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ATTITUDE STABILITY OF FLEXIBLE ASYMMETRIC DUAL-SPIN SPACECRAFT

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Abstract

This paper deals with the attitude motion of a dual-spin spacecraft which consists of an asymmetric rigid platform and an asymmetric rigid rotor connected together by a flexible joint. The equations of motion represent a flexible multiple-degree-of-freedom gyroscopic system with periodic coefficients due to asymmetries. The equations are analyzed for possible resonances and associated instability regions by using first asymptotic approximation. The resonances occur when either the rotor spin frequency or the platform spin frequency is in the neighborhood of a natural frequency or half the sum of any two natural frequencies. The instability regions associated with these resonances are given explicitly.

Nomenclature

I_{xp}, I_{yp}	=	platform transverse principle inertias about axes x_2 and y_2
I_{xr}, I_{yr}	=	rotor transverse principal inertias about axes x_4 and y_4
$\bar{I}_{xp}$	=	$I_{xp} + m\ell_p^2$
$\bar{I}_{yp}$	=	$I_{yp} + m\ell_p^2$
$\bar{I}_{xr}$	=	$I_{xr} + m\ell_r^2$
$\bar{I}_{yr}$	=	$I_{yr} + m\ell_r^2$
I_{tp}	=	$(\bar{I}_{xp} + \bar{I}_{yp})/2$
I_{tr}	=	$(\bar{I}_{xr} + \bar{I}_{yr})/2$
I_{pr}	=	$m\,\ell_p\,\ell_r$
K_x, K_y	=	stiffness in directions x_4' and y_4', respectively
K_m	=	$(K_x + K_y)/2$
ℓ_p	=	distance between points S and 0^1
ℓ_r	=	distance between points S and 0*
m	=	$m_p\, m_r/(m_p + m_r)$
m_p	=	platform mass
m_r	=	rotor mass
$0, 0^1, 0^*$	=	cm of spacecraft, platform and rotor, respectively
S	=	joint between platform and rotor
α	=	$(\bar{I}_{xp} - \bar{I}_{yp})/(\bar{I}_{xp} + \bar{I}_{yp})$ = platform inertia inequality factor
σ	=	$(\bar{I}_{xr} - \bar{I}_{yr})/(\bar{I}_{xr} + \bar{I}_{yr})$ = rotor inertia inequality factor
μ	=	$(K_x - K_y)/(K_x + K_y)$ = stiffness inequality factor
Ψ_x	=	$\theta_x + \phi_x$
Ψ_y	=	$\theta_y + \phi_y$
I_{zp}	=	platform axial inertia
I_{zr}	=	rotor axial inertia
ω_p	=	spin rate of the platform
ω_r	=	spin rate of the rotor
τ	=	$\tan^{-1}(\delta_y/\delta_x)$
τ_1	=	$\tan^{-1}\left[\frac{(I_{yr}-I_{zr})\Omega_x}{(I_{xr}-I_{zr})\Omega_y}\right]$
δ	=	$\sqrt{\delta_x^2 + \delta_y^2}$
Ω	=	$\sqrt{(I_{xr} - I_{zr})^2\,\Omega_y^2 + (I_{yr} - I_{zr})^2\,\Omega_x^2}$

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Introduction

Attitude stability of dual-spin spacecraft has been studied by several investigators[1,2,3]. Using the energy sink analysis, stability conditions have been derived as a function of spacecraft inertias and energy dissipations in the rotor and the platform. However, the emphasis in these studies has been on symmetrical spacecraft. This paper deals with attitude instability due to unequal transverse principal inertias and stiffness. Such effects have been extensively studied for gyroscopes and rotors[4,5]. The general equations of motion for such systems contain periodic coefficients. Reference 6 presents stability conditions for special cases an asymmetric platform with a symmetric rotor and a symmetric platform with an asymmetric rotor. For such cases, the equations of motion can be transformed into constant coefficient equations. In reference 7, the effects of momentum wheel stiffness inequalities are studied. In this paper, instability regions for a more general case of a dual-spin spacecraft, where asymmetries are assumed in both the platform and the rotor, are obtained.

The dual-spin spacecraft model consists of a rigid asymmetric platform connected to a rigid asymmetric rotor by a flexible joint. This model represents two classes of spacecraft. First, spin-stabilized spacecraft, such as INTELSAT VI, where the main body is spinning, providing gyroscopic stabilization, and a non spinning platform consisting of antennas which are oriented towards a fixed point on the earth. The second class consists of body-stabilized spacecraft in which the main body is non-spinning and the rotor consists of a moment wheel with compliant bearings, such as magnetic bearings. The equations of motion represent a multiple-degree-of-freedom gyroscopic system with periodic coefficients due to inertia and stiffness asymmetries. The gyroscopic equations are normalized by using the Meirovitch[8] method. The asymmetries are assumed to be small. The resulting equations are the equations of motion of a multiple-degree-of-freedom system under parametric excitation. The attitude instability regions are given explicitly by using first asymptotic approximation[9,10].

Spacecraft Model

An idealized model of a dual-spin spacecraft is shown in Figure 1. It consists of a platform P interconnected to a rotor R by a shaft and bearings. The rotor and the platform are assumed to be rigid. The flexibility is assumed in the shaft and the bearings. The platform and the rotor are assumed to be spinning at constant spin rates ω_p and ω_r, respectively. The axes $X_0Y_0Z_0$ are fixed in space with the origin at the CM of the system. The axes $X_2Y_2Z_2$ are obtained from $X_0Y_0Z_0$ by rotation θ_x about the X_0 axis and θ_y about the Y_1 axis. The axes $X_2'Y_2'Z_2'$ which are the principal inertia axes of the platform are fixed in the platform and rotating at spin rate ω_p about the Z_2 axis. The orientations of axes $X_4Y_4Z_4$, where Z_4 is the spin axis of the rotor, are

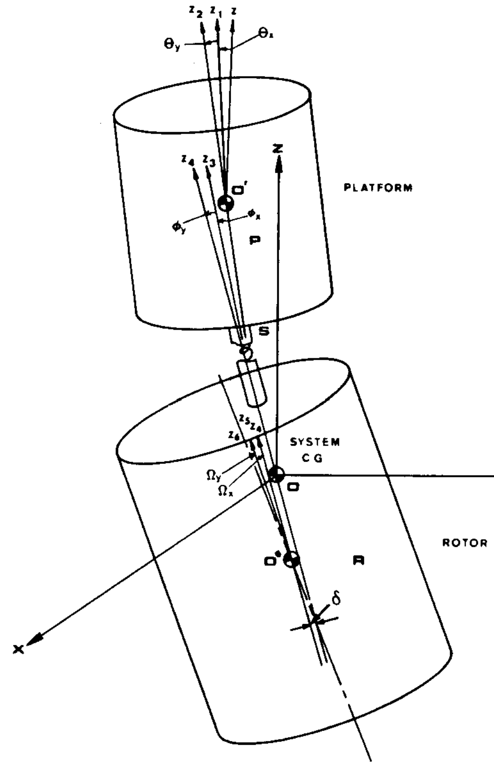

Figure 1. Dual-Spin Satellites

obtained by rotation ϕ_x about the X_2 axis and ϕ_y about the Y_3 axis. The axes $X_4'Y_4'Z_4'$ are fixed in the rotor and rotating at spin rate ω_r about Z_4 axis. The rotor is assumed to be statically and dynamically unbalanced. The principal inertia axes $X_6Y_6Z_6$ of the rotor are described with respect to $X_4'Y_4'Z_4'$ by rotation Ω_x about the X_4' axis and Ω_y about the Y_5 axis. The rotor CG offset, δ, has a component δ_x in X_4' axis and δ_y in the Y_4' axis. The CM's of the rotor, the platform, and the system are denoted as O^*, O^1, and O, respectively.

Equations of Motion

The equations of motion of the spacecraft model are obtained by using Lagrange's equations, as discussed in Reference 6. The higher order non-linear terms are neglected. Damping is not considered at this point. The equations of motion are:

$$M\ddot{X} + G\dot{X} + KX = F \qquad (1)$$

where generalized vector, X, is defined as follows:

$$X = \begin{Bmatrix} \theta_x \\ \theta_y \\ \Psi_x \\ \Psi_y \end{Bmatrix} \qquad (2)$$

The other matrices are defined as follows:

$$
M = \begin{bmatrix} I_{tp}(1 + \alpha\cos\omega_p t) & \alpha I_{tp} \sin 2\omega_p t & I_{pr} & 0 \\ & I_{tp}(1-\alpha\cos 2\omega_p t & 0 & I_{pr} \\ \text{Symmetric} & & I_{tr}(1+ & \sigma I_{tr} \sin 2\omega_r t \\ & & & I_{tr}(1-\alpha\cos 2\omega_r t) \end{bmatrix} \quad (3)
$$

$$
G = \begin{bmatrix} -2\alpha I_{tp}\omega_p \sin 2\omega_p t & I_{zp}\omega_p + 2\alpha I_{tp}\omega_p \cos 2\omega_p t & 0 & 0 \\ -I_{zp}\omega_p + 2 I_{tp}\omega_p \cos 2\omega_p t & 2\alpha I_{tp}\omega_p \sin 2\omega_p t \cos 2\omega_p t & 0 & 0 \\ 0 & 0 & -2\sigma I_{tr} \omega_r \sin 2\omega_r t & I_{zr}\omega_r + 2\sigma I_{tr}\omega_r \cos 2\omega_r t \\ 0 & 0 & -I_{zr}\omega_r + 2\sigma I_{tr}\omega_r \cos 2\omega_r t & 2\sigma I_{tr}\omega_r \sin 2\omega_r t \end{bmatrix} \quad (4)
$$

$$
K = \begin{bmatrix} K_m(1+\mu \cos 2\omega_r t) & \mu K_m \sin 2\omega_r t & -K_m(1+ m\cos 2\omega_r t) & -\mu k_m \sin 2\omega_r t \\ & K_m(1-\mu\cos 2\omega_r t) & -\mu K_m \sin 2\omega_r t & -K_m(1-\mu\cos 2\omega_r t) \\ & & K_m(1+\mu\cos 2\omega_r t) & \mu K_m \sin 2\omega_r t \\ \text{Symmetric} & & & K_m(1-\mu\cos 2\omega_r t) \end{bmatrix} \quad (5)
$$

$$
F = \begin{bmatrix} -m\,\delta\,\omega_r^2\,\ell_p \sin(\omega_r t + \tau) \\ m\,\delta\,\omega_r^2\,\ell_p \cos(\omega_r t + \tau) \\ -\Omega\,\omega_r^2 \sin(\omega_r t - \tau_1) - m\,\delta\omega_r^2\,\ell_r \sin(\omega_r t+\tau) \\ \Omega\,\omega_r^2 \cos(\omega_r t - \tau_1) + m\delta\omega_r^2\,\ell_r \cos(\omega_r t + \tau) \end{bmatrix} \quad (6)
$$

where M and K are symmetric matrices, G is a sum of a skew symmetric matrix and a symmetric matrix due to inertia inequalities. F is the force vector due to static and dynamic unbalance. Let us assume that stiffness and transverse inertia inequalities and static and dynamic unbalance are small and can be expressed as

$$
\begin{aligned} \alpha &= \varepsilon\,\bar{\alpha} \\ \sigma &= \varepsilon\,\bar{\sigma} \\ \mu &= \varepsilon\,\bar{\mu} \\ \delta &= \varepsilon\,\bar{\delta} \\ \Omega &= \varepsilon\,\bar{\Omega} \end{aligned} \quad (7)
$$

where ε is a small positive parameter. In this case, periodic terms due to the inequalities can be taken to the right hand side of the equation (1) and can be considered as parametric excitation. The matrix G becomes skew symmetric.

Normalization

In order to normalize these gyroscopic equations the Meirovitch method [8] is most suited. However, in order to use this method, the stiffness matrix K has to be non-singular and positive definite. In the present formulation, the stiffness matrix is singular and involves only $\Psi_x - \theta_x$ (ϕ_x) and $\Psi_y - \theta_y$ (ϕ_y). In order to use Meirovitch method of normalization, the following generalized coordinates are used.

$$
q = \begin{Bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\Psi}_x \\ \dot{\Psi}_y \\ \phi_x \\ \phi_y \end{Bmatrix} \quad (8)
$$

Using the Meirovitch method with these coordinates, the second order equations of motion are transformed into the following first order equations.

$$
\bar{M}\,\dot{q} + \bar{G}\,q = \varepsilon\Big[\sin 2\omega_p t \,\{F^1\dot{q} + F^2 q\} + \cos 2\omega_p t \,\{F^3\dot{q} + F^4 q\} + \sin 2\omega_r t \,\{F^5\dot{q} + F^6 q\} + \cos 2\omega_r t \,\{F^7\dot{q} + F^8 q\} + F^0\Big] \quad (9)
$$

where

$$
M = \begin{bmatrix} I_{tp} & 0 & I_{pr} & 0 & 0 & 0 \\ & I_{tp} & 0 & I_{pr} & 0 & 0 \\ & & I_{tr} & 0 & 0 & 0 \\ & \text{Symmetric} & & I_{tr} & 0 & 0 \\ & & & & k_m & 0 \\ & & & & & k_m \end{bmatrix} \quad (10)
$$

$$
G = \begin{bmatrix} 0 & I_{zp}\omega_p & 0 & 0 & -K_m & 0 \\ & 0 & 0 & 0 & 0 & -K_m \\ & & 0 & I_{zr}\omega_r & K_m & 0 \\ & & & 0 & 0 & K_m \\ & \text{Skew-Symmetric} & & & 0 & 0 \\ & & & & & 0 \end{bmatrix} \quad (11)
$$

The elements of the matrices in the right hand side of Eq. (9); F^1, F^2, F^3, F^4, F^5, F^6, F^7, and F^8; are all zeros with the following exceptions:

$$
\begin{aligned}
&-F^1(1,2) = -F^1(2,1) = -F^3(1,1) = F^3(2,2) = \alpha_1 \; ; \; F^2(1,1) = -F^2(2,2) = \\
&-F^4(1,2) = -F^4(2,1) = \alpha_2 \; ; \; -F^5(3,4) = -F^5(4,3) = -F^7(3,3) = F^7(4,4) = \sigma_1 \; ; \\
&F^6(3,3) = -F^6(4,4) = -F^8(3,4) = -F^8(4,3) = -\sigma_2 \; ; \; F^6(1,6) = F^6(2,5) = \\
&-F^6(3,6) = -F^6(4,5) = F^8(1,5) = -F^8(2,6) = -F^8(3,5) = F^8(4,6) = \mu_1
\end{aligned}
$$

$$F^{o} = \begin{Bmatrix} -F_2 \ell_p \sin(\omega_r t + \tau) \\ -F_2 \ell_p \cos(\omega_r t + \tau) \\ -F_3 \sin(\omega_r t + \alpha_3) \\ -F_3 \cos(\omega_r t + \alpha_3) \end{Bmatrix} \quad (12)$$

where

$$\begin{aligned} \alpha_1 &= \bar{\alpha}\, I_{tp} \\ \alpha_2 &= 2\bar{\alpha}\, \omega_p\, I_{tp} \\ \sigma_1 &= \bar{\sigma}\, I_{tr} \\ \sigma_2 &= 2\bar{\sigma}\, I_{tr}\omega_r \\ \mu_1 &= \bar{\mu}\, K_m \\ F_1 &= \Omega\, \omega_r^2 \\ F_2 &= m\, \delta\omega_r^2 \\ F_3 &= \sqrt{(F_1\cos\tau_1 + F_2\ell_r\cos\tau)^2 + (F_2\ell_r\sin\tau - F_1\sin\tau_1)^2} \\ \alpha_3 &= \tan^{-1}\left(\frac{F_2\ell_r\sin\tau - F_1\sin\tau_1}{F_1\cos\tau_1 + F_2\ell_r\cos\tau}\right) \end{aligned} \quad (13)$$

The eigenvalues and eigenvectors are obtained for the matrix $\bar{\bar{K}}$ which is defined as

$$\bar{\bar{K}} = \bar{M}^{-1}\bar{K} \quad (14)$$

where

$$\bar{K} = \bar{G}^T\bar{M}^{-1}\bar{G} \quad (15)$$

The eigenvalue solution of the matrix $\bar{\bar{K}}$ consists of 3 pairs of repeated eigen values ω_n (n=1,2,3) and 3 pairs of associated eigenvectors Y_n and Z_n (n=1,2,3).

$$\text{Let } [Y:Z] = [Y_1Y_2Y_3 : Z_1Z_2Z_3] \quad (16)$$

$$\omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & \omega_3 \end{bmatrix} \quad (17)$$

For a positive definite $\bar{M}$, which is true for the present system, the following orthogonality relations hold:

$$[Y:Z]^T \bar{M} [Y:Z] = I \quad (18)$$

$$[Y:Z]^T \bar{G} [Y:Z] = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \quad (19)$$

where I is a unit matrix

Let (20)

$$q = [Y:Z] \begin{Bmatrix} \xi \\ \eta \end{Bmatrix} \quad \text{where } \xi = \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix} \text{ and } \eta = \begin{Bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{Bmatrix}$$

Substituting q from equation (20) into equation (9), pre-multiplying by $[Y:Z]^T$, and using relationships (18) and (19), we get

$$I\begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix} + \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = \epsilon\Big[\sin 2\omega_p t\Big\{\bar{F}^1\begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix} + \bar{F}^2\begin{Bmatrix} \xi \\ \eta \end{Bmatrix}\Big\} + \cos 2\omega_p t\Big\{\bar{F}^3\begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix} + \bar{F}^4\begin{Bmatrix} \xi \\ \eta \end{Bmatrix}\Big\} + \sin 2\omega_r t\Big\{\bar{F}^5\begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix} + \bar{F}^6\begin{Bmatrix} \xi \\ \eta \end{Bmatrix}\Big\} + \cos 2\omega_r t\Big\{\bar{F}^7\begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix} + \bar{F}^8\begin{Bmatrix} \xi \\ \eta \end{Bmatrix}\Big\} + \bar{F}^o\Big] \quad (21)$$

$$\text{where } \bar{F}^j = [Y:Z]^T F^j [Y:Z], \quad \bar{F}^o = [Y:Z]^T F^o \quad (22)$$

where j=1,....,8

Let us define

$$\bar{F}^j = \begin{bmatrix} \bar{F}^j_{11} & \bar{F}^j_{12} \\ \bar{F}^j_{21} & \bar{F}^j_{22} \end{bmatrix} \quad j=1,\dots,8 \qquad \bar{F}^o = \begin{Bmatrix} \bar{F}^o_1 \\ \bar{F}^o_2 \end{Bmatrix} \quad (23)$$

Using equations (22) and (23), equation (21) can be rewritten as

$$I\dot{\xi} - \omega\eta = \epsilon\Big[\sin 2\omega_p t\Big\{\bar{F}^1_{11}\dot{\xi} + \bar{F}^1_{12}\dot{\eta} + \bar{F}^2_{11}\xi + \bar{F}^2_{12}\eta\Big\} + \cos 2\omega_p t\Big\{\bar{F}^3_{11}\dot{\xi} + \bar{F}^3_{12}\dot{\eta} + \bar{F}^4_{11}\xi + \bar{F}^4_{12}\eta\Big\} + \sin 2\omega_r t\Big\{\bar{F}^5_{11}\dot{\xi} + \bar{F}^5_{12}\dot{\eta} + \bar{F}^6_{11}\xi + \bar{F}^6_{12}\eta\Big\} + \cos 2\omega_r t\Big\{\bar{F}^7_{11}\dot{\xi} + \bar{F}^7_{12}\dot{\eta} + \bar{F}^8_{11}\xi + \bar{F}^8_{12}\eta\Big\} + \bar{F}^o_1\Big] \quad (24)$$

$$I\dot{\eta} + \omega\xi = \epsilon\Big[\sin 2\omega_p t\Big\{\bar{F}^1_{21}\dot{\xi} + \bar{F}^1_{22}\dot{\eta} + \bar{F}^2_{21}\xi + \bar{F}^2_{22}\eta\Big\} + \cos 2\omega_p t\Big\{\bar{F}^3_{21}\dot{\xi} + \bar{F}^3_{22}\dot{\eta} + \bar{F}^4_{21}\xi + \bar{F}^4_{22}\xi\Big\} + \sin 2\omega_r t\Big\{\bar{F}^5_{21}\dot{\xi} + \bar{F}^5_{22}\dot{\eta} + \bar{F}^6_{21}\xi + \bar{F}^6_{22}\eta\Big\} + \cos 2\omega_r t\Big\{\bar{F}^7_{21}\dot{\xi} + \bar{F}^7_{22}\dot{\eta} + \bar{F}^8_{21}\xi + \bar{F}^8_{22}\eta\Big\} + \bar{F}^o_2\Big] \quad (25)$$

The above two first order equations (24) and (25) are transformed into one second order equation by using the following steps: (a) differentiate equation (24) with respect to t; (b) multiply equation (25) by ω; (c) add the resulting equations from steps a and b, and (d) substitute η, $\dot{\eta}$ and $\ddot{\eta}$ in terms of ξ in the resulting equation from step c and neglect ε^2 and higher terms. By introducing damping in terms of modal damping in the resulting equation from step c, the final equation is

$$I\ddot{\xi} + \omega^2\xi = \varepsilon\Big[C\dot{\xi} + \sin 2\omega_p t(P^1\ddot{\xi} + P^2\dot{\xi} + P^3\xi) + \cos 2\omega_p t(P^4\ddot{\xi} + P^5\dot{\xi} + P^6\xi) + \sin 2\omega_r t(P^7\ddot{\xi} + P^8\dot{\xi} + P^9\xi) + \cos 2\omega_r t(P^{10}\ddot{\xi} + P^{11}\dot{\xi} + P^{12}\xi) + \bar{F}_1^o + \omega\bar{F}_2^o\Big] \quad (26)$$

where

$$C = \begin{bmatrix} -2\bar{\delta}_1\omega_1 & 0 & 0 \\ 0 & -2\bar{\delta}_2\omega_2 & 0 \\ 0 & 0 & -2\bar{\delta}_3\omega_3 \end{bmatrix} \quad (27)$$

$\delta_j = \varepsilon\bar{\delta}_j$ = critical dampings of the jth mode, j=1,2,3

$$\begin{aligned}
P^1 &= \bar{F}^1_{11} \\
P^2 &= \bar{F}^2_{11} - 2\omega_p\bar{F}^3_{11} + \omega\bar{F}^1_{21} - \omega\bar{F}^1_{12} + \omega^{-1}(-2\omega_p\bar{F}^4_{12} + \omega\bar{F}^2_{22}) \\
P^3 &= -2\omega_p\bar{F}^4_{11} + \omega\bar{F}^2_{21} - \omega(\bar{F}^2_{12} - 2\omega_p\bar{F}^3_{12} + \omega\bar{F}^1_{22}) \\
P^4 &= \bar{F}^3_{11} \\
P^5 &= 2\omega_p F^1_{11} + \bar{F}^4_{11} + \omega F^3_{21} + \omega^{-1}(2\omega_p\bar{F}^2_{12} + \omega\bar{F}^4_{22}) - \omega\bar{F}^3_{12} \\
P^6 &= 2\omega_p\bar{F}^2_{11} + \omega\bar{F}^4_{21} - \omega(2\omega_p\bar{F}^1_{12} + \bar{F}^4_{12} + \omega\bar{F}^3_{22}) \\
P^7 &= \bar{F}^5_{11} \\
P^8 &= \bar{F}^6_{11} - 2\omega_r\bar{F}^7_{11} + \omega\bar{F}^5_{21} - \omega\bar{F}^5_{12} + \omega^{-1}(-2\omega_r\bar{F}^8_{12} + \omega F^6_{22}) \\
P^9 &= -2\omega_r\bar{F}^8_{11} + \omega\bar{F}^6_{21} - \omega(\bar{F}^6_{12} - 2\omega_r\bar{F}^7_{12} + \omega\bar{F}^5_{22}) \\
P^{10} &= \bar{F}^7_{11} \\
P^{11} &= \bar{F}^8_{11} + 2\omega_r\bar{F}^5_{11} + \omega\bar{F}^7_{21} - \omega\bar{F}^7_{12} + \omega^{-1}(2\omega_r\bar{F}^6_{12} + \omega\bar{F}^8_{22}) \\
P^{12} &= 2\omega_r\bar{F}^6_{11} + \omega\bar{F}^8_{21} - \omega(\bar{F}^8_{12} + 2\omega_r\bar{F}^5_{12} + \omega\bar{F}^7_{22})
\end{aligned} \quad (28)$$

Instability Regions

In the right hand side of Eq. (26), the last two terms produce forced vibrations and the other terms generate parametric resonances. Hsu[9] has carried out the first approximation analysis of similar equations and determined the instability criteria explicitly. The instability may occur for the following reasonance conditions:

$$\omega_p \simeq \frac{\omega_j + \omega_k}{2} \quad (29)$$

or $$\omega_r \simeq \frac{\omega_j + \omega_k}{2}, \; j, \; k=1,2,3 \quad (30)$$

The stability conditions for these resonances are as follows:

<u>Case I</u>

$$\omega_p = \frac{\omega_j + \omega_k}{2} + \varepsilon\bar{\lambda}, \; j = k \quad (31)$$

where $\varepsilon\bar{\lambda} = \lambda$

The spacecraft will be unstable if

$$\left[\frac{\nu + (\nu^2 + \beta^2)^{1/2}}{2}\right]^{1/2} > 2(\bar{\delta}_k\omega_k + \bar{\delta}_j\omega_j) \quad (32)$$

where

$$\nu = 16\bar{\lambda}^2 + \frac{1}{\omega_k\omega_j}(d_1d_2 + d_3d_4) \quad (33)$$

$$\beta = \frac{1}{\omega_k\omega_j}(d_3d_2 - d_1d_4) \quad (34)$$

$$\begin{aligned}
d_1 &= P^6_{kj} - P^4_{kj}\omega_j^2 - P^2_{kj}\omega_j \\
d_2 &= P^6_{jk} - P^4_{jk}\omega_k^2 - P^2_{jk}\omega_k \\
d_3 &= P^3_{kj} - P^1_{kj}\omega_j^2 + P^5_{kj}\omega_j \\
d_4 &= P^3_{jk} - P^1_{jk}\omega_k^2 + P^5_{jk}\omega_k
\end{aligned} \quad (35)$$

<u>Case II</u>

$$\omega_r = \frac{\omega_j + \omega_k}{2} + \varepsilon\bar{\lambda}, \; j = k \quad (36)$$

where $\varepsilon\bar{\lambda} = \lambda$

The spacecraft will be unstable if

$$\left[\frac{\nu + (\nu^2 + \beta^2)^{1/2}}{2}\right]^{1/2} > 2(\bar{\delta}_k\omega_k + \bar{\delta}_j\omega_j) \quad (37)$$

where

$$\begin{aligned}
\nu &= 16\bar{\lambda}^2 + \frac{1}{\omega_j\omega_k}(d_5d_6 + d_7d_8) \\
\beta &= \frac{1}{\omega_k\omega_j}(d_7d_6 - d_5d_8) \\
d_5 &= P^{12}_{kj} - P^{10}_{kj}\omega_j^2 - P^8_{kj}\omega_j \\
d_6 &= P^{12}_{jk} - P^{10}_{jk}\omega_k^2 - P^8_{jk}\omega_k \\
d_7 &= P^9_{kj} - P^7_{kj}\omega_j^2 + P^{11}_{kj}\omega_j \\
d_8 &= P^9_{jk} - P^7_{jk}\omega_k^2 + P^{11}_{jk}\omega_k
\end{aligned} \quad (38)$$

Case III

$$\omega_p \simeq \omega_k \quad (k = 1,2,3) \qquad (39)$$

The spacecraft will be unstable if

$$\omega_k + \Delta > \omega_r > \omega_k - \Delta \qquad (40)$$

where

$$\Delta = \frac{\varepsilon}{4\omega_k}\left[(P^{6}_{kk} - P^{4}_{kk}\omega_k^{2} - P^{2}_{kk}\,\omega_k)^2 + (P^{3}_{kk} - P^{1}_{kk}\omega_k^{2} + P^{5}_{kk}\omega_k)^2 - 16\,\bar{\delta}_k^2\omega_k^4\right]^{1/2} \qquad (41)$$

Case IV

$$\omega_r \simeq \omega_k \quad (k = 1,2,3) \qquad (42)$$

The spacecraft will be unstable if

$$\omega_k + \Delta > \omega_r < \omega_k - \Delta \qquad (43)$$

where

$$\Delta = \frac{\varepsilon}{4\omega_k}\left[(P^{12}_{kk} - P^{10}_{kk}\,\omega_k^{2} - P^{8}_{kk}\,\omega_k)^2 + (P^{9}_{kk} - P^{7}_{kk}\omega_k^{2} + P^{11}_{kk}\omega_k)^2 - 16\,\bar{\delta}_k^2\omega_k^4\right]^{1/2} \qquad (44)$$

The above instability conditions are obtained from the first **asymptotic** approximation theory and are therefore called "first approximation instability regions." Another important approximation in the above analysis is in the representation of damping which is introduced as modal damping.

Numerical Examples

In this paper, two numerical examples have been analyzed. The first example represents a dual-spin stabilized spacecraft and the second example represents a body-stabilized spacecraft with a fixed momentum whed.

First Example: Dual-spin stabilized spacecraft.

The following parameters have been used in this example for numerical calculations:

M_p = 274.8 kg	I_p = 120.34 kg.m^2
M_r = 445.1 kg	I_{tr} = 263.47 kg.m^2
ℓ_p = 0.51 m	I_{zr} = 246.92 kg.m^2
ℓ_r = 0.823 m	K_m = 1.44x10^5 N.m/rad
ω_p = 0	δ = Ω=0

These mass properties approximate those of INTELSAT IV Spacecraft. The stiffness, k_m, corresponds to the stiffness of the BAPTA (Bearing and Power Transfer Assembly). The parameters are the same as those used in Reference 6 in order to make a comparison between an exact solution and a first asymptotic approximation.

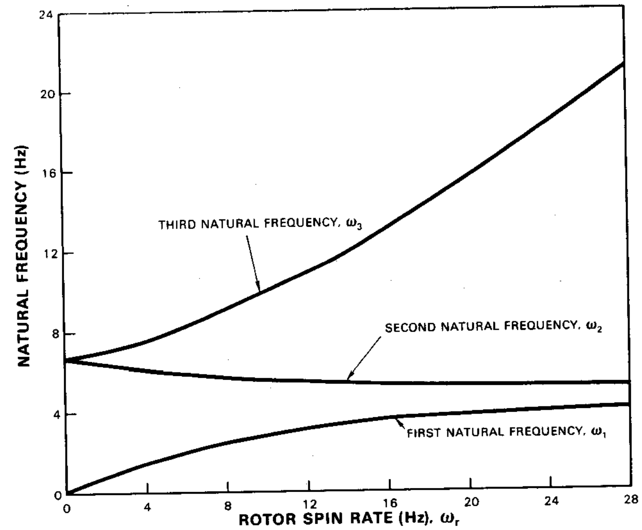

FIGURE 2. NATURAL FREQUENCIES FOR DUAL-SPIN STABILIZED SPACECRAFT

Figure 2 presents the natural frequencies of the system. The first natural frequency is closely associated with the rigid body nutation frequency. It is, however, lower due to joint flexibility. In the second natural mode, the gyroscopic stiffness opposes the structural stiffness because the precession and the spin rate directions are opposite. Hence, the second natural frequency decreases with the increase in the spin rate. In the third natural mode, opposite is the case, i.e., natural frequency increases with the increase in the rotor spin rate.

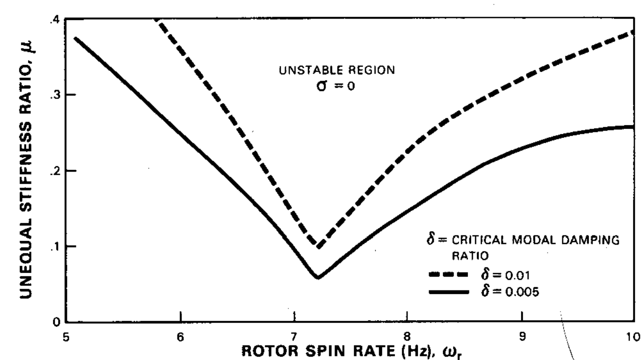

FIGURE 3. UNSTABLE REGION FOR RESONANCE CONDITION $\omega_r = \frac{1}{2}(\omega_2 + \omega_3)$ FOR DUAL-SPIN STABILIZED SPACECRAFT

As discussed in the previous section, the inequality in the transverse principal inertias and the transverse joint stiffness can create attitude instablity for **resonances** ω_p or $\omega_r \simeq 1/2\,(\omega_j + \omega_k)$ or ω_k. In the present example, attitude instability can occur in the neighborhood of ω_r = 3.6 Hz ($\omega_r \simeq 1/2(\omega_1 + \omega_2)$); ω_r = 5.2 Hz ($\omega_r \simeq \omega_2$); ω_r = 7.25 Hz ($\omega_r \simeq 1/2(\omega_2 + \omega_3)$), and ω_r = 10 Hz ($\omega_r \simeq \omega_3$). Figure 3 shows the instability region due to inequality in joint transverse stiffness for resonance conditions $\omega_r \simeq 1/2(\omega_2 + \omega_3)$. In the unstable region, the nutation angle increases exponentially. In Reference 6, the unstable regions for the same parameters are obtained by calculating eigen values of the system. In this paper, however, the possible instability regions are predicted on the basis of resonance conditions and the instability regions are calculated explicitly. The agreement in the current results and the results of Reference 7 is good. The operating rotor spin rate (0.833 Hz) of the INTELSAT IV spacecraft is well within the stable region.

Second Example: Body-stabilized spacecraft

The following parameters have been used in this example for numerical calculations.

M_p = 950 kg	I_p = 1675 kg-m^2
M_r = 10 kg	I_{tr} = 0.038 kg-m^2
ℓ_p = 0	I_{zr} = 0.060 kg-m^2
ℓ_r = 0	K_m = 902 N.m/rad
$\omega_p = \delta = \Omega = 0$	α = 0.2

The mass properties are close to those of INTELSAT V spacecraft. The momentum wheel is, however, assumed to have more flexible bearings, magnetic bearings instead of ball bearings.

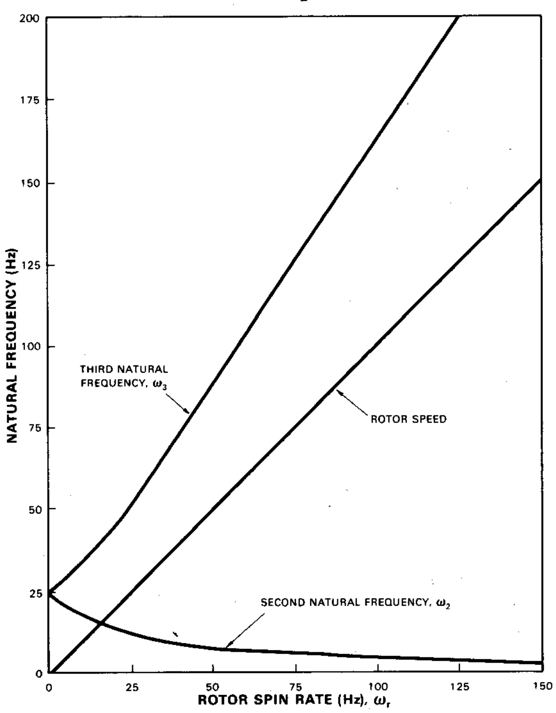

FIGURE 4. NATURAL FREQUENCIES OF BODY-STABILIZED SPACECRAFT

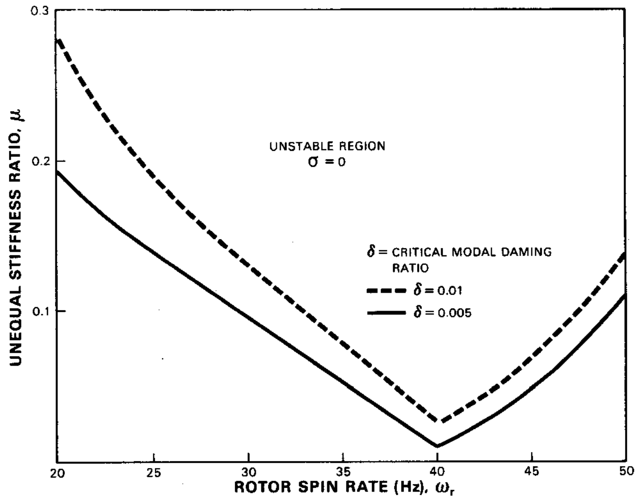

FIGURE 5. UNSTABLE REGION FOR RESONANCE CONDITION $\omega_r = 1/2\,(\omega_2 + \omega_3)$ FOR BODY-STABILIZED SPACECRAFT

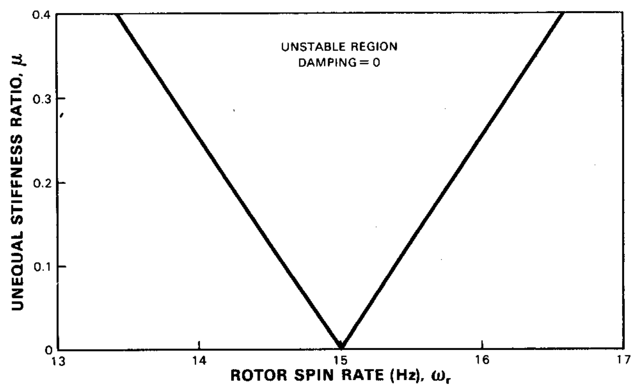

FIGURE 6. UNSTABLE REGION FOR RESONANCE CONDITION $\omega_r = \omega_2$ FOR BODY-STABILIZED SPACECRAFT

Figure 4 presents the natural frequencies of the system. The first natural frequency, nutation frequency, is several orders of magnitude lower than the other natural frequencies. The possible resonances are at ω_r = 9 Hz ($\omega_r \simeq 1/2(\omega_1 + \omega_2)$); ω_r = 15 Hz ($\omega_r \simeq \omega_2$); ω_r = 25 Hz ($\omega_r \simeq 1/2(\omega_3 + \omega_1)$) and ω_r = 40 Hz ($\omega_r \simeq 1/2(\omega_2 + \omega_3)$). Figures 5 and 6 show instability regions due to inequality in transverse joint stiffness for resonances $\omega_r \simeq 1/2(\omega_2 + \omega_3)$ and $\omega_r \simeq \omega_2$, respectively. The nominal operating speed is 130 Hz (7,800 RPM). So during spin-up the momentum wheel passes through these instability regions.

Summary and Conclusions

The equations of motion of an assymmetric dual-spin spacecraft are presented. An asymmetric rigid platform is assumed to be connected to an asymmetric and statically and dynamically unbalanced rigid rotor by a flexible joint. The resulting equations are the equations of motion of a multiple-degree-of-freedom gyroscopic system with periodic coefficients. The periodic coefficients are contributed by inequality in the rotor transverse principal inertias, inequality in the platform transverse principal inertias, and the transverse stiffness inequality of the flexible joint. The gyroscopic equations are normalized by using the Meirovitch method. The possible resonances and the associated instability conditions are obtained by using asymptotic approximation. The resonances occur when either the platform spin frequency or the rotor spin frequency is in the neighborhood of any natural frequency or in the neighborhood of half the sum of any two natural frequencies. The stability criteria for these resonances are given explicitly. In the analysis, damping is introduced as modal damping. A more detailed analysis should represent damping in terms of rotor damping, platform damping, and joint damping.

Two numerical examples, representing a dual-spin stabilized and a body-stabilized spacecraft with fixed magnetic bearing momentum wheel, have been analyzed. The natural frequencies, possible resonances and instability regions have been determined for these examples. For the dual-spin stabilized spacecraft, the operating rotor spin rate is well below the unstable region. In the body-stabilized spacecraft, the momentum wheel passes through the instability regions during spin-up. Hence, asymmetries in magnetic bearing stiffness can cause attitude instability.

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