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An Efficient Algorithm for Vibration Suppresion to Meet Pointing Requirements of Optical Payloads

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For several future imaging spacecraft, vibration isolation and suppression of its optical payloads have become a challenging problem. These spacecraft have increased performance requirements for the payload, resulting in increased fine steering and vibration isolation requirements. The vibration sources on the spacecraft, however, are increased due to the new large flexible structures and addition of rotating devices. One promising way to address both issues is through the Stewart platform. By using a hexapod with six actuated struts, it is possible to achieve both fine pointing and vibration control. Traditional vibration control algorithms rely on the knowledge of the plant and are usually computationally intensive. This paper applies a computationally efficient vibration control to the Stewart hexapod problem. This algorithm does not rely on the knowledge of the plant other than to find the adaptation rate coefficient and performs well on highly non-linear plants. A convergence analysis is presented. Results are shown for a voice-coil actuated hexapod.

Introduction

ALTHOUGH military and scientific satellites have been carrying optical payloads for some time, the vibration requirements have increased on current and future spacecrafts. Passive-only mounts have been used for isolation on most satellites, but lower frequencies and higher vibration levels require softer isolators. The design of these softer isolators is complicated due to high launch loads. In addition, passive isolators do not solve all problems: they are incapable of isolating the vibration comming trough the umbilical or suppress vibration produced by the payload itself (cryocoolers, for example). Combined with the vibration isolation problem, these optical payloads have fine pointing requirements, which become difficult to achieve by the spacecraft. One solution is to have fast steering mirrors to reduce requirements on the spacecraft bus.

Active vibration/steering solutions have been pursued to address the passive systems limitations. One very promising configuration for vibration isolation is the Stewart Platform.[1] Although it was introduced in 1965, its application to vibration isolation was only possible after the introduction of the fast DSP chips. The most popular vibration control algorithm, the Extended-X LMS, was presented in 1981 by Widrow et al.[2] and Burguess[3] (working independently). Most studies on vibration isolation using the Stewart platform did not appear until mid-90's due to limitations in computing power.

In 1997, Rahman, Spanos and Laskin proposed a hexapod-based vibration isolation and steering system.[4] The vibration isolation/steering device consisted of a Stewart hexapod using voice-coil actuators, with position control and vibration suppression covering different frequency ranges. Stephen G. Edwards obtained good results with the Clear Box method,[5] which had better performance than the standard Multiple Error LMS[6] in several cases.

At the *Spacecraft Research Design Center*, active vibration isolation by using smart struts is an active area of research. One of the topics of research is the vibration isolation using a Stewart hexapod (Ultra-Quitet Platform —UQP). This hexapod, built by CSA, Inc.,

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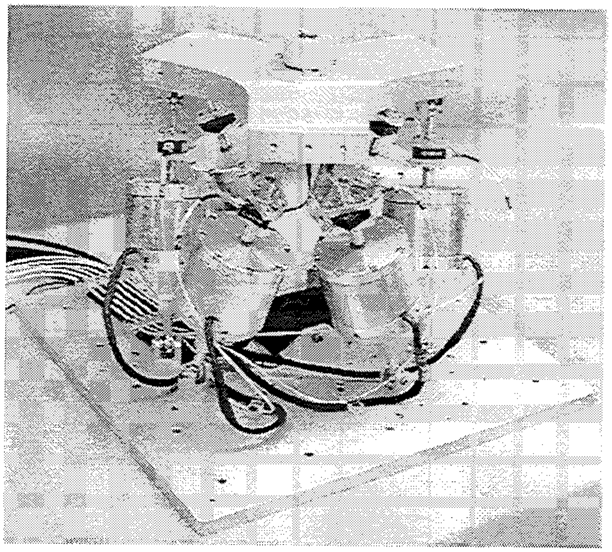

Figure 1 The Precision Pointing Hexapod[11]

has six actuated struts equipped with piezoeletric actuators and geophone sensors. The struts are mounted in a cubic configuration in order to minimize the cross-coupling.[5,7] The actuators can provide $50\mu m$ of travel and have a bandwidth above 1KHz. A dynamic shaker is mounted below the bottom plate (off-centered) to provide the vibration disturbance. Using the UQP, Edwards, Chen, Agrawal and Longman worked on the Clear Box Method, Extended-Error LMS, Repetitive Control and other methods.[5,8–10] Several other algorithms were also implemented for studies and comparison.

All the studied methods required intense computation for adequate performance. Since computing power is directly related to electric power (which relates to launching weight), finding a computationally efficient algorithm is important for space applications.

Recently, the SRDC received a new hexapod — the Precision Pointing Hexapod (PPH) shown on Fig. 1.[11] It have accelerometers instead of geophones as sensors and voice coils instead of the piezoeletric actuators. The new actuators can provide more than $\pm 5mm$ of travel, allowing the platform to tilt about $\pm 2.5^o$ and twist of $\pm 10^o$. Six eddy current sensors are used to provide the position information. A custom adapter allows the shaker to be mounted in different positions. This new hexapod have been used as an experimental setup for research in precision pointing and vibration control of sensitive payloads. The research goal is to develop a steering/vibration supression control system for use with optical payloads. This paper presents results of the work on vibration control. It assumes multi-tonal vibration and presents a computationally efficient control algorithm for the vibration suppression problem.

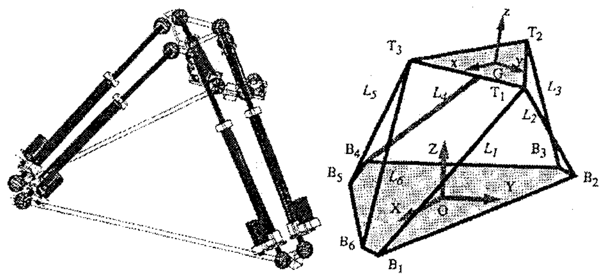

Figure 2 The platform view in 3D

The hexapod

Description

The Stewart platform, also known as parallel manipulator, is composed of two plates connected by some links. One of the plates is considered to be the reference (base) and the other (top) is the one for which the attitude and position are desired to be controlled. A diagram of such a platfrom is shown on Fig. 2.[12] The platform can be controlled by changing the length of the actuators.

The main components of the platform are:

- *Top plate.* Is the portion that has its attitude controller.
- *Bottom base.* Is the plane to which the actuators are connected.
- *Links.* The platform has six links between the top and bottom plates. Each link end is connected using a ball joint.
- *Accelerometers.* Mounted inline with each of the six links, on the moving part of the actuator shaft.
- *Actuators.* Voice coils actuators on each of the six links.
- *Disturbance generator.* Mounted on a custom bracket, the shaker (Aura AST-1B-4) provides the vibration source and can provide more than $56mg$ of vibration (at $50Hz$). Another one can be mounted on top of the platform and provide more than $500mg$ of vibration (at $50Hz$).
- *Eddy current sensor.* Six of them, mounted in pairs in symmetric positions of the top plate. They are used to provide position information.
- *Target.* Each eddy current sensor has a matching metallic target.
- *Floating table.* The table on which the hexapod is mounted. It is kept floating on air cushions in order to minimize the influence of the ground vibrations.
- *Passive isolation.* Isolates the bottom plate from the base at higher frequencies.

Actuators

This platform is well suited for position and vibration control. The actuators, with a stroke of more than $\pm 5mm$ can provide more than 2.5^o of tilting and 10^o of twisting. Capable of delivering up to $40N$ of dynamic force and up to $70N$ of static force, the actuators are adequate to control large levels of vibration. The unwanted consequence of the long-stroke actuators used in the PPH is the introduction of nonlinearities in the system.

Also, the large angles of tilt and twist achievable by the platform led to the use of ball-joint conections between the struts and both bases. These joints introduce backslash to the actuator's travel and are an important source of nonlinearity. The most relevant aspect is that the nonlinearities are very important for the PPH and therefore any algorithm that is to be implemented must perform well in the presence of a nonlinear plant.

Accelerometers

The platform makes use of accelerometers mainly for vibration control purposes. Although the plant is completely observable only by using position sensors, they are not adequate for vibration control. The accelerometers being used are able to measure frequencies down to DC level. Since the DC acceleration does not impact spacecraft pointing performance or vibration suppression, a digital filter was added in software to eliminate this component. Antialiasing filters were also added before feeding the signal to the AD converter.

Controller

The optical payload problem can be decomposed in two distinct parts: pointing and vibration. The use of the hexapod has the potential of performing both tasks at the same time. This work concentrates on tonal vibration suppression. The sources of this type of vibration include reaction wheels, fluid pumps, cryocoolers, etc. All of these operate at frequencies usually much higher than the ones needed for pointing. Therefore, it is reasonable to come up with a control law that separates the position and the vibration control in the frequency domain. This has been done in Raman et al in 1997.[4]

Controlling vibration using a hexapod is not a trivial task. Treating the system as six independent SISO systems is one of the used approaches.[4,13,14] Others, realizing the highly coupled nature of the hexapod (even in the cubic configuration)[15] approached the problem using MIMO controllers. Edwards,[5] Chen[8] and others have pursued this approach with promising results.

Most vibration control algorithms make use of a model of the system. For a system as complex as a hexapod, the model itself makes these methods computationally intensive. Others, like the Clear Box Method, do not require the knowledge of the model *a priori*, but rely on even more computational effort than LMS-based methods.

The PPH, due to its unique characteristics, is highly nonlinear. Therefore, any vibration control method

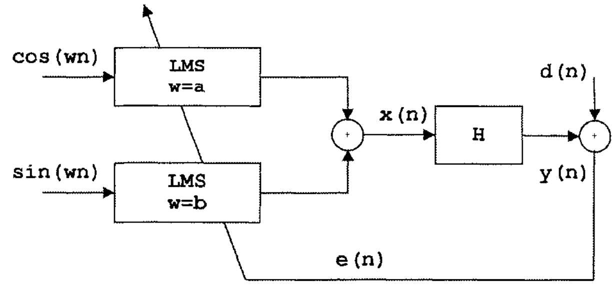

Figure 3 Adaptive Notch Canceller

to be used must tolerate highly nonlinear plants. It is important to point out that the authors could not find any work being done on hexapods which addresses specifically the nonlinearity issue. Even more generic theoretical work on vibration isolation/suppression rarely deal the nonlinearity issue.

Proposed controller

In 1998, Bertran and Montoro[16] proposed an *Adaptive Notch Canceller.* Their work originated from the need to suppress vibration originated by rotating machinery. The proposed controller needed, for the studied case, only two assumptions: stable linear SISO plant and tonal disturbances with known frequencies. A block diagram describing the controller is shown in Fig. 3.

Assuming that the plant H is linear, then for any sinusoidal signal d_n it is possible to find a sinusoidal input x_n such that $y_n = -d_n$. Usually one would write the input as follows:

$$x_n = X\cos(\omega_d n + \beta_x) \tag{1}$$

There are several algorithms that can find the optimal value of x_n that minimize $e_n = y_n + d_n$, but almost all assume that y_n is a linear combination of the parameters. The input Eq. (1) can be changed to the equivalent form:

$$x_n = a\cos(\omega_d n) + b\sin(\omega_d n) \tag{2}$$

Assuming that $H(\omega_d) = \alpha e^{j\beta}$, the output y_n can then be written as:

$$y_n = a\alpha\cos(\omega_d n + \beta) + b\alpha\sin(\omega_d n + \beta) \tag{3}$$

Using this form the output y_n is linear in the parameters a and b and most adaptive algorithms can be used to find a and b. This work uses the LMS algorithm.

The main characteristics of this controller are:

- *Simplicity.* This filter requires very little computing power and uses a "LMS" filter of order one no matter how complex the plant is.
- *Expandible.* By using the superposition principle (the plant is assumed linear), one can stack several controllers at diferent frequencies.

Convergence analysis

Bertran and Montoro did derive the stability analysis for the studied case, but the stability assumed a particular plant and thus it is not appopriate for the Stewart Platform problem. Therefore, a more general approach is presented in the following discussion.

The basic equations that define the controller are:

$$
\begin{aligned}
d_n &= D\cos(\omega_d n + \gamma) && (4)\\
H(j\omega_d) &= \alpha e^{j\beta} && (5)\\
x_n &= a_n \cos(\omega_d n) + b_n \sin(\omega_d n) && (6)\\
y_n &= \begin{array}{l} a_n\alpha\cos(\omega_d n + \beta) + \\ b_n\alpha\sin(\omega_d n + \beta)\end{array} && (7)\\
a_{n+1} &= a_n + \mu e_n \cos(\omega_d n) && (8)\\
b_{n+1} &= b_n + \mu e_n \sin(\omega_d n) && (9)\\
e_n &= y_n + d_n && (10)
\end{aligned}
$$

The weight update equations can therefore be written as:

$$
\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = \mathbf{F}_{n,\omega_d} \begin{bmatrix} a_n \\ b_n \end{bmatrix} + \begin{bmatrix} \mu\cos(\omega_d n) \\ \mu\sin(\omega_d n) \end{bmatrix} d_n \qquad (11)
$$

whith:

$$
\begin{aligned}
F_{1,1} &= \frac{1}{2}\mu\alpha\left(\cos(\beta + 2\omega_d n) + \cos(\beta)\right) + 1\\
F_{1,2} &= \frac{1}{2}\mu\alpha\left(\sin(\beta + 2\omega_d n) + \sin(\beta)\right)\\
F_{2,1} &= \frac{1}{2}\mu\alpha\left(\sin(\beta + 2\omega_d n) - \sin(\beta)\right)\\
F_{2,2} &= -\frac{1}{2}\mu\alpha\left(\cos(\beta + 2\omega_d n) + \cos(\beta)\right) + 1
\end{aligned}
$$

Proof of convergence for matched frequencies

Let, for the convergence analysis, x_n and d_n be rewritten as:

$$
\begin{aligned}
x_n &= A_n e^{j\omega n} && (12)\\
d_n &= D e^{j(\omega n + \gamma)} && (13)
\end{aligned}
$$

Equations (8) and (9) can also be written in the complex form. By combining both, one gets:

$$
A_{n+1} = A_n + \mu e_n e^{-j\omega n} \qquad (14)
$$

Substituting equations (5) and (12) on Eq. (10), the error can be written as:

$$
e_n = A_n \alpha e^{j(\omega n + \beta)} + d_n \qquad (15)
$$

Then, using Eq. (15) on Eq. (14):

$$
\begin{aligned}
A_{n+1} &= A_n + \mu\left(A_n\alpha e^{j(\omega n+\beta)} + d_n\right)e^{-j\omega n}\\
&= \left(1 + \mu\alpha e^{j\beta}\right)A_n + d_n e^{-j\omega n} && (16)
\end{aligned}
$$

Now, using Eq. (13) on Eq. (16):

$$
A_{n+1} = \left(1 + \mu\alpha e^{j\beta}\right)A_n + De^{j\gamma} \qquad (17)
$$

This equation is stable if:

$$
\left|1 + \mu\alpha e^{j\beta}\right| < 1 \qquad (18)
$$

If a proper value of μ is chosen, the value of A_n will approach a constant when the time index n approaches infinite. Therefore Eq. (14) can be written:

$$
\begin{aligned}
A_{n+1} &= A_n + \mu e_n e^{-j\omega n}\\
\mu e_n e^{-j\omega n} &= A_{n+1} - A_n\\
e_n e^{-j\omega n} &= 0 \text{ (when } n \to \infty)\\
&\Rightarrow\\
\lim_{n\to\infty} e_n &= 0
\end{aligned}
$$

Please note that this result is valid for small values of μ so that the propagation time imposed by the plant is neglectible.

Optimal Weights

The exact cancellation of the error only occurs when there is an exact match between the frequency fed to the controller and the disturbance frequency. It is desired to know the effect of the frequency mismatch on the weights of the filters.

Writing d_n and x_n in the complex form and adding an error in frequency, one can write:

$$
\begin{aligned}
e_n &= d_n + y_n\\
&= De^{j((\omega_d+\delta\omega)n+\gamma)} + A_n e^{j\omega_d n} H(\omega_d)\\
&= De^{j((\omega_d+\delta\omega)n+\gamma)} + \alpha A_n e^{j(\omega_d n+\beta)} && (19)
\end{aligned}
$$

Forcing the error to be zero, one can then evaluate A_n:

$$
\begin{aligned}
De^{j\omega_d n + j\delta\omega n + \gamma} &= -\alpha A_n^* e^{j\omega_d n + beta}\\
&\Rightarrow\\
A_n^* &= \frac{D}{\alpha} e^{j(\delta\omega n + \gamma - \beta + \pi)} && (20)
\end{aligned}
$$

The real coefficients a_n and b_n are obtained rewriting Eq. (20) into the real form:

$$
\begin{aligned}
a_n^* &= -\frac{D}{\alpha}\cos(\delta\omega n + \gamma - \beta)\\
b_n^* &= -\frac{D}{\alpha}\sin(\delta\omega n + \gamma - \beta) && (21)
\end{aligned}
$$

Equation (21) represents the optimal weights. If the actual weights followed the optimal ones, then exact cancellation would be obtained. In practice this situation is not possible when $\delta\omega \neq 0$ due to the propagation time imposed by the plant H.

It is also important to realize that in the event of a frequency mismatch, the weights will cycle with zero mean. If the frequencies are matched ($\delta\omega = 0$), then the coefficients will converge to a constant value. Since the weight update Eq. (11) is linear, then the proposed adaptative method will converge to the true solution.

Stability in the presence of mismatched frequency

It has been already shown that the error for a matched vibration supressor is zero (assuming that there is no noise). It is important to verify that the steady state error remains bounded when there is a frequency mismatch.

Equation (17) shows that the weights will not diverge if an appropriate value of μ is chosen so that $\|1+\mu\alpha e^{j\beta}\| < 1$. This guarantees exponentical convergence and bounded-input/bounded-output stability.

Main characteristics

Some of the main characteristics of this controller are outlined below:

- Equation (11) is linear. In the event of matched frequencies the algorithm will converge to the true solution.
- The algorithm is *very* efficient. It requires only about *seven* floating point operations per iteration (two summations, five products and two trigonometric evaluations).
- The amount of computation is not increased due to the plant complexity (order).
- The amount of computation increases linearly with the number of frequencies.
- The algorithm is particularly sensitive to frequency mismatch. The error will remain bounded for bounded inputs, but there will be no exact cancelation for the general case.

Experimental results

First, a sinusoidal signal is sent to all actuators in order to verify the effects of the plant nonlinearities. Next, two experiments are presented. On the first one the shaker is driven with a single sinusoid. The second one shows the algorithm performance when controlling close tones.

Plant response

The actuators were fed with a single tone ($40Hz$), all in phase. The output of the plant is shown on Fig. 4. As one can see, robustness to nonlinearities is very important for any vibration suppression algorithm to be used on the PPH.

Fundamental and harmonic

The shaker was mounted off-centered, with the vibration along the *z-axis*, at $40Hz$. Six identical controllers were configured to suppress the vibration at $40Hz$ and $80Hz$. The controller sampling rate was $1KHz$.

Figures 5 and 6 show the power spectrum of the uncontrolled and controlled levels of vibration on the *z-axis*. Table 1 summarizes the results for the frequencies $40Hz$, $80Hz$ and $120Hz$ (uncontrolled).

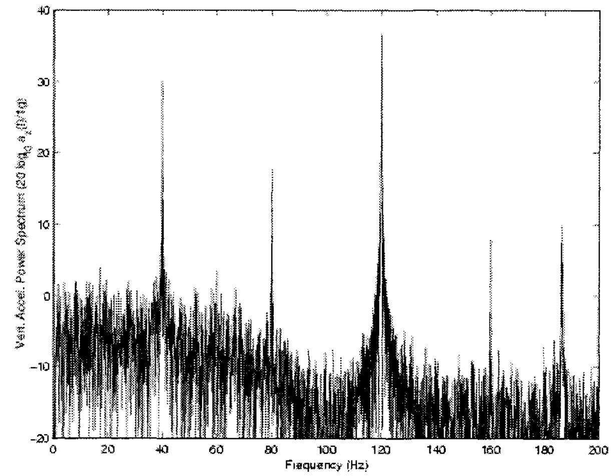

Figure 4 Plant response to sinusoidal input

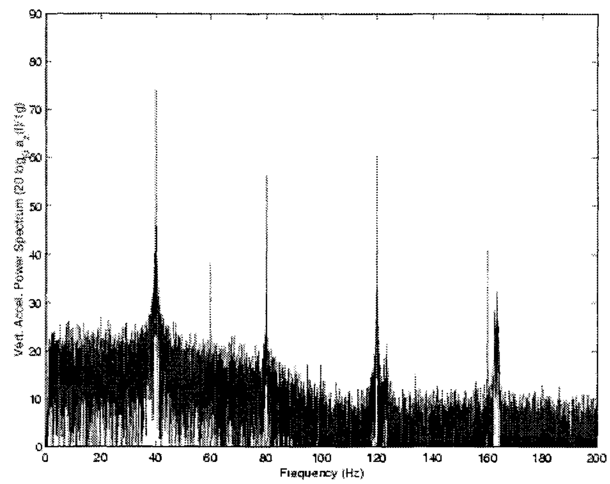

Figure 5 Single tone and harmonic — no control

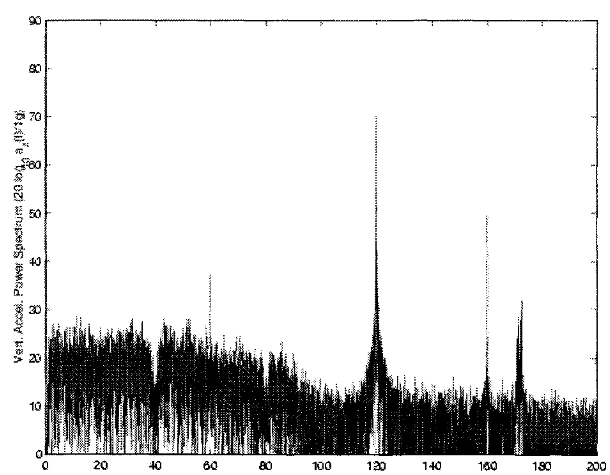

Figure 6 Single tone and harmonic — controlled

Frequency	non-controlled	controlled	reduction
$40Hz$	74.3	7.9	66.4
$80Hz$	56.3	16.1	40.2
$120Hz$	60.3	70.3	-10

Table 1 Decentralized controller ($40Hz$ and $80Hz$) $(20\log_{10}(accel/1g))$

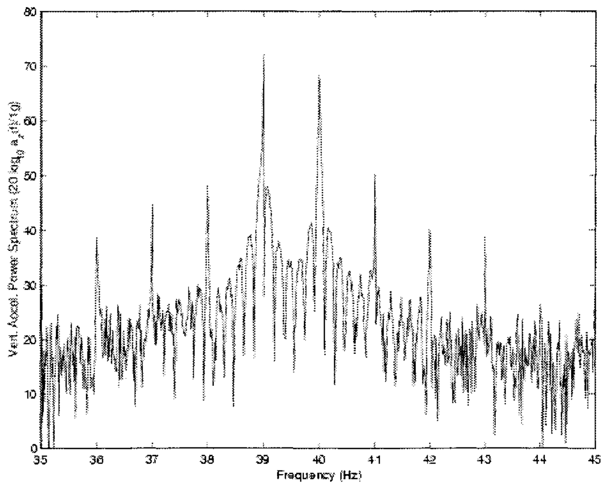

Figure 7 Close tones — no control

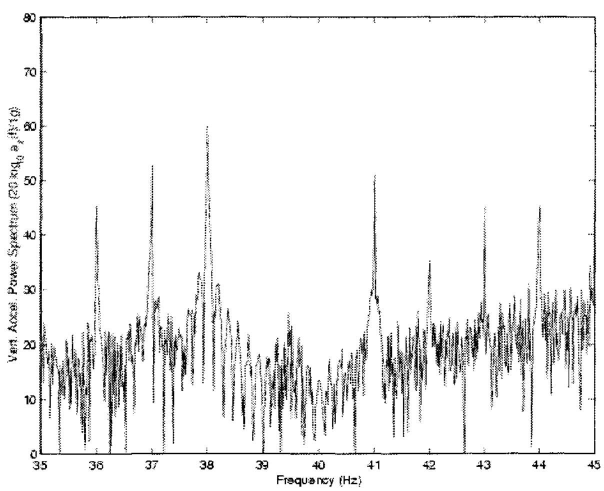

Figure 8 Close tones — controlled

Two close tones

Using the same configuration as in the previous experiment, the shaker was excited with the frequencies $39Hz$ and $40Hz$, both with same amplitude. A controller was used to suppress both frequencies. Figures 7 and 8 show the power spectrum of the controlled and uncontrolled levels of vibration on the z-*axis*. Table 2 summarizes the results.

Comments

It is important to mention that the controller was successful in reducing the disturbances at the assigned frequencies down to the noise floor. This result was achieved even when the two tones were very close (39

Frequency	non-controlled	controlled	reduction
$37Hz$	44.7	52.7	-8.0
$38Hz$	48.1	59.7	-11.6
$39Hz$	72.2	12.1	60.1
$40Hz$	68.2	9.1	59.1
$41Hz$	50.1	51.1	-1.0
$42Hz$	40.2	35.4	-4.8

Table 2 Decentralized controller (close tones) $(20\log_{10}(accel/1g))$

and $40Hz$). Unfortunatelly other harmonics did increase in the process. As expected, this effect was not noticed on simulations of linear plants.

Although the experiment was successful in testing the algorithm for vibration suppression, it uncovered some limitations of the hardware setup. Two of them are the most important:

- *Accelerometers' range.* The accelerometers used in this platform cover the range $\pm 2g$, including DC. Although the DC was filtered out, it became clear that the range was not appropriate for space applications. A fairly large amount of vibration was necessary in order to obtain satisfactory response from the accelerometers.
- *Actuator free-play.* In order to minimize this problem, all joints were pre-loaded. New flexible joints will replace the current ones.

Summary

This paper applies a computationally efficient vibration suppression algorithm, Adaptive Notch Canceller, for vibration suppression in spacecraft. A proof of convergence was presented for matched frequencies. It was also shown that the algorithm will not diverge in the case of mismatched frequencies. The algorithm performed very well on a highly nonlinear MIMO plant.

The very little computational power required and the level of supression achieved make this method very well suited for supression of tonal vibrations in spacecraft. This is especially important when one considers that the power consumption increases with computational capability. Future work includes finding the bounds of μ for a generic plant and the influence of measurement noise.

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