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ANGULAR RATE ESTIMATION FOR GYROLESS SATELLITE ATTITUDE CONTROL

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This paper presents the results of a study to develop and evaluate an attitude and attitude rate estimation technique for a multi-body spacecraft that includes a real time angular rate calculation from the system dynamic model and a Kalman filter estimator with attitude sensor updates provided by star trackers. The performances of this method, dynamic gyro, are compared with that of rate gyros and the effects of primary error sources in the dynamic model are analyzed. It is shown that the corrections provided by a star tracker based Kalman filter make the system robust to measurement and parameter knowledge error sources. This method provides an imperfect but operative means of estimating multi-body spacecraft angular rates. This method is ideally suited to a spacecraft designed specifically for its implementation with precise internal sensors and mechanism to monitor spacecraft parameters and integrated external torque estimation modeling.

1. INTRODUCTION

In a spacecraft attitude control system with high pointing requirements, the attitude determination systems have relied primarily on rate gyros, star trackers, and Kalman Filter for attitude and attitude rate estimation. The rate gyros have bias errors and star trackers have measurement gaps. The common practice is that during star tracker measurement gaps, rate gyros are used to determine attitude and attitude rates. When the star tracker measurements is taken, using Kalman Filter, the attitude estimation is corrected and rate gyros bias is updated. However, rate gyros have a tendency to degrade or fail in orbit. As an example, gyros have degraded/failed on several spacecraft, such as NASA Skylab, the International Ultraviolet Explorer, and Hubble Space Telescope. In such cases, there is a need to estimate angular rate in the absence of rate gyros. For some spacecraft designs, rate gyros are too expensive. Therefore, it will be highly desirable to have gyro less attitude determination system.

Historically, sensitivity and bandwidth limitations of available star sensors have precluded their use as primary sensors for attitude rate determination. Using the recent advancements in star sensor technology, Ref. 1 proposes the implementation of star sensor to estimate attitude and attitude rate. The paper defines the requirements such implementation will impose on the star sensors and error state Kalman filter used to estimate spacecraft quaternion and its angular rate. Reference 2 presents a real-time predictive filter for spacecraft attitude estimation without the utilization of rate gyros. The formulation uses only attitude sensors, such as three-axis magnetometers, sun sensors, and star trackers. This technique has been used on the Solar Anomalous Magnetospheric Particle Explorer (SAMPEX) spacecraft. Reference 3 presents a sequential nonlinear estimator for satellite attitude and attitude rate estimation by utilizing vector observations. This method is claimed to have several advantages. First, the acquired vector measurements are directly processed to extract attitude and attitude rate information, thus avoiding the computation of temporal derivatives of these noisy measurements. Second, no use is made of spacecraft dynamic model, which is frequently considered to be highly uncertain. Finally, the algorithm directly determines attitude matrix. Reference 4 presents a nonlinear estimator for reconstructing the angular rate of a spacecraft without rate gyros. The angular rate estimator structure is similar to a Kalman filter. The estimated angular rate is propagated based on spacecraft model. It is assumed that the disturbance torques are smaller and at lower frequency in comparison to control torques and not included in the estimation of the angular rate. The angular rates are updated periodically using the measurements from onboard attitude sensors.

Another method, proposed by Aerospace, is to estimate spacecraft angular rate through direct calculation from a dynamic model of the system. Information from internal sensors that detect relative orientations and rates of momentum exchanges devices and appendages are used to determine component angular momentum and moments of inertia. Control torques and estimated disturbance are integrated to capture external dynamic effects. The dynamic model continuously tracks the total

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momentum and inertia dyadic from which the angular rate can be calculated. The accuracy of the rate estimate is dependent on the quality of the dynamic model and sensor information available. Error sources include imperfect knowledge of the system and component inertia dyadic and relative angular position and rate data from the internal sensors. Errors are also introduced in the modelling of external disturbances.

This paper presents the results of a study to develop and evaluate a attitude and attitude rate estimation technique for a multi-body spacecraft that incorporates a real time angular rate calculation from the system dynamic model, dynamic gyro, and a Kalman filter estimator with attitude sensor updates provided by star trackers. The Bifocal Relay Mirror spacecraft, Ref. 5, is used as an example for multi-body spacecraft to evaluate the dynamic gyro. The performance of the developed gyro less attitude determination system is compared to a conventional gyro-based system that uses the same Kalman filter and attitude updates. The impact of several dynamic model error sources on the performance of dynamic gyro is analyzed.

2. DYNAMICS OF BIFOCAL RELAY MIRROR SPACECRAFT

Bifocal Relay Mirror spacecraft is composed of two optically coupled telescopes used to redirect the laser light from ground-based, aircraft-based or spacecraft based lasers to distant points on the earth or in space. The receiver telescope captures the incoming energy from a laser transmitter system while a separate transmitter telescope directs the laser beam at the desired target.

The dynamic model of bifocal relay mirror spacecraft is shown in Fig. 1. It consists of two bodies, transmit telescope and receive telescope. The receive telescope rotates with respect to the transmit telescope about a single axis. The center of mass (c.m.) of the receive telescope is on the rotation axis. Therefore, the c.m.of the system is fixed during the relative motion of the receive telescope.

The coordinate system x, y, z is fixed in the transmit telescope with x-axis parallel to rotation axis of the receive telescope, z as telescope axis and y is normal to x and z such that the x, y, z coordinate system is right handed mutually orthogonal frame. The origin of the coordinate system is at the c.m. of the transmit telescope. Unit vectors along x, y, and z are $\vec{i}, \vec{j}, and\ \vec{k}$ respectively. The coordinate system x′, y′, z′, is fixed in the receive telescope with x′ axis as the rotation axis, parallel to x axis, z′ as telescope and y′ is normal of x′ and z′ such that x′, y′, z′ coordinate system is right handed mutually orthogonal frame with origin at the c.m. of the receive telescope. The coordinate system x′, y′, z′ are obtained from the coordinate system x, y, z by rotation α about x-axis. The equations of motion of the system are written in the coordinate frame x, y, z with the origin at the c.m. of the system. The spacecraft is assumed to be rigid.

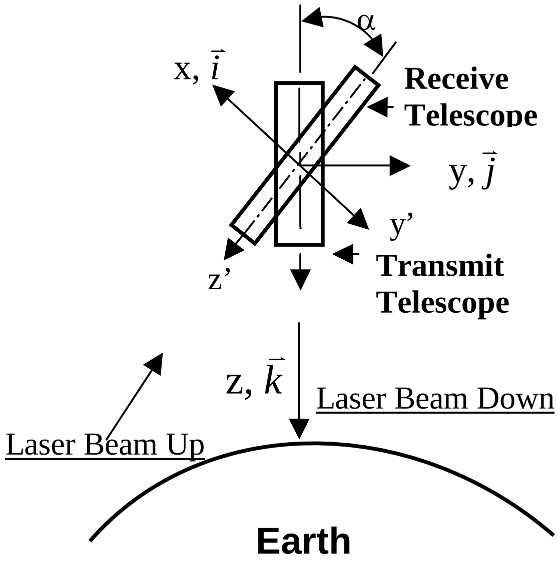

Figure 1. Bifocal Relay Mirror Spacecraft Dynamic Model

2.1. Equations of Motion

In a general case, the rotational equation of a body about an arbitrary point P is given by

$$M_p = \dot{H}_p - \dot{\rho}_c \times m \dot{r}_c \tag{1}$$

Where
M_p = total sum of external forces about P
H_p = angular momentum of the body about P
ρ_c = Vector from P to C.M. of the body
m = mass of the body
$\dot{r}_c$ = Velocity of center of mass of the body

If the point P is at c.m. of the body, then $\rho_c = 0$, and

$$M = \dot{H}\,|_I \tag{2}$$

For the bifocal relay mirror spacecraft, the point P is at the c.m. of the system, therefore Eq. (2) is applicable. The angular momentum of the spacecraft, H_S, can be written as follows:

$$H_S = H + H_{rel} + H_W \tag{3}$$

where
H_S = angular momentum of the spacecraft.
H = angular momentum of the spacecraft by neglecting the contribution by the relative motion of the receive telescope and reaction wheels with respect to transmit telescope

H_{rel} = angular momentum due to relative motion of the receive telescope
H_w = angular momentum due to relative motion of the reaction wheels.

The next step in the derivation is to determine these angular momentums.

Inertia

Let I_T be the inertia matrix of the transmit telescope about its c.m. in coordinate frame x, y, and z; I_R be the inertia matrix of the receive telescope in the coordinate frame x, y, and z; and I_S be the spacecraft inertia matrix about it c.m. in the coordinate frame x, y, and z.

Angular Velocities

The angular velocity of the transmit telescope

$$\omega_T = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{4}$$

The relative angular velocity of the receive telescope with respect to transmit telescope ω_{rel} is given by

$$\omega_{rel} = \begin{bmatrix} \dot{\alpha} \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

The angular velocity of the receive telescope, ω_R, is

$$\omega_R = \omega_T + \omega_{rel} = \begin{bmatrix} \omega_x + \dot{\alpha} \\ \omega_y \\ \omega_z \end{bmatrix} \tag{6}$$

Angular Momentums

The angular momentum H is given by

$$H = I_s \omega_T \tag{7}$$

The relative angular momentum H_{rel} is given by

$$H_{rel} = I_R \omega_{rel} \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (3), we get

$$H_S = I_S \omega_T + I_R \omega_{rel} + H_w \tag{9}$$

Using Eq. (2), the equation of motion of the spacecraft is given by

$$\begin{aligned} M &= \dot{H}_s\big|_I \\ &= \dot{H}_s\big|_T + \omega_T \times H_s \end{aligned} \tag{10}$$

where
$\dot{H}_s\big|_I$ = rate of change in the inertial frame
$\dot{H}_s\big|_T$ = rate of change in the transmit telescope frame

Substituting Eq. (9) into Eq. (10), we get

$$\begin{aligned} M &= \dot{I}_S \omega_T + I_S \dot{\omega}_T + I_R \dot{\omega}_{rel} + \dot{H}_W \\ &+ \omega_T \times \left[I_S \omega_T + I_R \omega_{rel} + H_W \right] \end{aligned} \tag{11}$$

It should be noted that I_S is a function of α, and therefore time dependent. Equation (11) can be rewritten as

$$\begin{aligned} &\frac{d}{dt}\left[I_S \omega_T + I_R \omega_{rel} + H_W \right] = \\ &M - \omega_T \times \left[I_S \omega_T + I_R \omega_{rel} + H_W \right] \end{aligned} \tag{12}$$

The formulation of Eq. (12) is used for MATLAB/SIMULINK simulation. The spacecraft is subjected to large angle maneuvers. Therefore quaternion formulation is better suited and is used.

3. SPACECRAFT CONTROL

For the spacecraft attitude control, reaction wheels are used as actuators with magnetic torque rods to desaturate the reaction wheels. The primary sensors are rate gyros to determine spacecraft angular rates and star trackers to determine spacecraft attitude. The rate gyros have bias errors and star trackers have measurement gaps. During the measurement gap for the star trackers, rate gyros are used to determine angular rates and angular position. When the star trackers measurements are taken, using Kalman Filter, the angular position is corrected and rate gyro biases are updated. During the simulations, disturbance torques are assumed from gravity gradient and earth's magnetic field. In this paper we will compare the performance of rate gyros with dynamic gyro.

3.1. Gravity Gradient Torque

The gravity gradient torque M_G is given by

$$M_G = \frac{3\mu}{R_0^3} \begin{bmatrix} C_{13} \\ C_{23} \\ C_{33} \end{bmatrix} \times \begin{bmatrix} I_{xx} & -I_{xy} & -I_{sz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} C_{13} \\ C_{23} \\ C_{33} \end{bmatrix} \tag{13}$$

where
μ = gravitational constant

R_0 = orbit radius
C_{ij} = elements of transformation matrix from orbit frame to body frame
I_{ij} = elements of body inertia matrix

3.2. Magnetic Moment

Magnetic moment is given by

$$M_m = m \times B \qquad (14)$$

Where
M_m = magnetic moment (N.m)
m = spacecraft magnetic dipole (a.m^2)
B = earth's magnetic field vector (N/a.m)

The earth's magnetic field is given by

$$B = \frac{K_{mag}}{R^3}\left[3(M.R)R - M\right] \qquad (15)$$

where
$K_{mag} = 7.943 \times 10^{15}$ N.m^2/a^2
R = earth c.m. to spacecraft c.m. (m) vector
M= unit dipole vector

The earth's magnetic field component in orbit coordinates is given by

$$B_{xo} = \frac{K_{mag}}{R^3}\begin{bmatrix} \cos\ \alpha(\cos\ \varepsilon\ \sin\ i - \sin\ \varepsilon\ \cos\ i\ \cos\ u) + \\ -\sin\ \alpha\ \sin\ \varepsilon\ \sin\ u \end{bmatrix}$$

$$B_{YO} = \frac{K_{mag}}{R^3}\left[-cos\ \varepsilon\ cos\ i - sin\ \varepsilon\ sin\ i\ cos\ u\right]$$

$$B_{ZO} = \frac{K_{mag}}{R^3}\begin{bmatrix} 2\ sin\ \alpha(Cos\ \varepsilon\ Sin\ i - sin\ \varepsilon\ cos\ i\ cos\ u) + \\ +2\ cos\ \alpha\ sin\ \varepsilon\ sin\ u \end{bmatrix} \qquad (16)$$

where
$\alpha =$ true anomaly of the spacecraft
ε = magnetic dipole tilt from north pole
i = orbit inclination
u = right ascension angle of magnetic dipole with respect to right ascension of the orbit normal.

The earth's magnetic field vector in body frame is given by

$$B_B = {}^{B}C^{O}\ B_O \qquad (17)$$

where ${}^{B}C^{O}$ is transformation matrix from orbit frame to body frame.

Magnetic Disturbance Moment

The magnetic disturbance moment is given by

$$M_{md} = m_d \times B_B \qquad (18)$$

where
m_d = spacecraft magnetic dipole

Magnetic Control Moment

Magnetic control is used to desaturate the reaction wheels. The command for magnetic dipole of the torque rods or coils, m_c is given by

$$m_c = -K_{cmag}\,(B_B \times h) \qquad (19)$$

where
h = angular momentum of the wheel
K_{cmag} = gain

3.3. Wheel Control Laws

The wheel control laws are as follows

$$\begin{aligned} \dot{H}_{W1} &= 2\ \ k_1 q_{1E} q_{4E} + k_{1d}\omega_{1E} \\ \dot{H}_{W2} &= 2\ \ k_2 q_{2E} q_{4E} + k_{2d}\omega_{2E} \\ \dot{H}_{W3} &= 2\ \ k_3 q_{3E} q_{4E} + k_{3d}\omega_{3E} \end{aligned} \qquad (20)$$

where q_E is quaternion error, ω_E is the error between commanded angular rate and measured angular rate, and k_i and k_{id} are the gains for the control.

3.4. Feed Forward control

The feed forward control torque, M_{fd}, is given by

$$\begin{aligned} M_{fd} &= \dot{I}_S\omega_T + I_S\dot{\omega}_T + I_R\dot{\omega}_{rel} \\ &+\omega_T \times \left[I_S\omega_T + I_R\omega_{rel} + H_W\right] \end{aligned} \qquad (21)$$

It should be noted that I_S is time dependent.

3.5. Kalman Filter

As discussed earlier, spacecraft uses rate gyros in the transmit telescope to determine angular rates and star trackers to determine angular position. The rate gyros have bias errors and star trackers have measurement gaps. The common practice is that during measurement gaps for the star trackers, rate gyros are used to determine angular rates and angular position. When the star tracker measurement is taken, using Kalman Filter, the angular position is corrected and rate gyro bias is updated.

The basic of discrete filter is as follows. Let the system be defined by

$$\begin{aligned} x(k+1) &= \phi(k+1,k)x(k) \\ &\quad + \Delta(k+1,k)u(k) + W(k) \\ z(k) &= H(k)x(k) + V(k) \end{aligned} \qquad (22)$$

Let us define co-variances as

$$\begin{aligned} E\left[V(k)V^T(j)\right] &= R(k)\delta_{kj} \\ E\left[W(k)W^T(j)\right] &= Q(k)\delta_{kj} \end{aligned} \qquad (23)$$

The covariance of estimation error matrix is

$$P(k,k) \triangleq E\left[e(k,k)e^{T}(k,k)\right] \qquad (24)$$

The estimator correction is

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)\left[z(k) - H(k)\hat{x}(k/k-1)\right]$$
$$\hat{x}(k/k-1) = \phi(k,k-1)x(k-1/k-1) + \Delta(k,k-1)U(k-1) \qquad (25)$$

Here $\hat{x}(k/k)$ is optimal estimate of x (k) given observations at times up to and including k and $\hat{x}(k/k-1)$ is optimal prediction of x (k) given observations at times up to and including k-1.

In Kalman Filter, gain K (k) is determined to minimize the sum of the main diagonal of the matrix P (k, k). The gain and covariance equations are given by

$$K(k) = P(k/k-1)H^{T}(k)\left[H(k)P(k/k-1)H^{T}(k) + R(k)\right]^{-1}$$
$$P(k/k) = \left[I - K(k)H(k)\right]P(k/k-1)$$
$$P(k+1/k) = \phi(k+1,k)P(k,k)\phi^{T}(k+1,k) + Q(k) \qquad (26)$$

In our system, there are six state variable, three attitude errors in angles with respect to inertial frame θ_E and three gyro bias errors b_E. The state vector is

$$x = \begin{bmatrix} \theta_{1E} \\ \theta_{2E} \\ \theta_{3E} \\ b_{1E} \\ b_{2E} \\ b_{3E} \end{bmatrix} = \begin{bmatrix} \theta_E \\ b_E \end{bmatrix} \qquad (27)$$

It is assumed that there are constant bias errors. The state equation is

$$\begin{bmatrix} d\theta_E / dt \\ db_E / dt \end{bmatrix} = \begin{bmatrix} O_{3x3} & {}^{I}C^{T} \\ O_{3x3} & O_{3x3} \end{bmatrix} \begin{bmatrix} \theta_E \\ b_E \end{bmatrix} = f(t)x \qquad (28)$$

where ${}^{I}C^{T}$ is transformation matrix from transmit telescope frame to inertial frame, or transpose of ${}^{T}C^{I}$.

The state transition matrix ϕ is

$$\phi_k = e^{f(t)\Delta t} = \begin{bmatrix} I_{3x3} & {}^{I}C_k^{T}\Delta t \\ O_{3x3} & I_{3x3} \end{bmatrix} \qquad (29)$$

The star trackers produce horizontal and vertical outputs (H,V) corresponding to the position of the star on the detector array. The tracker measurements in a vector form is

$$S_m = \begin{bmatrix} h \\ V \\ 1 \end{bmatrix} \qquad (30)$$

with normalization, the vector is

$$s_m = \frac{S_m}{\left|S_m\right|} \qquad (31)$$

The star in inertial space has gone through a star identification process and has been compensated for annual and vehicular aberration to yield a unit vector SI in inertial frame. Transforming it into star tracker frame, the predicted vector is

$$s_p = {}^{S}C^{T}\ {}^{T}\hat{C}^{I}\ S_I \qquad (32)$$

where ${}^{S}C^{T}$ is transformation matrix from transmit telescope to star tracker. It should be noted that ${}^{T}\hat{C}^{I}$ is an estimate. The measurement residual error is

$$z = E(s_m - s_p) \qquad (33)$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Next we want to express the measurement residual error in terms of attitude errors θ_E. Let ${}^{T}C^{I}$ be the correct transformation, then

$${}^{T}C^{I} = {}^{T}\hat{C}^{I}\left[1 - S(\theta_E)\right] \qquad (34)$$

where S is skew symmetric operator and $S(\theta_E)$ is given by

$$S(\theta_E) = \begin{bmatrix} 0 & -\theta_{3E} & \theta_{2E} \\ \theta_{3E} & 0 & -\theta_{1E} \\ -\theta_{2E} & \theta_{1E} & 0 \end{bmatrix} \qquad (35)$$

Then sm can be written as

$$\begin{aligned} s_m &= {}^{S}C^{T}\ {}^{T}C^{I}\ S_I \\ &= {}^{S}C^{T}\ {}^{T}\hat{C}^{I}\left[1 - S(\theta_E)\right]S_I \end{aligned} \qquad (36)$$

Substituting Eqs. (32) and (36) into Eq. (33), we get

$$\begin{aligned} z &= -E\,{}^{S}C^{T}\;{}^{T}\hat{C}^{I}\;S(\theta_E)\,S_I \\ &= E\;{}^{S}C^{T}\;{}^{T}\hat{C}^{I}\;S(S_I)\,\theta_E \\ &= \left[E\;{}^{S}C^{T}\;{}^{T}\hat{C}^{I}\;S(S_I)\;0_{2x3}\right]x \end{aligned} \tag{37}$$

Therefore

$$H = \left[E\;{}^{S}C^{T}\;{}^{T}\hat{C}^{I}\;S(S_I)\;0_{2x3}\right] \tag{38}$$

The Kalman Filter update is

$$x = \begin{bmatrix} \theta_E \\ b_E \end{bmatrix} = Kz \tag{39}$$

The initial state estimate is zero. The attitude angle correction is changed to quaternion correction as follows

$$q_E = Sin\left(\frac{|\theta_E|}{2}\right)\left[\frac{\theta_{1E}\vec{i} + \theta_{2E}\vec{j} + \theta_{3E}\vec{k}}{|\theta_E|}\right] + Cos\left(\frac{|\theta_E|}{2}\right) \tag{40}$$

Where

$$|\theta_E| = \sqrt{\theta_{1E}^2 + \theta_{2E}^2 + \theta_{3E}^2}$$

The corrected quaternion is

$$q_{new} = q_{old}\;q_E \tag{41}$$

The above equation implies quaternion multiplication.

4. DYNAMIC ANGULAR RATE CALCULATION

The continuous dynamic equations of motion for the Bifocal Relay Mirror spacecraft are derived in section 2. These equations produce the spacecraft angular rate from external control and disturbance moments applied to the body. A similar discrete model can be applied in the spacecraft attitude processor software to produce a real time calculated estimate of the angular rate, referred to as the dynamic gyro. The angular rate generated by this method can be used as a substitution for conventional gyroscope outputs. Attitude determination based on the dynamic gyro can be implemented as a back up failure mode or a primary operating mode to increase the expected lifetime of the satellite gyroscopes.

4.1. Discrete Equations of Motion

The discretized equations of motion are derived from

$$\Delta \mathrm{H_s} = \sum \mathrm{M}\,\Delta \mathrm{t} \tag{42}$$

where $\sum \mathrm{M}$ is the sum of external moments applied to the spacecraft including controls and modeled disturbances. This allows the total system angular momentum to be tracked with

$$\mathrm{H_s}(k+1) = \mathrm{H_s}(k) + \Delta \mathrm{H_s} \tag{43}$$

Subtracting the relative momentum of the reaction wheels and secondary body produces

$$\mathrm{H} = \mathrm{H_s} - \mathrm{H_w} - \mathrm{H_{rel}} \tag{44}$$

The calculated spacecraft angular rate is then given by

$$\omega_{\mathrm{T}} = \mathrm{I_s^{-1}H} \tag{45}$$

4.2. Momentum Correction From Kalman Filter Updates

The gyro bias error states, $\tilde{\mathrm{b}}$, are interpreted as spacecraft body rate errors. Using the calculated spacecraft inertia matrix, $\mathrm{I_s}$, a correction to the system angular momentum can be generated by

$$\Delta \mathrm{H_{corr}} = \mathrm{I_s}\,\tilde{\mathrm{b}} \tag{46}$$

The Kalman filter momentum correction is applied as if the error in the dynamic gyro is attributable to the total spacecraft body. The relative momentum terms from the secondary body and the reaction wheels are treated as if they are without error.

4.3. Error Sources

After initial calibration, attitude determination error in gyro-based is almost entirely attributable to a single set of imperfect gyroscope rate sensors. As long as gyro data does not become erratic, a Kalman estimator based on a slowly changing rate bias plant model produces an effective attitude determination system even with relatively noisy rate inputs.

The error in rate calculations from dynamic modeling, on the other hand, is due to numerous factors and is much harder to characterize. Since the dynamic calculation is produced from total system momentum tracking, any error in knowledge of external torques directly correlates to rate error. Errors in system or component moments of inertia have the same effect. The data from all moving appendages and momentum exchange devices are critical to the accuracy of the rate calculation. It is important that all known biases be removed from sensor data and calculated input errors since the Kalman filter estimator is based on the assumption of uncorrelated zero-mean Gaussian noise. Even if all input parameters were known exactly the discrete modeling of the spacecraft dynamics introduces some error.

External Control and Disturbance Torques

In the dynamic gyro, known externally applied moments are integrated in the system angular momentum calculations. These include control moments other than those imparted by momentum exchange devices as well as modeled disturbance torques. For the Bifocal Relay Mirror satellite, the gravity gradient torque is the most significant disturbance and can be modeled as an input to increase the accuracy of the dynamic rate calculation.

Reaction Wheel Relative Momentum

The relative momentum of each reaction wheel is given by its orientation within the spacecraft, the inertia of its rotor and the wheel spin rate. The imperfect sensor measurements from the reaction wheel tachometers introduce errors in system momentum calculation. Relative orientation angles of reaction wheels are fixed and errors can be corrected through calibration. Orientations of control moment gyros (CMGs), however, are variable. Since these devices usually have high momentum, small gimbal resolver errors can have a significant impact on total system momentum calculations. In this simulation, times varying artificial alignment errors are applied to the reaction wheel momentum measurements to observe the effects of CMGs alignment errors.

Moment of Inertia Calculations

The inertia matrix also depends on internal sensor input from position encoders or potentiometers for relative angular orientation of appendages. The model of the potentiometer that measures the relative angle of the receive telescope includes quantization effects and additive noise. If appendage relative motion is slow or component moments of inertia are small, it may not be necessary to update the system inertia dyadic at the bandwidth of the attitude processor. The affects of inertia update rate are evaluated.

5. SIMULATION RESULTS

The spacecraft dynamics, attitude determination, and control laws discussed in the previous sections were implemented on MATLAB/SIMULINK model. The following parameters are used for the simulation.

The simulation time period is 500 seconds. The simulation solver method is ode5 (Dormand-Prince), and the solver fixed step size is 0.05 seconds. The secular torque magnitude is 1e-4 Nm. The transmit telescope inertia: I_{xx} = 2,997 kgm^2, I_{yy} = 3,164 kgm^2, and I_{zz} = 882 kgm^2; receive telescope inertia: $I_{x'x'}$ = 1,721 kgm^2, $I_{y'y'}$ = 1,560 kgm^2, and $I_{z'z'}$=183 kgm^2. The reaction wheel gains are k = [3000, 7000, 4500] and k_d = [1000, 2000, 1000]. The control law delay for initial determination errors is 30 seconds. The rate gyros static rate biases are 1e-4*[-1,1.5,1] rad/sec. The initial errors are: for quaternion [0.008, 0.012, -0.008] and for angular rate errors [-0.001,0.001,0.002] rad/sec. For the reaction wheels, the maximum torque is 1 Nm.

The command attitude profile is shown in Figure 2. The left figure shows the transmit telescope rotation in quaternions and the right figure shows the relative angle rotation of the receive telescope. The profile resembles the maneuver required to maintain transmit and receive telescopes pointing during an overhead pass to conduct laser relay operations. The majority of the maneuver is performed in the spacecraft pitch axis, q_2, as both telescopes orient to points at fixed ground sites. The orbit altitude is 715 km with an inclination of 40 degrees. Based on the ground site separation distance and orbital altitude, the largest relative angle is about 30 degrees during a near overhead pass between the uplink and downlink ground sites.

The magnitude of the total spacecraft angular momentum during the maneuvering profile is plotted in Figure 3. The baseline simulation for Dynamic Gyro does not include gravity gradient torques. In order to observe the performance of the attitude determination without updates, a 200 second star gap is simulated starting 100 seconds into the run. As a worst–case analysis, the star gap occurs during the peak maneuvering time of the satellite including the rotation of the secondary body. For simulation, rate gyros are replaced by dynamic gyro. The accuracy of the angular rate calculation is entirely dependent upon the ability of the dynamic gyro to track the total spacecraft angular momentum. The error in the magnitude of the total system momentum compared to the simulated actual momentum is shown in Fig.4. Figure 5 shows the comparison of the dynamic gyro and gyro based attitude determination system. At the end of the gap, the error in the dynamic gyro based system is about five times that of the gyro-based system. Figure 6 shows the increase in performance of the dynamic gyro based attitude determination system when the gravity gradient moment is modeled. At the end of the star gap, the attitude errors are comparable to the gyro-based system. The total system angular momentum error is much smaller during the star gap. Figure 7shows the effect of periodic alignment error of the net angular momentum of the reaction wheels with a magnitude of approximately 0.5 degrees. A significant increase in attitude error estimation is developed during the star gap. The dynamic gyro does not track the system angular momentum as well even during continuous star coverage. Figure 8 shows the effects of decreasing the inertia calculation frequency from

20 Hz rate of the dynamic gyro to 10 Hz. The quaternion error profile is significantly altered but the magnitude of the error is only slightly increased. The momentum error in the dynamic gyro takes longer to correct after the star gap since the Kalman filter attempt to correct for all errors as if they were due to spacecraft momentum

6. CONCLUSIONS

Dynamic gyro, estimating spacecraft angular rate from dynamic model of the system, provides an imperfect but operative means of estimating multi-body spacecraft angular rates when the data from rate gyro sensors are not available. The dynamic gyro performance is compared with the performance of rate gyros and the effects of primary error sources in the dynamic model are investigated. It is shown that the corrections provided by a star tracker based Kalman filter make the system robust to measurement and parameter knowledge error sources. Significant improvement in attitude determination performance is realized when the disturbance torques are modeled. The other primary error sources include alignment error of momentum exchange control devices, inertia update frequency, and relative angle and rate knowledge of slewing appendages. Error effects are amplified during the star gaps when no corrections to the dynamic model are available.

This attitude determination concept is ideally suited to a spacecraft designed specifically for its implementation with precise internal sensors and mechanism to monitor spacecraft parameters and external torque modeling.

7. REFERENCES

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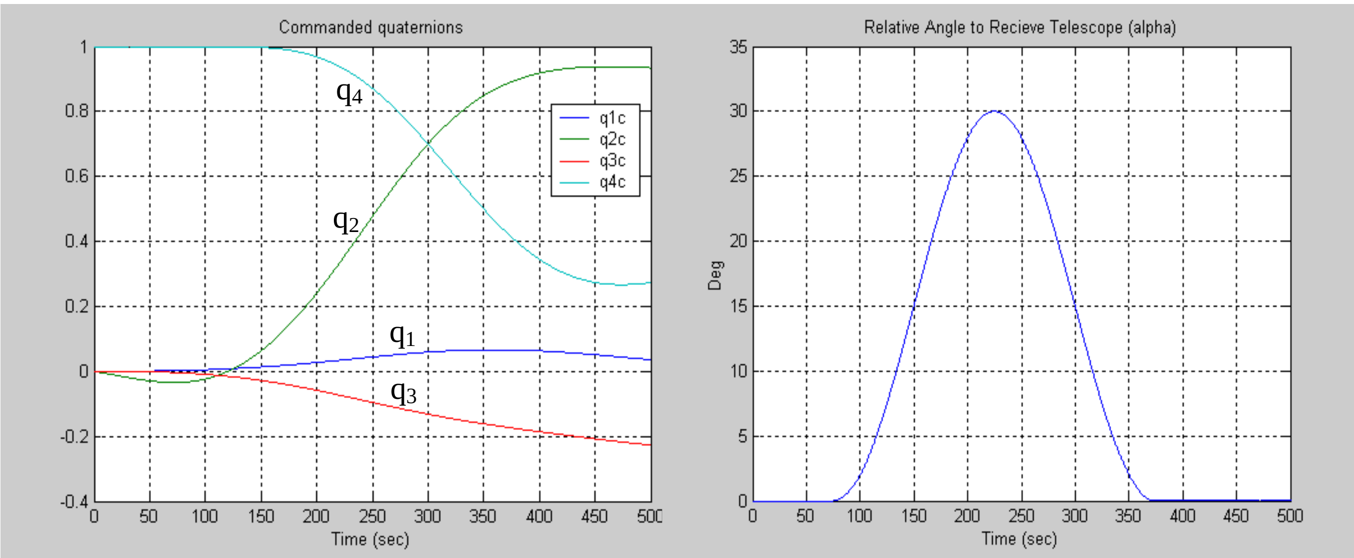

Figure 2 Command Attitude Profile During Tracking Maneuver

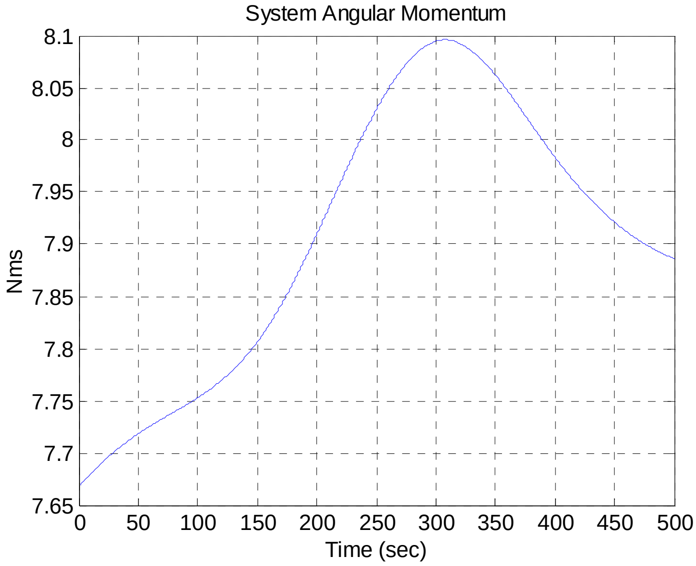

Figure 3 Total Spacecraft Angular Momentum Profile

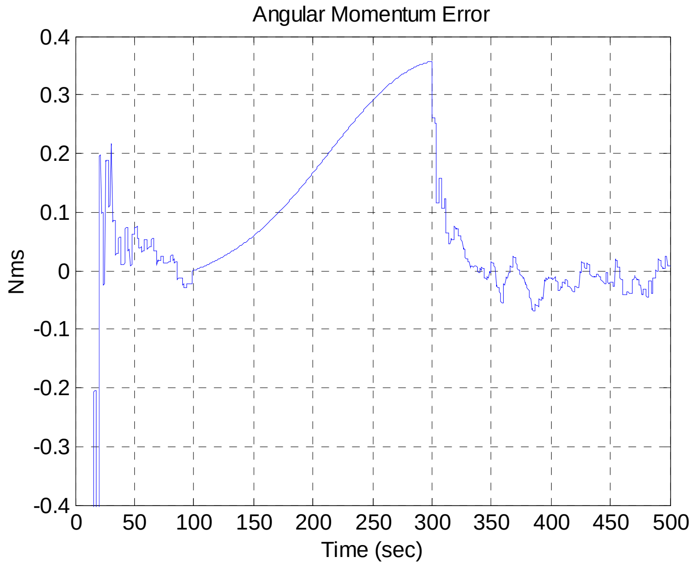

Figure 4 Baseline Dynamic Gyro Angular Momentum Error

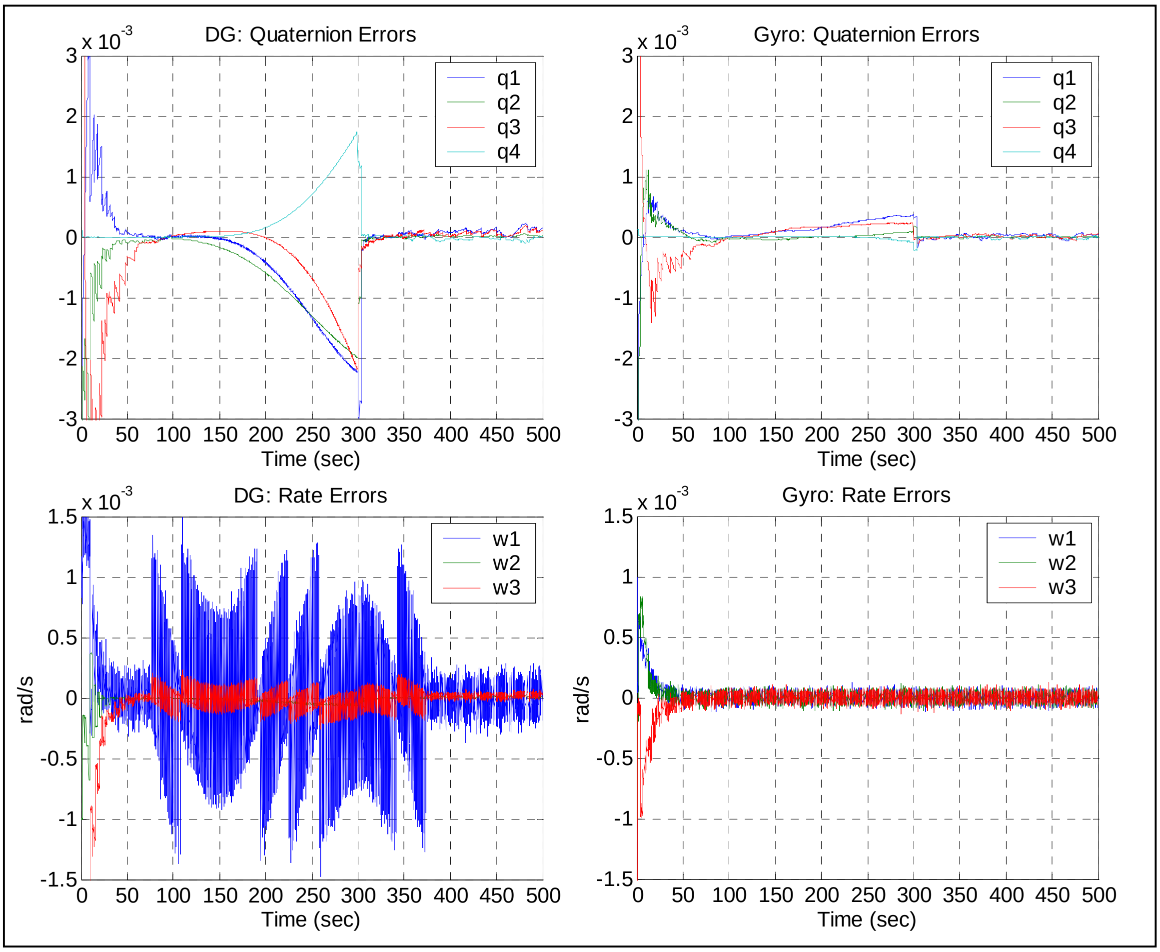

Figure 5 Estimated Attitude Errors Using Rate Gyros

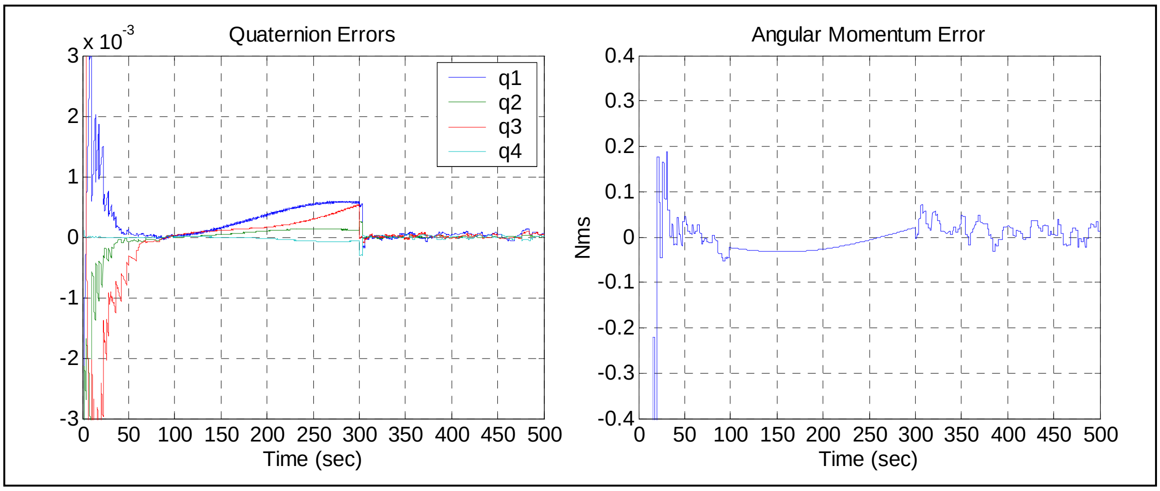

Figure 6 Errors with Gravity Gradient Disturbance Modeled in the Dynamic Gyro

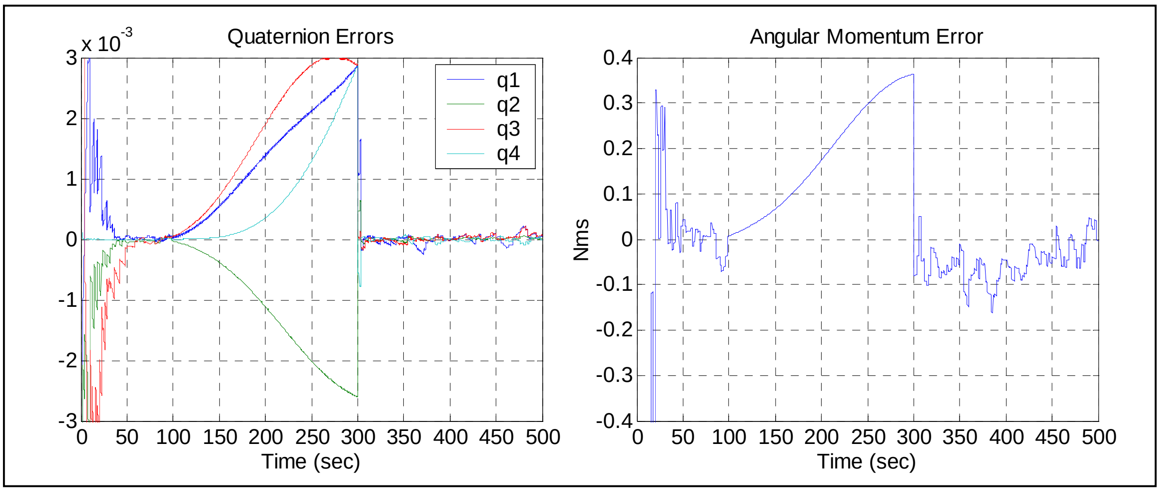

Figure 7 Effects of Reaction Wheel Alignment Error on the Dynamic Gyro Performance

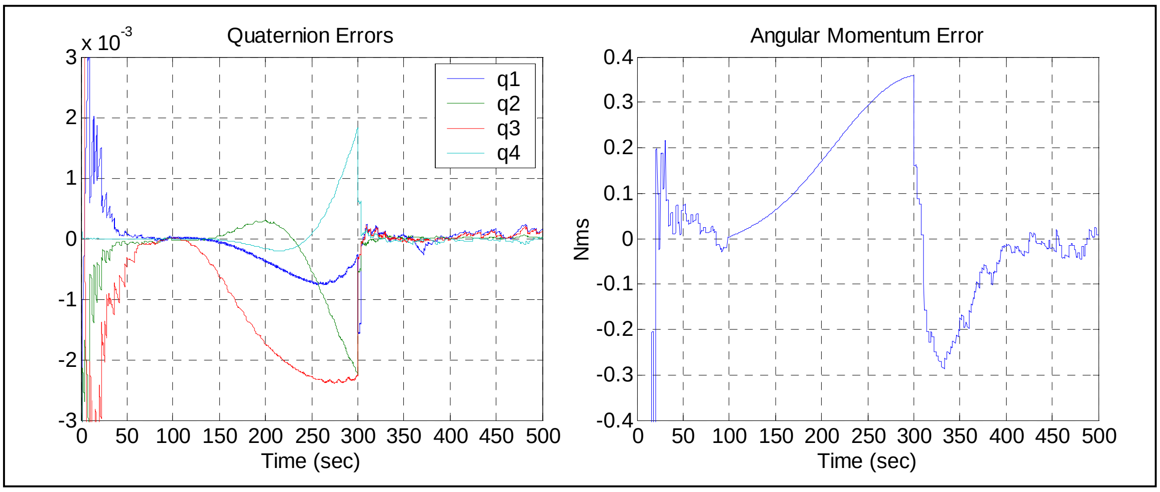

Figure 8 Effects of Reducing Inertia Update Rate on Dynamic Gyro Performance.