaled model tests to determine liquid dynamic characteristics. The boundary-layer model was also used to determine liquid dynamic characteristics. The boundary-layer model was also used for the scaled test parameters to analytically predict liquid dynamic characteristics. Analytical results are given in Table 2. The parameters used were tank diameter = 0.146 m, radial distance of the tank center from spin axis = 0.218 m, liquid density = 1 g/ml, and viscosity = 0.009 cm^2/s. The damping ratio is defined as a ratio of damping to critical damping. Some of the experimental data are presented in Ref. 1, and regarding the INTELSAT boundary-layer model it is stated in the paper that "Unpublished finite element eigenmode modeling for the off-axis case at INTELSAT is beginning to compare well with measured data, promising that spinning liquid modal tests may soon follow rather than lead analysis—as is the practice in the more established area of structural mechanics."

Nutation Time Constant

During several mission phases of INTELSAT VI, active nutation control is used. To design the active nutation control properly, the nutation dedamping due to liquid slosh needs to be determined accurately. Air-bearing tests have been performed on the INTELSAT VI program to determine its nutation time constant. The critical parameters, inertia ratio and fill fraction, are kept the same and other parameters are scaled due to limitations of test vehicles and chamber. In-orbit nutation time constants are extrapolated from the measured nutation time constant by using dimensional analysis.

Anomalous Resonances

On-orbit liquid slosh tests on INTELSAT IV revealed sharp reduction of nutation time constants for certain tank fill fractions and inertia ratios. Air-bearing tests were later performed and these validated the in-orbit results. These conditions due to resonances in liquid could not be explained by linear models because the slosh frequencies were significantly higher than the nutation driving frequency. These resonances were termed as anomalous resonances.

INTELSAT VI air-bearing liquid slosh test results have also exhibited sharp reductions in nutation time constants for certain fill fractions and inertial ratios, defined as ratio of moment of inertia about spin axis to that about transverse axis. To validate the boundary-layer finite element model, it was used to analytically predict nutation time constants for these parameters. Nutation time constants are determined by solving Eq. (6) for eigenvalues,[12] nutation frequencies and nu-

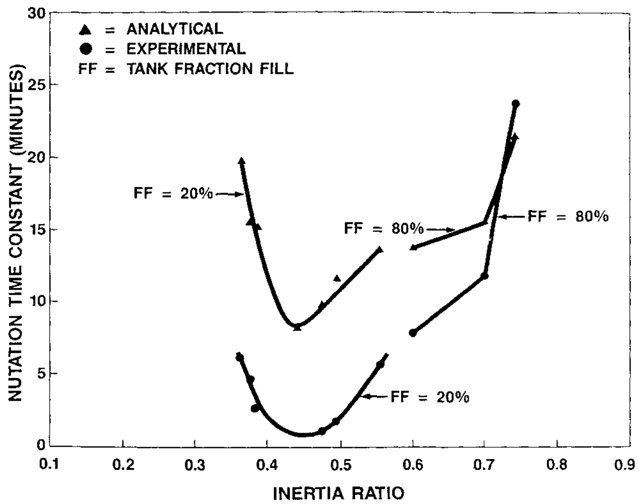

Fig. 5 Nutation time constant.

tation time constants, of the spacecraft liquid system. Figure 5 shows the plots of nutation time constants determined analytically and experimentally for tank fill fractions of 20 and 80%. Both experimental and analytical results predict a sharp drop in nutation time constants for the same inertia ratio (0.45) and fill fraction (20%). The analytical values of nutation time constant are, however, higher than experimental results. This resonance condition could be due to excitation of inertial modes close to the nutation driving frequency.

VI. Summary

The boundary-layer model to predict dynamic characteristics of liquid motion in a spinning spacecraft with partially filled tanks is presented. The solution is obtained by solving three boundary-value problems: an inviscid fluid problem, a boundary-layer problem, and a viscous correction problem. The boundary-layer equations are solved analytically and the inviscid and viscous correction solutions are obtained by using finite element methods. The finite element computer program has been developed by MBB, Germany, under an INTELSAT contract. This program calculated the liquid natural frequencies and mode shapes, torque exerted by the liquid on the spacecraft, energy dissipation rates, spacecraft nutation frequencies, and time constants. The computer program has been used extensively on the INTELSAT VI program.

The results indicate that the liquid motion in rotating bodies can be considered as a combination of two types of natural oscillations: sloshing modes and inertial modes. The numerical results show two slosh modes, first azimuth and elevation modes, and lower modes to be inertial modes and higher modes to be a combination of inertial and slosh modes. Therefore, only these two slosh modes can be accurately represented by pendulum models.

The analytical results predict a sharp drop in nutation time constant for certain inertia ratios and tank fill fractions. This phenomenon, known as anomalous resonance, was also present during INTELSAT IV in-orbit liquid slosh tests and ground air-bearing tests for INTELSAT IV and VI. The nutation time constant depends on two liquid characteristics, energy dissipation, and resonance of normal modes due to nutation driving frequency. Since the slosh frequencies are significantly higher than the nutation driving frequencies, it appears that the resonance condition is due to excitation of inertial modes. Further work is required to identify the inertial modes which contribute to this resonance.

VII. Conclusions

The boundary-layer model accurately predicts liquid natural frequencies and mode shapes. The analytical results show that in general a liquid mode will be a combination of slosh and inertial modes. Only very few modes can be considered as pure slosh modes, such as first azimuth and first elevation modes in the example presented in this paper. Therefore, only these slosh modes can be represented accurately by equivalent pendulum models. The boundary-layer model also predicted a sharp reduction of nutation time constant for certain fill fractions and inertia ratios as revealed in ground and on-orbit liquid slosh data. It appears to be a liquid resonance due to excitation of certain inertia mode(s). Further work is necessary to explain this phenomenon.

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