Supplementary Materials: Multivariate Rational Approximation

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SM1. Number of Pole-like Points and Errors

In Fig. 7, we showed the average number of pole-like points found over all functions in Table A.5 in each rational approximation for different noise levels when the interpolation data was sampled using d-LHS. Additionally, in Fig. 7, we compared the quality of the four approximation approaches for five typical test functions from Table A.5 whose interpolation data is sampled using d-LHS. We summarized these results over all functions in Table 2. In this section we present detailed results of the pole-like points and error (due to poles, not due to poles and total testing error) found in the approximation approaches of all the functions from Table A.5. These detailed results are given for when the interpolation data is sampled using the three strategies of SG, LHS, d-LHD. The results are presented for noise-free data ($\epsilon = 0$) as well as for data with the relative noise level of $\epsilon = 10^{-6}$ and $\epsilon = 10^{-2}$.

The number of pole-like points and error results for approximating noise-free interpolation data of all functions from Table A.5 are shown in Table SM1.1. In this table, $p(x)$ is the polynomial approximation, (b) $r_1(x)$ is the rational approximation using Algorithm 3.1 without degree reduction, (c) $r_2(x)$ is the rational approximation using Algorithm 3.1 with the degree reduction described in Algorithm 3.3, and (d) $r_3(x)$ is the rational approximation using Algorithm 4.1. The number of pole-like points found on the face and inside the domain (see Eq. (17)) is given. The error due to poles (see Eq. (18)), not due to poles (see Eq. (19)), and total testing error (see Eq. (13)) is also given. Because pole-like points and errors related to these points are only applicable for rational approximations, we only give these results for $r_1(x), r_2(x)$, and $r_3(x)$, and put a "'-" in its place for $p(x)$ instead. *Function No.* is the number of the function from Table A.5. *Sample Type* is the name of the sampling strategy used to pick interpolation points from the domain. *AM* is the arithmetic mean and *MD* is the median of the results from running the experiment over LHS and d-LHD samples with different random seeds. Since SG is deterministic in picking points from the domain, a "-" is placed for *MD* whenever the sampling strategy is SG. Similarly, the timing and iteration results for approximating noisy data of $\epsilon = 10^{-6}$ and $\epsilon = 10^{-2}$ is given in Table SM1.2 and Table SM1.3, respectively.

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Table SM1.1: Number of pole-like points found on the face and inside the domain as well as errors found in all four approximations of functions from Table A.5. The interpolation data for these functions are noise-free, i.e., $\epsilon = 0$. Number of pole-like points and the error due to poles and not due to poles are given for $r_1(x), r_2(x)$, and $r_3(x)$ only. A "-" is used in its place for $p(x)$. The number of pole-like points, error due to poles and not due to poles is given for the threshold value of $t = 10^2$. *Function No.* is the number of the function from Table A.5. *Sample Type* is the name of the sampling strategy used to pick interpolation points from the domain. *AM* is the arithmetic mean and *MD* is the median. Since SG is deterministic in picking points from the domain, a "-" is placed for *MD* whenever the sampling strategy is SG.

			$\epsilon = 0$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.1	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	4.64	-	4.64	-	5.30E+02	-	-	-
		Δ_r	4.64	-	4.64	-	5.30E+02	-	71.33	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0.60	0	0.60	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.20	0	0.20	0	0	0	-	-
		$E_{r,10^2}$	1.84E+03	0	1.84E+03	0	0	0	-	-
		$E'_{r,10^2}$	1.05E+03	8.76E+02	1.05E+03	8.76E+02	1.02E+02	1.01E+02	-	-
		Δ_r	2.35E+03	1.02E+03	2.35E+03	1.02E+03	1.02E+02	1.01E+02	2.20E+02	1.99E+02
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.80	2	1.80	2	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	1.10E+04	2.98E+03	1.10E+04	2.98E+03	0	0	-	-
		$E'_{r,10^2}$	1.58E+03	1.34E+03	1.58E+03	1.34E+03	9.73	8.13	-	-
		Δ_r	1.13E+04	3.27E+03	1.13E+04	3.27E+03	9.73	8.13	94.71	91.86
A.5.2	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	0.29	-	0.29	-	1.88E+02	-	-	-
		Δ_r	0.29	-	0.29	-	1.88E+02	-	8.53	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	2.20	1	2.20	1	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	1.52E+03	1.12E+02	1.52E+03	1.12E+02	0	0	-	-
		$E'_{r,10^2}$	82.22	34.11	82.22	34.11	5.36	4.70	-	-
		Δ_r	1.53E+03	1.24E+02	1.53E+03	1.24E+02	5.36	4.70	27.85	25.42
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0.20	0	0.20	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.20	0	0.20	0	0	0	-	-
		$E_{r,10^2}$	8.44E+03	0	8.44E+03	0	0	0	-	-
		$E'_{r,10^2}$	62.36	53.56	62.36	53.56	1.28	1.06	-	-
		Δ_r	8.47E+03	60.38	8.47E+03	60.38	1.28	1.06	15.15	15.27
A.5.3	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	3.68	-	3.68	-	4.66E+02	-	-	-
		Δ_r	3.68	-	3.68	-	4.66E+02	-	1.76E+02	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	6.92	4.67	6.92	4.67	4.63	4.02	-	-
		Δ_r	6.92	4.67	6.92	4.67	4.63	4.02	1.68E+03	1.52E+03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	4.69	4.29	4.69	4.29	5.84	6.81	-	-
		Δ_r	4.69	4.29	4.69	4.29	5.84	6.81	2.76E+02	3.00E+02

			$\epsilon = 0$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.4	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	2.12E-10	-	2.12E-10	-	1.52E+02	-	-	-
		Δ_r	2.12E-10	-	2.12E-10	-	1.52E+02	-	4.56E-12	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	4.99E-10	4.82E-10	4.99E-10	4.82E-10	2.65E-06	3.06E-06	-	-
		Δ_r	4.99E-10	4.82E-10	4.99E-10	4.82E-10	2.65E-06	3.06E-06	1.91E-12	1.62E-12
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.83E-10	1.64E-10	1.83E-10	1.64E-10	1.61E-07	1.16E-07	-	-
		Δ_r	1.83E-10	1.64E-10	1.83E-10	1.64E-10	1.61E-07	1.16E-07	1.03E-12	1.05E-12
A.5.5	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	9.28	-	9.28	-	3.38E+02	-	-	-
		Δ_r	9.28	-	9.28	-	3.38E+02	-	10.76	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0.40	0	0.40	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.40	2	1.40	2	0	0	-	-
		$E_{r,10^2}$	8.73E+03	2.65E+03	8.73E+03	2.65E+03	0	0	-	-
		$E'_{r,10^2}$	9.01E+02	8.11E+02	9.01E+02	8.11E+02	14.65	13.65	-	-
		Δ_r	8.97E+03	2.77E+03	8.97E+03	2.77E+03	14.65	13.65	44.84	45.07
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.60	1	0.60	1	0	0	-	-
		$E_{r,10^2}$	5.71E+02	8.45E+02	5.71E+02	8.45E+02	0	0	-	-
		$E'_{r,10^2}$	8.55E+02	8.53E+02	8.55E+02	8.53E+02	5.17	4.71	-	-
		Δ_r	1.13E+03	1.17E+03	1.13E+03	1.17E+03	5.17	4.71	12.59	10.69
A.5.6	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.09E-10	-	1.09E-10	-	1.14E+02	-	-	-
		Δ_r	1.09E-10	-	1.09E-10	-	1.14E+02	-	0.03	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.93E-10	1.68E-10	1.93E-10	1.68E-10	2.61E-07	2.09E-08	-	-
		Δ_r	1.93E-10	1.68E-10	1.93E-10	1.68E-10	2.61E-07	2.09E-08	0.13	0.10
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.71E-07	8.43E-11	1.71E-07	8.43E-11	2.96E-08	3.14E-08	-	-
		Δ_r	1.71E-07	8.43E-11	1.71E-07	8.43E-11	2.96E-08	3.14E-08	0.05	0.05

			$\epsilon = 0$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.7	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	8.31E-09	-	8.31E-09	-	4.49E+03	-	-	-
		Δ_r	8.31E-09	-	8.31E-09	-	4.49E+03	-	1.39E+03	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	3.82E-08	8.04E-09	3.82E-08	8.04E-09	1.69E-04	9.92E-07	-	-
		Δ_r	3.82E-08	8.04E-09	3.82E-08	8.04E-09	1.69E-04	9.92E-07	3.49E+03	3.25E+03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.96E-08	2.10E-08	1.96E-08	2.10E-08	1.60E-06	1.48E-09	-	-
		Δ_r	1.96E-08	2.10E-08	1.96E-08	2.10E-08	1.60E-06	1.48E-09	2.34E+03	2.02E+03
A.5.8	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	2.84E-08	-	2.84E-08	-	1.18E+04	-	-	-
		Δ_r	2.84E-08	-	2.84E-08	-	1.18E+04	-	1.17E+04	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	5.30E-06	1.54E-08	5.30E-06	1.54E-08	0.03	0.02	-	-
		Δ_r	5.30E-06	1.54E-08	5.30E-06	1.54E-08	0.03	0.02	2.18E+04	2.15E+04
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.54E-08	1.50E-09	1.54E-08	1.50E-09	7.99E-05	6.01E-07	-	-
		Δ_r	1.54E-08	1.50E-09	1.54E-08	1.50E-09	7.99E-05	6.01E-07	1.95E+04	1.93E+04
A.5.9	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	2.74E-08	-	2.74E-08	-	2.23E+02	-	-	-
		Δ_r	2.74E-08	-	2.74E-08	-	2.23E+02	-	19.50	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	3.37E-09	3.60E-09	3.37E-09	3.60E-09	4.64E-04	5.15E-08	-	-
		Δ_r	3.37E-09	3.60E-09	3.37E-09	3.60E-09	4.64E-04	5.15E-08	51.94	52.33
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.70E-08	8.98E-09	1.70E-08	8.98E-09	3.34E-04	2.00E-04	-	-
		Δ_r	1.70E-08	8.98E-09	1.70E-08	8.98E-09	3.34E-04	2.00E-04	32.50	32.78

			$\epsilon = 0$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.10	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.84E-09	-	1.84E-09	-	84.36	-	-	-
		Δ_r	1.84E-09	-	1.84E-09	-	84.36	-	0.65	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	9.13E-11	7.64E-11	9.13E-11	7.64E-11	3.48E-09	3.19E-09	-	-
		Δ_r	9.13E-11	7.64E-11	9.13E-11	7.64E-11	3.48E-09	3.19E-09	2.38	2.23
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.77E-10	1.26E-10	1.77E-10	1.26E-10	7.62E-10	6.56E-10	-	-
		Δ_r	1.77E-10	1.26E-10	1.77E-10	1.26E-10	7.62E-10	6.56E-10	1.02	1.05
A.5.11	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	2.02E-11	-	2.02E-11	-	29.50	-	-	-
		Δ_r	2.02E-11	-	2.02E-11	-	29.50	-	0.10	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.39E-10	1.47E-10	1.39E-10	1.47E-10	1.80E-09	1.03E-09	-	-
		Δ_r	1.39E-10	1.47E-10	1.39E-10	1.47E-10	1.80E-09	1.03E-09	0.94	0.65
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.28E-10	8.91E-11	1.28E-10	8.91E-11	2.40E-09	1.29E-09	-	-
		Δ_r	1.28E-10	8.91E-11	1.28E-10	8.91E-11	2.40E-09	1.29E-09	0.21	0.22
A.5.12	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	3.25E-10	-	3.25E-10	-	2.28E+02	-	-	-
		Δ_r	3.25E-10	-	3.25E-10	-	2.28E+02	-	4.71	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	5.90E-11	2.99E-12	5.90E-11	2.99E-12	1.94E-07	1.44E-07	-	-
		Δ_r	5.90E-11	2.99E-12	5.90E-11	2.99E-12	1.94E-07	1.44E-07	24.90	25.44
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	7.66E-09	1.23E-12	7.66E-09	1.23E-12	5.47E-04	2.85E-04	-	-
		Δ_r	7.66E-09	1.23E-12	7.66E-09	1.23E-12	5.47E-04	2.85E-04	8.34	8.52

			$\epsilon = 0$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.13	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	5.19E-10	-	5.19E-10	-	99.75	-	-	-
		Δ_r	5.19E-10	-	5.19E-10	-	99.75	-	3.83	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.42E-10	1.56E-12	1.42E-10	1.56E-12	1.59E-07	1.67E-07	-	-
		Δ_r	1.42E-10	1.56E-12	1.42E-10	1.56E-12	1.59E-07	1.67E-07	17.32	15.09
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.96E-11	6.21E-13	1.96E-11	6.21E-13	7.94E-04	1.79E-07	-	-
		Δ_r	1.96E-11	6.21E-13	1.96E-11	6.21E-13	7.94E-04	1.79E-07	6.89	7.44
A.5.14	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.62E-13	-	1.62E-13	-	64.88	-	-	-
		Δ_r	1.62E-13	-	1.62E-13	-	64.88	-	2.53	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	9.82E-10	8.40E-11	9.82E-10	8.40E-11	1.22E-07	5.97E-08	-	-
		Δ_r	9.82E-10	8.40E-11	9.82E-10	8.40E-11	1.22E-07	5.97E-08	8.34	8.80
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	6.49E-09	8.18E-13	6.49E-09	8.18E-13	4.48E-04	6.34E-09	-	-
		Δ_r	6.49E-09	8.18E-13	6.49E-09	8.18E-13	4.48E-04	6.34E-09	4.17	4.32
A.5.15	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	5.87E-10	-	5.87E-10	-	23.14	-	-	-
		Δ_r	5.87E-10	-	5.87E-10	-	23.14	-	7.71	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	5.91E-10	5.96E-10	5.91E-10	5.96E-10	4.02E-03	4.56E-03	-	-
		Δ_r	5.91E-10	5.96E-10	5.91E-10	5.96E-10	4.02E-03	4.56E-03	33.09	28.69
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	3.83E-10	3.48E-10	3.83E-10	3.48E-10	4.57E-03	4.89E-03	-	-
		Δ_r	3.83E-10	3.48E-10	3.83E-10	3.48E-10	4.57E-03	4.89E-03	9.31	9.30

			$\epsilon = 0$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.16	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.26E+03	-	1.26E+03	-	1.06E+04	-	-	-
		Δ_r	1.26E+03	-	1.26E+03	-	1.06E+04	-	2.61E+04	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	54.60	62	54.60	62	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	4.80	4	4.80	4	0	0	-	-
		$E_{r,10^2}$	8.46E+06	7.76E+06	8.46E+06	7.76E+06	0	0	-	-
		$E'_{r,10^2}$	5.27E+05	5.24E+05	5.27E+05	5.24E+05	4.00E+04	4.06E+04	-	-
		Δ_r	8.48E+06	7.79E+06	8.48E+06	7.79E+06	4.00E+04	4.06E+04	3.61E+04	3.59E+04
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	33	32	33	32	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.60	1	0.60	1	0	0	-	-
		$E_{r,10^2}$	9.63E+06	4.30E+06	9.63E+06	4.30E+06	0	0	-	-
		$E'_{r,10^2}$	4.07E+05	4.16E+05	4.07E+05	4.16E+05	1.50E+04	1.41E+04	-	-
		Δ_r	9.65E+06	4.31E+06	9.65E+06	4.31E+06	1.50E+04	1.41E+04	3.43E+04	3.53E+04
A.5.17	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	55.65	-	55.65	-	70.38	-	-	-
		Δ_r	55.65	-	55.65	-	70.38	-	6.49	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	2.20	1	2.20	1	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.20	1	1.20	1	0	0	-	-
		$E_{r,10^2}$	2.42E+03	4.32E+02	2.42E+03	4.32E+02	0	0	-	-
		$E'_{r,10^2}$	1.68E+02	1.43E+02	1.68E+02	1.43E+02	2.22	2.13	-	-
		Δ_r	2.47E+03	4.53E+02	2.47E+03	4.53E+02	2.22	2.13	10.22	9.91
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	2.40	2	2.40	2	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2.20	2	2.20	2	0	0	-	-
		$E_{r,10^2}$	2.52E+03	1.41E+03	2.52E+03	1.41E+03	0	0	-	-
		$E'_{r,10^2}$	1.73E+02	1.72E+02	1.73E+02	1.72E+02	1.43	1.39	-	-
		Δ_r	2.56E+03	1.42E+03	2.56E+03	1.42E+03	1.43	1.39	8.04	8.02
A.5.18	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	3.39E+02	-	3.39E+02	-	3.19E+02	-	-	-
		Δ_r	3.39E+02	-	3.39E+02	-	3.19E+02	-	2.70E+02	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	2.40	2	2.40	2	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2.40	2	2.40	2	0	0	-	-
		$E_{r,10^2}$	1.88E+04	8.35E+03	1.88E+04	8.35E+03	0	0	-	-
		$E'_{r,10^2}$	1.96E+03	1.89E+03	1.96E+03	1.89E+03	3.47E+02	3.50E+02	-	-
		Δ_r	1.91E+04	8.48E+03	1.91E+04	8.48E+03	3.47E+02	3.50E+02	5.14E+02	4.84E+02
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	3.20	3	3.20	3	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2.60	3	2.60	3	0	0	-	-
		$E_{r,10^2}$	3.15E+04	1.35E+04	3.15E+04	1.35E+04	0	0	-	-
		$E'_{r,10^2}$	2.11E+03	2.05E+03	2.11E+03	2.05E+03	3.08E+02	3.02E+02	-	-
		Δ_r	3.20E+04	1.36E+04	3.20E+04	1.36E+04	3.08E+02	3.02E+02	4.48E+02	4.17E+02

			$\epsilon = 0$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.19	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	6	-	6	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	145	-	145	-	0	-	-	-
		$E_{r,10^2}$	1.76E+05	-	1.76E+05	-	0	-	-	-
		$E'_{r,10^2}$	1.24E+04	-	1.24E+04	-	1.17E+03	-	-	-
		Δ_r	1.77E+05	-	1.77E+05	-	1.17E+03	-	3.61E+02	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	182	141	182	141	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	182	148	182	148	0	0	-	-
		$E_{r,10^2}$	8.69E+06	7.21E+05	8.69E+06	7.21E+05	0	0	-	-
		$E'_{r,10^2}$	1.96E+04	1.78E+04	1.96E+04	1.78E+04	1.10E+03	1.12E+03	-	-
		Δ_r	8.69E+06	7.21E+05	8.69E+06	7.21E+05	1.10E+03	1.12E+03	2.42E+03	2.29E+03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.30E+02	62	1.30E+02	62	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.51E+02	141	1.51E+02	141	0	0	-	-
		$E_{r,10^2}$	2.53E+06	3.74E+05	2.53E+06	3.74E+05	0	0	-	-
		$E'_{r,10^2}$	1.60E+04	1.47E+04	1.60E+04	1.47E+04	7.46E+02	7.38E+02	-	-
		Δ_r	2.53E+06	3.74E+05	2.53E+06	3.74E+05	7.46E+02	7.38E+02	3.98E+02	4.05E+02
A.5.20	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	2.67E-09	-	2.67E-09	-	2.62E+02	-	-	-
		Δ_r	2.67E-09	-	2.67E-09	-	2.62E+02	-	8.40E-12	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	5.88E-10	6.31E-10	5.88E-10	6.31E-10	8.81E-09	6.44E-09	-	-
		Δ_r	5.88E-10	6.31E-10	5.88E-10	6.31E-10	8.81E-09	6.44E-09	9.47E-12	7.28E-12
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	7.25E-10	9.56E-10	7.25E-10	9.56E-10	4.69E-06	2.08E-06	-	-
		Δ_r	7.25E-10	9.56E-10	7.25E-10	9.56E-10	4.69E-06	2.08E-06	2.34E-12	2.33E-12

Table SM1.2: Number of pole-like points found on the face and inside the domain as well as errors found in all four approximations of functions from Table A.5. The interpolation data for these functions have a relative noise level of $\epsilon = 10^{-6}$. Number of pole-like points and the error due to poles and not due to poles are given for $r_1(x), r_2(x)$, and $r_3(x)$ only. A "-" is used in its place for $p(x)$. The number of pole-like points, error due to poles and not due to poles is given for the threshold value of $t = 10^2$. *Function No.* is the number of the function from Table A.5. *Sample Type* is the name of the sampling strategy used to pick interpolation points from the domain. *AM* is the arithmetic mean and *MD* is the median. Since SG is deterministic in picking points from the domain, a "-" is placed for *MD* whenever the sampling strategy is SG.

			$\epsilon = 10^{-6}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.1	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	9.14	-	9.14	-	5.30E+02	-	-	-
		Δ_r	9.14	-	9.14	-	5.30E+02	-	71.33	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.40	1	1.40	1	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.40	0	0.40	0	0	0	-	-
		$E_{r,10^2}$	1.20E+04	8.30E+03	1.20E+04	8.30E+03	0	0	-	-
		$E'_{r,10^2}$	1.28E+03	9.11E+02	1.28E+03	9.11E+02	1.02E+02	1.00E+02	-	-
		Δ_r	1.21E+04	8.31E+03	1.21E+04	8.31E+03	1.02E+02	1.00E+02	2.20E+02	1.99E+02
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	1	0	1	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.40	0	0.40	0	0	0	-	-
		$E_{r,10^2}$	2.17E+03	0	2.17E+03	0	0	0	-	-
		$E'_{r,10^2}$	1.85E+03	2.01E+03	1.85E+03	2.01E+03	9.72	8.13	-	-
		Δ_r	3.20E+03	2.01E+03	3.20E+03	2.01E+03	9.72	8.13	94.71	91.86
A.5.2	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	0.29	-	0.29	-	1.88E+02	-	-	-
		Δ_r	0.29	-	0.29	-	1.88E+02	-	8.53	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	1.20	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	1.20	0	0	0	-	-
		$E_{r,10^2}$	0	0	4.96E+02	0	0	0	-	-
		$E'_{r,10^2}$	39.37	18.22	91.49	16.17	5.36	4.70	-	-
		Δ_r	39.37	18.22	5.09E+02	16.17	5.36	4.70	27.85	25.42
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0.60	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.20	0	0	0	0	0	-	-
		$E_{r,10^2}$	3.29E+02	0	0	0	0	0	-	-
		$E'_{r,10^2}$	73.46	49.32	1.12	0.98	1.29	1.05	-	-
		Δ_r	3.66E+02	1.06E+02	1.12	0.98	1.29	1.05	15.15	15.27
A.5.3	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	3.68	-	3.68	-	4.66E+02	-	-	-
		Δ_r	3.68	-	3.68	-	4.66E+02	-	1.76E+02	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	23.40	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	80.40	37	0	0	-	-
		$E_{r,10^2}$	0	0	2.83E+05	9.45E+03	0	0	-	-
		$E'_{r,10^2}$	6.90	4.67	7.15E+02	7.89E+02	4.62	4.02	-	-
		Δ_r	6.90	4.67	2.83E+05	9.49E+03	4.62	4.02	1.68E+03	1.52E+03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	6.38E+02	543	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	2.05E+03	1921	0	0	-	-
		$E_{r,10^2}$	0	0	4.76E+05	5.40E+05	0	0	-	-
		$E'_{r,10^2}$	4.69	4.29	5.14E+03	5.64E+03	5.84	6.81	-	-
		Δ_r	4.69	4.29	4.76E+05	5.40E+05	5.84	6.81	2.76E+02	3.00E+02

			$\epsilon = 10^{-6}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.4	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.85	-	4.03E-04	-	1.52E+02	-	-	-
		Δ_r	1.85	-	4.03E-04	-	1.52E+02	-	7.25E-04	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.11	0.75	9.01E-04	8.06E-04	3.76E-03	3.01E-03	-	-
		Δ_r	1.11	0.75	9.01E-04	8.06E-04	3.76E-03	3.01E-03	8.80E-03	5.65E-03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	2.01	0.65	6.57E-04	6.61E-04	1.61E-03	1.76E-03	-	-
		Δ_r	2.01	0.65	6.57E-04	6.61E-04	1.61E-03	1.76E-03	1.59E-03	1.59E-03
A.5.5	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	16.98	-	2.44E+03	-	3.38E+02	-	-	-
		Δ_r	16.98	-	2.44E+03	-	3.38E+02	-	10.75	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0.20	0	1.45E+02	104	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.60	0	2.28E+02	221	0	0	-	-
		$E_{r,10^2}$	9.14E+02	0	2.49E+06	8.86E+05	0	0	-	-
		$E'_{r,10^2}$	9.17E+02	8.90E+02	1.43E+04	1.53E+04	14.66	13.67	-	-
		Δ_r	1.56E+03	1.10E+03	2.49E+06	8.86E+05	14.66	13.67	44.84	45.06
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	58.60	26	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.60	2	47.20	21	0	0	-	-
		$E_{r,10^2}$	1.85E+03	2.55E+03	6.21E+05	8.96E+04	0	0	-	-
		$E'_{r,10^2}$	8.90E+02	7.22E+02	7.93E+03	5.50E+03	5.17	4.71	-	-
		Δ_r	2.19E+03	2.64E+03	6.21E+05	8.98E+04	5.17	4.71	12.59	10.69
A.5.6	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	7.57	-	1.78E+02	-	1.14E+02	-	-	-
		Δ_r	7.57	-	1.78E+02	-	1.14E+02	-	0.03	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.20	0	0	0	0	0	-	-
		$E_{r,10^2}$	59.08	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.25	0.97	0.44	9.41E-04	2.47E-03	2.31E-03	-	-
		Δ_r	60.17	0.99	0.44	9.41E-04	2.47E-03	2.31E-03	0.13	0.10
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	0.32	0.29	1.05E+02	19.71	1.39E-03	1.26E-03	-	-
		Δ_r	0.32	0.29	1.05E+02	19.71	1.39E-03	1.26E-03	0.05	0.05

			$\epsilon = 10^{-6}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.7	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	79	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	54	-	0	-	-	-
		$E_{r,10^2}$	0	-	4.52E+06	-	0	-	-	-
		$E'_{r,10^2}$	0.75	-	9.73E+04	-	4.49E+03	-	-	-
		Δ_r	0.75	-	4.52E+06	-	4.49E+03	-	1.39E+03	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	19.80	8.93	0.06	0.05	0.85	0.28	-	-
		Δ_r	19.80	8.93	0.06	0.05	0.85	0.28	3.49E+03	3.25E+03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	23.72	11.02	3.81E-03	3.43E-03	0.94	7.98E-03	-	-
		Δ_r	23.72	11.02	3.81E-03	3.43E-03	0.94	7.98E-03	2.34E+03	2.02E+03
A.5.8	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	0.03	-	1.66E+04	-	1.18E+04	-	-	-
		Δ_r	0.03	-	1.66E+04	-	1.18E+04	-	1.17E+04	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	8.80	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	4.20	0	0	0	-	-
		$E_{r,10^2}$	0	0	1.53E+06	0	0	0	-	-
		$E'_{r,10^2}$	5.87E+02	1.86E+02	5.88E+04	0.91	76.54	6.38	-	-
		Δ_r	5.87E+02	1.86E+02	1.53E+06	0.91	76.54	6.38	2.18E+04	2.15E+04
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	6.39	1.05	0.03	0.03	0.14	0.14	-	-
		Δ_r	6.39	1.05	0.03	0.03	0.14	0.14	1.95E+04	1.93E+04
A.5.9	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	4.48	-	5.06E-04	-	2.23E+02	-	-	-
		Δ_r	4.48	-	5.06E-04	-	2.23E+02	-	19.49	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	1.34E+02	38.93	2.03E-03	2.20E-03	0.01	0.01	-	-
		Δ_r	1.34E+02	38.93	2.03E-03	2.20E-03	0.01	0.01	51.94	52.33
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	2.13	2.45	7.89E-04	8.11E-04	4.86E-03	4.95E-03	-	-
		Δ_r	2.13	2.45	7.89E-04	8.11E-04	4.86E-03	4.95E-03	32.50	32.78

			$\epsilon = 10^{-6}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.10	SG	$\lvert W_{r,10^2}^{(fc)}\rvert$	0	-	135	-	0	-	-	-
		$\lvert W_{r,10^2}^{(in)}\rvert$	0	-	1850	-	0	-	-	-
		$E_{r,10^2}$	0	-	2.04E+05	-	0	-	-	-
		$E'_{r,10^2}$	0.14	-	4.40E+03	-	84.36	-	-	-
		Δ_r	0.14	-	2.04E+05	-	84.36	-	0.65	-
	LHS	$\lvert W_{r,10^2}^{(fc)}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W_{r,10^2}^{(in)}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	0.41	0.26	0.08	9.16E-03	8.96E-04	9.05E-04	-	-
		Δ_r	0.41	0.26	0.08	9.16E-03	8.96E-04	9.05E-04	2.38	2.23
	d-LHD	$\lvert W_{r,10^2}^{(fc)}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W_{r,10^2}^{(in)}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	0.62	0.20	1.77E-04	1.59E-04	2.67E-04	2.33E-04	-	-
		Δ_r	0.62	0.20	1.77E-04	1.59E-04	2.67E-04	2.33E-04	1.02	1.05
A.5.11	SG	$\lvert W_{r,10^2}^{(fc)}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W_{r,10^2}^{(in)}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	0.03	-	2.03E+02	-	29.50	-	-	-
		Δ_r	0.03	-	2.03E+02	-	29.50	-	0.10	-
	LHS	$\lvert W_{r,10^2}^{(fc)}\rvert$	0	0	163	157	0	0	-	-
		$\lvert W_{r,10^2}^{(in)}\rvert$	0	0	125	139	0	0	-	-
		$E_{r,10^2}$	0	0	1.61E+05	5.29E+04	0	0	-	-
		$E'_{r,10^2}$	0.47	0.11	1.65E+03	1.64E+03	9.07E-04	8.63E-04	-	-
		Δ_r	0.47	0.11	1.61E+05	5.29E+04	9.07E-04	8.63E-04	0.94	0.65
	d-LHD	$\lvert W_{r,10^2}^{(fc)}\rvert$	0	0	89.20	32	0	0	-	-
		$\lvert W_{r,10^2}^{(in)}\rvert$	0	0	89.40	54	0	0	-	-
		$E_{r,10^2}$	0	0	6.42E+04	4.10E+04	0	0	-	-
		$E'_{r,10^2}$	0.09	0.04	1.29E+03	9.39E+02	5.83E-04	5.02E-04	-	-
		Δ_r	0.09	0.04	6.43E+04	4.10E+04	5.83E-04	5.02E-04	0.21	0.22
A.5.12	SG	$\lvert W_{r,10^2}^{(fc)}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W_{r,10^2}^{(in)}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	8.89E-04	-	36.51	-	2.28E+02	-	-	-
		Δ_r	8.89E-04	-	36.51	-	2.28E+02	-	4.71	-
	LHS	$\lvert W_{r,10^2}^{(fc)}\rvert$	0	0	53.60	0	0	0	-	-
		$\lvert W_{r,10^2}^{(in)}\rvert$	0	0	45.80	0	0	0	-	-
		$E_{r,10^2}$	0	0	3.37E+04	0	0	0	-	-
		$E'_{r,10^2}$	8.93	0.46	1.81E+03	6.25E-03	0.01	6.06E-03	-	-
		Δ_r	8.93	0.46	3.37E+04	6.25E-03	0.01	6.06E-03	24.90	25.45
	d-LHD	$\lvert W_{r,10^2}^{(fc)}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W_{r,10^2}^{(in)}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	0.51	0.16	32.93	6.90E-04	2.62E-03	1.70E-03	-	-
		Δ_r	0.51	0.16	32.93	6.90E-04	2.62E-03	1.70E-03	8.34	8.52

			$\epsilon = 10^{-6}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.13	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	0.36	-	5.34E+02	-	99.75	-	-	-
		Δ_r	0.36	-	5.34E+02	-	99.75	-	3.83	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	8.60	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	8.60	0	0	0	-	-
		$E_{r,10^2}$	0	0	1.33E+04	0	0	0	-	-
		$E'_{r,10^2}$	2.46	0.33	3.79E+02	3.41E-03	5.04E-03	4.19E-03	-	-
		Δ_r	2.46	0.33	1.33E+04	3.41E-03	5.04E-03	4.19E-03	17.32	15.09
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	58.20	57	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	35.60	37	0	0	-	-
		$E_{r,10^2}$	0	0	5.88E+04	5.30E+04	0	0	-	-
		$E'_{r,10^2}$	0.13	0.08	1.97E+03	1.98E+03	6.62E-04	6.53E-04	-	-
		Δ_r	0.13	0.08	5.88E+04	5.30E+04	6.62E-04	6.53E-04	6.89	7.44
A.5.14	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	0.16	-	1.89E-04	-	64.88	-	-	-
		Δ_r	0.16	-	1.89E-04	-	64.88	-	2.53	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	10.80	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	5.20	0	0	0	-	-
		$E_{r,10^2}$	0	0	3.46E+03	0	0	0	-	-
		$E'_{r,10^2}$	4.33	0.17	4.15E+02	1.94E-03	0.01	2.26E-03	-	-
		Δ_r	4.33	0.17	3.48E+03	1.94E-03	0.01	2.26E-03	8.34	8.80
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	0.13	0.04	3.20E-04	2.79E-04	6.43E-04	5.05E-04	-	-
		Δ_r	0.13	0.04	3.20E-04	2.79E-04	6.43E-04	5.05E-04	4.17	4.32
A.5.15	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	0.34	-	5.96E-05	-	23.14	-	-	-
		Δ_r	0.34	-	5.96E-05	-	23.14	-	7.71	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	0.20	0.11	6.41E-05	6.41E-05	1.24E-03	1.55E-03	-	-
		Δ_r	0.20	0.11	6.41E-05	6.41E-05	1.24E-03	1.55E-03	33.09	28.69
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	0.09	0.05	5.46E-05	4.87E-05	4.98E-03	3.92E-03	-	-
		Δ_r	0.09	0.05	5.46E-05	4.87E-05	4.98E-03	3.92E-03	9.31	9.30

			$\epsilon = 10^{-6}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.16	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.26E+03	-	1.26E+03	-	1.06E+04	-	-	-
		Δ_r	1.26E+03	-	1.26E+03	-	1.06E+04	-	2.61E+04	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	54.60	62	55.80	64	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	5	5	5.80	5	0	0	-	-
		$E_{r,10^2}$	7.83E+06	8.23E+06	8.24E+06	8.23E+06	0	0	-	-
		$E'_{r,10^2}$	5.30E+05	5.45E+05	5.47E+05	5.45E+05	4.02E+04	4.03E+04	-	-
		Δ_r	7.85E+06	8.25E+06	8.26E+06	8.25E+06	4.02E+04	4.03E+04	3.61E+04	3.59E+04
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	32.60	32	32.60	32	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.60	1	0.60	1	0	0	-	-
		$E_{r,10^2}$	2.74E+08	4.17E+06	2.74E+08	4.17E+06	0	0	-	-
		$E'_{r,10^2}$	4.10E+05	4.16E+05	4.10E+05	4.16E+05	1.45E+04	1.34E+04	-	-
		Δ_r	2.74E+08	4.19E+06	2.74E+08	4.19E+06	1.45E+04	1.34E+04	3.43E+04	3.53E+04
A.5.17	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	75.93	-	75.93	-	70.38	-	-	-
		Δ_r	75.93	-	75.93	-	70.38	-	6.49	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.40	2	1.40	2	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.60	0	0.60	0	0	0	-	-
		$E_{r,10^2}$	4.10E+02	2.82E+02	4.10E+02	2.82E+02	0	0	-	-
		$E'_{r,10^2}$	1.64E+02	1.53E+02	1.64E+02	1.53E+02	2.22	2.13	-	-
		Δ_r	4.79E+02	3.62E+02	4.79E+02	3.62E+02	2.22	2.13	10.22	9.91
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.20	1	1.20	1	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.80	2	1.80	2	0	0	-	-
		$E_{r,10^2}$	5.94E+02	4.67E+02	5.94E+02	4.67E+02	0	0	-	-
		$E'_{r,10^2}$	1.67E+02	1.64E+02	1.67E+02	1.64E+02	1.43	1.39	-	-
		Δ_r	6.23E+02	5.15E+02	6.23E+02	5.15E+02	1.43	1.39	8.04	8.02
A.5.18	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	3.39E+02	-	3.39E+02	-	3.19E+02	-	-	-
		Δ_r	3.39E+02	-	3.39E+02	-	3.19E+02	-	2.70E+02	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	2.40	2	2	2	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2	1	1.80	1	0	0	-	-
		$E_{r,10^2}$	4.88E+03	4.77E+03	3.93E+03	3.75E+03	0	0	-	-
		$E'_{r,10^2}$	2.04E+03	2.05E+03	1.70E+03	1.78E+03	3.46E+02	3.47E+02	-	-
		Δ_r	5.37E+03	5.19E+03	4.40E+03	4.62E+03	3.46E+02	3.47E+02	5.14E+02	4.84E+02
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	3.40	4	0.20	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2.60	3	0.20	0	0	0	-	-
		$E_{r,10^2}$	7.78E+03	6.75E+03	1.44E+03	0	0	0	-	-
		$E'_{r,10^2}$	2.10E+03	2.18E+03	4.66E+02	3.82E+02	3.10E+02	3.00E+02	-	-
		Δ_r	8.13E+03	6.89E+03	1.72E+03	4.62E+02	3.10E+02	3.00E+02	4.48E+02	4.17E+02

			$\epsilon = 10^{-6}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.19	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	6	-	44	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	144	-	49	-	0	-	-	-
		$E_{r,10^2}$	1.72E+05	-	9.36E+04	-	0	-	-	-
		$E'_{r,10^2}$	1.24E+04	-	9.91E+03	-	1.17E+03	-	-	-
		Δ_r	1.73E+05	-	9.41E+04	-	1.17E+03	-	3.61E+02	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.82E+02	141	1.82E+02	141	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	182	147	182	147	0	0	-	-
		$E_{r,10^2}$	1.42E+06	9.28E+05	1.42E+06	9.28E+05	0	0	-	-
		$E'_{r,10^2}$	1.96E+04	1.77E+04	1.96E+04	1.77E+04	1.10E+03	1.12E+03	-	-
		Δ_r	1.42E+06	9.29E+05	1.42E+06	9.29E+05	1.10E+03	1.12E+03	2.42E+03	2.29E+03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.30E+02	62	47	43	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.50E+02	141	64.20	49	0	0	-	-
		$E_{r,10^2}$	7.85E+07	6.25E+05	2.08E+05	1.60E+05	0	0	-	-
		$E'_{r,10^2}$	1.61E+04	1.47E+04	1.07E+04	9.96E+03	7.46E+02	7.38E+02	-	-
		Δ_r	7.85E+07	6.25E+05	2.08E+05	1.60E+05	7.46E+02	7.38E+02	3.98E+02	4.05E+02
A.5.20	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	2.42	-	1.77E+02	-	2.62E+02	-	-	-
		Δ_r	2.42	-	1.77E+02	-	2.62E+02	-	1.50E-03	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	1.19E+02	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	27	0	0	0	-	-
		$E_{r,10^2}$	0	0	5.04E+05	0	0	0	-	-
		$E'_{r,10^2}$	4.18	1.10	4.22E+03	1.18E+03	6.13E-03	5.89E-03	-	-
		Δ_r	4.18	1.10	5.05E+05	1.18E+03	6.13E-03	5.89E-03	0.01	0.01
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	0	0	0	0	0	0	-	-
		$E'_{r,10^2}$	5.65	5.01	2.17E+02	2.06E+02	2.43E-03	2.18E-03	-	-
		Δ_r	5.65	5.01	2.17E+02	2.06E+02	2.43E-03	2.18E-03	3.29E-03	3.23E-03

Table SM1.3: Number of pole-like points found on the face and inside the domain as well as errors found in all four approximations of functions from Table A.5. The interpolation data for these functions have a relative noise level of $\epsilon = 10^{-2}$. Number of pole-like points and the error due to poles and not due to poles are given for $r_1(x), r_2(x)$, and $r_3(x)$ only. A "-" is used in its place for $p(x)$. The number of pole-like points, error due to poles and not due to poles is given for the threshold value of $t = 10^2$. *Function No.* is the number of the function from Table A.5. *Sample Type* is the name of the sampling strategy used to pick interpolation points from the domain. *AM* is the arithmetic mean and *MD* is the median. Since SG is deterministic in picking points from the domain, a "-" is placed for *MD* whenever the sampling strategy is SG.

			$\epsilon = 10^{-2}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.1	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	1	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	2.06E+03	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.03E+03	-	1.36E+03	-	5.31E+02	-	-	-
		Δ_r	2.30E+03	-	1.36E+03	-	5.31E+02	-	73.22	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	100	103	1.61E+02	157	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	30	19	29	25	0	0	-	-
		$E_{r,10^2}$	1.56E+06	3.73E+05	9.12E+05	5.71E+05	0	0	-	-
		$E'_{r,10^2}$	1.52E+04	1.74E+04	1.70E+04	2.10E+04	1.42E+03	1.16E+03	-	-
		Δ_r	1.56E+06	3.74E+05	9.12E+05	5.72E+05	1.42E+03	1.16E+03	2.68E+02	2.82E+02
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	19.80	2	7.20	10	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	4	2	9.20	10	0	0	-	-
		$E_{r,10^2}$	4.29E+04	1.05E+04	3.60E+04	2.96E+04	0	0	-	-
		$E'_{r,10^2}$	5.48E+03	3.01E+03	4.98E+03	6.06E+03	74.48	43.42	-	-
		Δ_r	4.35E+04	1.10E+04	3.65E+04	3.04E+04	74.48	43.42	96.56	96.42
A.5.2	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	22	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	20	-	4	-	0	-	-	-
		$E_{r,10^2}$	6.44E+03	-	2.89E+02	-	0	-	-	-
		$E'_{r,10^2}$	5.90E+02	-	6.16E+02	-	1.88E+02	-	-	-
		Δ_r	6.47E+03	-	6.80E+02	-	1.88E+02	-	10.73	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	15	4	3.89E+02	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	22.20	20	9.54E+02	0	0	0	-	-
		$E_{r,10^2}$	8.39E+03	2.28E+03	4.21E+05	0	0	0	-	-
		$E'_{r,10^2}$	5.62E+02	5.48E+02	2.73E+03	8.83E+02	34.32	30.95	-	-
		Δ_r	8.44E+03	2.31E+03	4.22E+05	8.83E+02	34.32	30.95	50.56	29.35
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	34.80	36	41.20	28	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	17.40	17	403	347	0	0	-	-
		$E_{r,10^2}$	1.80E+04	7.47E+03	5.58E+04	4.66E+04	0	0	-	-
		$E'_{r,10^2}$	7.11E+02	7.10E+02	2.21E+03	2.07E+03	13.30	12.65	-	-
		Δ_r	1.80E+04	7.50E+03	5.59E+04	4.66E+04	13.30	12.65	15.82	15.73
A.5.3	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	2	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	18	-	0	-	0	-	-	-
		$E_{r,10^2}$	1.49E+04	-	0	-	0	-	-	-
		$E'_{r,10^2}$	5.13E+02	-	1.93E+02	-	4.66E+02	-	-	-
		Δ_r	1.49E+04	-	1.93E+02	-	4.66E+02	-	1.78E+02	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	61.20	7	5.99E+02	381	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.09E+02	97	3.30E+03	3408	0	0	-	-
		$E_{r,10^2}$	1.18E+05	4.18E+04	1.13E+06	6.87E+05	0	0	-	-
		$E'_{r,10^2}$	1.10E+03	1.14E+03	4.79E+03	6.23E+03	1.29E+02	90.02	-	-
		Δ_r	1.18E+05	4.18E+04	1.13E+06	6.87E+05	1.29E+02	90.02	1.67E+03	1.53E+03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	34.60	5	3.82E+02	416	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.57E+02	107	3.29E+03	3586	0	0	-	-
		$E_{r,10^2}$	1.23E+05	2.34E+04	5.26E+05	4.00E+05	0	0	-	-
		$E'_{r,10^2}$	1.16E+03	1.04E+03	5.97E+03	6.23E+03	48.09	47.02	-	-
		Δ_r	1.23E+05	2.34E+04	5.26E+05	4.00E+05	48.09	47.02	2.77E+02	3.00E+02

			$\epsilon = 10^{-2}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.4	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	5	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	17	-	0	-	0	-	-	-
		$E_{r,10^2}$	1.87E+04	-	0	-	0	-	-	-
		$E'_{r,10^2}$	4.63E+02	-	4.02	-	1.51E+02	-	-	-
		Δ_r	1.87E+04	-	4.02	-	1.51E+02	-	7.25	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	9	6	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	34.40	41	0	0	0	0	-	-
		$E_{r,10^2}$	1.17E+04	5.18E+03	0	0	0	0	-	-
		$E'_{r,10^2}$	5.97E+02	6.14E+02	9.10	8.15	9.02	8.07	-	-
		Δ_r	1.17E+04	5.24E+03	9.10	8.15	9.02	8.07	88.04	56.50
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	11.40	5	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	40.80	38	0	0	0	0	-	-
		$E_{r,10^2}$	1.40E+05	1.25E+04	0	0	0	0	-	-
		$E'_{r,10^2}$	7.28E+02	7.22E+02	6.57	6.62	6.57	6.61	-	-
		Δ_r	1.40E+05	1.25E+04	6.57	6.62	6.57	6.61	15.88	15.90
A.5.5	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	1	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	71	-	467	-	0	-	-	-
		$E_{r,10^2}$	4.79E+04	-	3.69E+05	-	0	-	-	-
		$E'_{r,10^2}$	6.38E+03	-	2.27E+04	-	3.33E+02	-	-	-
		Δ_r	4.83E+04	-	3.70E+05	-	3.33E+02	-	19.82	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	17.40	10	2.29E+02	174	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	12.20	9	396	349	0	0	-	-
		$E_{r,10^2}$	1.51E+05	2.18E+04	6.85E+05	4.04E+05	0	0	-	-
		$E'_{r,10^2}$	3.68E+03	3.72E+03	1.88E+04	1.70E+04	71.00	73.10	-	-
		Δ_r	1.51E+05	2.19E+04	6.85E+05	4.04E+05	71.00	73.10	1.48E+02	64.95
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	5.40	3	3.54E+02	263	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	7.20	6	3.10E+02	247	0	0	-	-
		$E_{r,10^2}$	2.02E+04	1.18E+04	1.30E+06	8.12E+05	0	0	-	-
		$E'_{r,10^2}$	2.84E+03	2.99E+03	1.99E+04	2.03E+04	23.94	22.17	-	-
		Δ_r	2.05E+04	1.22E+04	1.30E+06	8.12E+05	23.94	22.17	27.99	27.66
A.5.6	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	4	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	37	-	0	-	0	-	-	-
		$E_{r,10^2}$	3.47E+04	-	0	-	0	-	-	-
		$E'_{r,10^2}$	4.79E+02	-	2.07E+02	-	1.14E+02	-	-	-
		Δ_r	3.47E+04	-	2.07E+02	-	1.14E+02	-	5.63	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	12	13	1.31E+02	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	22.60	17	27.20	0	0	0	-	-
		$E_{r,10^2}$	6.61E+03	6.66E+03	1.48E+04	0	0	0	-	-
		$E'_{r,10^2}$	5.65E+02	5.07E+02	8.13E+02	2.23E+02	10.93	10.81	-	-
		Δ_r	6.64E+03	6.70E+03	1.50E+04	2.23E+02	10.93	10.81	67.85	45.15
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	32.80	2	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	30.80	19	0	0	0	0	-	-
		$E_{r,10^2}$	2.16E+04	3.91E+03	0	0	0	0	-	-
		$E'_{r,10^2}$	6.38E+02	4.72E+02	1.50E+02	1.49E+02	5.47	5.16	-	-
		Δ_r	2.17E+04	3.93E+03	1.50E+02	1.49E+02	5.47	5.16	10.12	9.14

			$\epsilon = 10^{-2}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.7	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	122	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	38	-	0	-	-	-
		$E_{r,10^2}$	0	-	2.50E+06	-	0	-	-	-
		$E'_{r,10^2}$	6.86E+03	-	1.28E+05	-	4.50E+03	-	-	-
		Δ_r	6.86E+03	-	2.50E+06	-	4.50E+03	-	1.42E+03	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	25	3	11	10	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.20	2	57.80	48	0	0	-	-
		$E_{r,10^2}$	1.18E+06	6.70E+04	2.04E+06	2.27E+06	0	0	-	-
		$E'_{r,10^2}$	4.07E+04	1.89E+04	8.04E+04	8.76E+04	1.38E+03	6.47E+02	-	-
		Δ_r	1.20E+06	7.01E+04	2.04E+06	2.27E+06	1.38E+03	6.47E+02	3.48E+03	3.23E+03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	7.40	3	155	129	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2.40	2	20.80	19	0	0	-	-
		$E_{r,10^2}$	8.60E+04	9.24E+04	1.75E+07	2.75E+06	0	0	-	-
		$E'_{r,10^2}$	3.01E+04	2.40E+04	1.39E+05	1.33E+05	1.25E+02	1.02E+02	-	-
		Δ_r	9.22E+04	9.71E+04	1.75E+07	2.75E+06	1.25E+02	1.02E+02	2.33E+03	2.04E+03
A.5.8	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	58	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	4	-	0	-	-	-
		$E_{r,10^2}$	0	-	5.71E+06	-	0	-	-	-
		$E'_{r,10^2}$	2.80E+02	-	2.81E+05	-	1.17E+04	-	-	-
		Δ_r	2.80E+02	-	5.71E+06	-	1.17E+04	-	1.18E+04	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	39.60	51	10	10	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	5	5	8.20	8	0	0	-	-
		$E_{r,10^2}$	2.29E+07	1.16E+07	2.66E+06	1.39E+06	0	0	-	-
		$E'_{r,10^2}$	2.42E+05	2.90E+05	1.63E+05	1.60E+05	2.51E+04	2.39E+04	-	-
		Δ_r	2.30E+07	1.16E+07	2.67E+06	1.40E+06	2.51E+04	2.39E+04	2.19E+04	2.14E+04
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	0.20	0	4.40	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	0	0	0	0	0	-	-
		$E_{r,10^2}$	8.27E+03	0	3.46E+05	0	0	0	-	-
		$E'_{r,10^2}$	3.31E+04	4.75E+04	6.46E+04	2.75E+04	1.36E+03	6.26E+02	-	-
		Δ_r	3.62E+04	5.62E+04	3.77E+05	2.75E+04	1.36E+03	6.26E+02	1.94E+04	1.92E+04
A.5.9	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	1	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	3	-	0	-	0	-	-	-
		$E_{r,10^2}$	2.23E+04	-	0	-	0	-	-	-
		$E'_{r,10^2}$	6.17E+03	-	25.24	-	2.24E+02	-	-	-
		Δ_r	2.31E+04	-	25.24	-	2.24E+02	-	40.29	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	12	9	79.40	106	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	7	6	18.80	24	0	0	-	-
		$E_{r,10^2}$	5.70E+04	3.13E+04	3.30E+05	2.06E+05	0	0	-	-
		$E'_{r,10^2}$	7.76E+03	7.22E+03	1.38E+04	2.26E+04	1.09E+02	1.07E+02	-	-
		Δ_r	5.79E+04	3.20E+04	3.31E+05	2.07E+05	1.09E+02	1.07E+02	1.30E+02	1.30E+02
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	7.20	5	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	3.80	4	0	0	0	0	-	-
		$E_{r,10^2}$	1.45E+05	3.11E+04	0	0	0	0	-	-
		$E'_{r,10^2}$	5.12E+03	5.51E+03	26.58	26.85	42.88	45.56	-	-
		Δ_r	1.45E+05	3.16E+04	26.58	26.85	42.88	45.56	64.17	62.26

			$\epsilon = 10^{-2}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.10	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	2	-	526	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	7	-	1210	-	0	-	-	-
		$E_{r,10^2}$	1.14E+03	-	1.71E+05	-	0	-	-	-
		$E'_{r,10^2}$	3.46E+02	-	4.03E+03	-	84.20	-	-	-
		Δ_r	1.19E+03	-	1.71E+05	-	84.20	-	1.82	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	8.40	7	2.25E+02	221	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	5.40	5	9.33E+02	916	0	0	-	-
		$E_{r,10^2}$	3.26E+03	1.79E+03	2.23E+05	2.36E+05	0	0	-	-
		$E'_{r,10^2}$	3.23E+02	3.34E+02	3.38E+03	3.24E+03	7.34	7.39	-	-
		Δ_r	3.29E+03	1.81E+03	2.23E+05	2.36E+05	7.34	7.39	20.45	17.06
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	9.20	6	3.83E+02	377	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	5.60	5	1.15E+03	1222	0	0	-	-
		$E_{r,10^2}$	6.83E+03	2.36E+03	3.12E+06	3.66E+05	0	0	-	-
		$E'_{r,10^2}$	3.45E+02	3.27E+02	3.91E+03	4.01E+03	2.62	2.65	-	-
		Δ_r	6.86E+03	2.42E+03	3.12E+06	3.66E+05	2.62	2.65	4.88	4.47
A.5.11	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	3	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1	-	2	-	0	-	-	-
		$E_{r,10^2}$	1.55E+02	-	2.07E+03	-	0	-	-	-
		$E'_{r,10^2}$	1.92E+02	-	3.02E+02	-	29.27	-	-	-
		Δ_r	2.47E+02	-	2.09E+03	-	29.27	-	1.36	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	4.80	4	1.58E+02	66	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	4	2	2.36E+02	146	0	0	-	-
		$E_{r,10^2}$	1.54E+03	1.45E+03	1.27E+05	3.33E+04	0	0	-	-
		$E'_{r,10^2}$	2.53E+02	2.61E+02	1.71E+03	1.41E+03	3.90	3.82	-	-
		Δ_r	1.57E+03	1.48E+03	1.27E+05	3.34E+04	3.90	3.82	10.03	10.26
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	2.20	2	85.40	36	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	3	3	89.60	48	0	0	-	-
		$E_{r,10^2}$	1.03E+03	7.98E+02	2.48E+04	1.75E+04	0	0	-	-
		$E'_{r,10^2}$	2.28E+02	2.44E+02	1.26E+03	9.52E+02	1.75	1.76	-	-
		Δ_r	1.07E+03	8.36E+02	2.48E+04	1.75E+04	1.75	1.76	2.28	2.35
A.5.12	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	158	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	8	-	0	-	-	-
		$E_{r,10^2}$	0	-	7.10E+04	-	0	-	-	-
		$E'_{r,10^2}$	7.50	-	5.12E+03	-	2.28E+02	-	-	-
		Δ_r	7.50	-	7.12E+04	-	2.28E+02	-	9.46	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	17.40	3	142	139	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1	0	73.40	68	0	0	-	-
		$E_{r,10^2}$	3.75E+04	1.79E+04	3.70E+05	1.27E+05	0	0	-	-
		$E'_{r,10^2}$	1.22E+03	7.31E+02	5.91E+03	5.96E+03	77.34	74.61	-	-
		Δ_r	3.77E+04	1.80E+04	3.70E+05	1.27E+05	77.34	74.61	41.11	37.06
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	5.40	4	86.40	59	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.80	1	4.20	3	0	0	-	-
		$E_{r,10^2}$	1.46E+04	2.28E+03	2.57E+05	1.04E+05	0	0	-	-
		$E'_{r,10^2}$	8.51E+02	8.55E+02	3.03E+03	3.42E+03	11.85	11.01	-	-
		Δ_r	1.47E+04	2.50E+03	2.57E+05	1.04E+05	11.85	11.01	11.03	10.19

			$\epsilon = 10^{-2}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.13	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	1	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2	-	0	-	0	-	-	-
		$E_{r,10^2}$	1.09E+03	-	0	-	0	-	-	-
		$E'_{r,10^2}$	3.59E+02	-	5.56E+02	-	99.83	-	-	-
		Δ_r	1.15E+03	-	5.56E+02	-	99.83	-	5.16	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	53.80	20	32	30	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	12.60	7	28.60	24	0	0	-	-
		$E_{r,10^2}$	1.95E+04	5.32E+03	3.12E+04	2.16E+04	0	0	-	-
		$E'_{r,10^2}$	1.50E+03	1.20E+03	1.57E+03	1.61E+03	31.33	28.80	-	-
		Δ_r	1.96E+04	5.45E+03	3.13E+04	2.17E+04	31.33	28.80	26.21	29.40
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.40	1	25	23	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	3.40	3	15.60	14	0	0	-	-
		$E_{r,10^2}$	3.94E+03	4.85E+03	1.46E+04	1.09E+04	0	0	-	-
		$E'_{r,10^2}$	4.04E+02	4.35E+02	1.37E+03	1.36E+03	6.51	6.29	-	-
		Δ_r	3.97E+03	4.86E+03	1.47E+04	1.09E+04	6.51	6.29	8.86	8.52
A.5.14	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	1	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	5	-	0	-	0	-	-	-
		$E_{r,10^2}$	1.65E+03	-	0	-	0	-	-	-
		$E'_{r,10^2}$	6.12E+02	-	3.74	-	64.95	-	-	-
		Δ_r	1.76E+03	-	3.74	-	64.95	-	3.31	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.40	0	99	132	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2	1	42.80	48	0	0	-	-
		$E_{r,10^2}$	1.79E+03	1.15E+03	4.14E+04	4.86E+04	0	0	-	-
		$E'_{r,10^2}$	4.12E+02	4.51E+02	1.90E+03	3.05E+03	15.50	9.98	-	-
		Δ_r	1.96E+03	1.23E+03	4.14E+04	4.87E+04	15.50	9.98	19.52	17.93
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	4.60	1	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1	0	0	0	0	0	-	-
		$E_{r,10^2}$	4.05E+03	5.25E+02	0	0	0	0	-	-
		$E'_{r,10^2}$	3.66E+02	3.16E+02	1.60E+02	2.55E+02	3.71	3.54	-	-
		Δ_r	4.13E+03	5.55E+02	1.60E+02	2.55E+02	3.71	3.54	5.45	4.62
A.5.15	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1	-	0	-	0	-	-	-
		$E_{r,10^2}$	1.02E+02	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.35E+02	-	6.27	-	23.13	-	-	-
		Δ_r	1.70E+02	-	6.27	-	23.13	-	7.76	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	2	2	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2.40	1	0	0	0	0	-	-
		$E_{r,10^2}$	1.67E+03	6.06E+02	0	0	0	0	-	-
		$E'_{r,10^2}$	2.17E+02	2.24E+02	1.39	0.25	1.18	1.22	-	-
		Δ_r	1.71E+03	6.56E+02	1.39	0.25	1.18	1.22	33.72	30.66
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.20	0	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2.40	3	0	0	0	0	-	-
		$E_{r,10^2}$	3.54E+02	3.54E+02	0	0	0	0	-	-
		$E'_{r,10^2}$	1.60E+02	1.37E+02	1.59	0.21	0.56	0.59	-	-
		Δ_r	3.92E+02	4.24E+02	1.59	0.21	0.56	0.59	9.39	9.39

			$\epsilon = 10^{-2}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.16	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.45E+03	-	6.00E+04	-	1.06E+04	-	-	-
		Δ_r	1.45E+03	-	6.00E+04	-	1.06E+04	-	2.61E+04	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	48.40	53	0.60	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	6.60	6	2.60	2	0	0	-	-
		$E_{r,10^2}$	6.82E+06	5.40E+06	1.03E+06	4.11E+05	0	0	-	-
		$E'_{r,10^2}$	5.77E+05	6.01E+05	1.27E+05	1.38E+05	4.18E+04	4.12E+04	-	-
		Δ_r	6.85E+06	5.44E+06	1.06E+06	4.34E+05	4.18E+04	4.12E+04	3.60E+04	3.58E+04
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	23.60	17	2	2	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0.40	0	0.60	0	0	0	-	-
		$E_{r,10^2}$	5.16E+06	4.93E+06	1.37E+06	2.46E+05	0	0	-	-
		$E'_{r,10^2}$	3.77E+05	3.46E+05	1.28E+05	1.05E+05	1.74E+04	1.52E+04	-	-
		Δ_r	5.19E+06	4.96E+06	1.40E+06	2.91E+05	1.74E+04	1.52E+04	3.44E+04	3.52E+04
A.5.17	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	23	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	21	-	0	-	0	-	-	-
		$E_{r,10^2}$	3.09E+03	-	0	-	0	-	-	-
		$E'_{r,10^2}$	6.67E+02	-	59.14	-	70.35	-	-	-
		Δ_r	3.16E+03	-	59.14	-	70.35	-	7.40	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	34.60	28	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	15	14	0	0	0	0	-	-
		$E_{r,10^2}$	9.71E+03	7.70E+03	0	0	0	0	-	-
		$E'_{r,10^2}$	6.72E+02	6.67E+02	38.18	56.23	12.27	12.43	-	-
		Δ_r	9.74E+03	7.73E+03	38.18	56.23	12.27	12.43	25.94	25.30
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	11.40	12	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	9.20	9	0	0	0	0	-	-
		$E_{r,10^2}$	1.81E+04	7.36E+03	0	0	0	0	-	-
		$E'_{r,10^2}$	4.56E+02	4.69E+02	68.12	66.95	6.82	6.21	-	-
		Δ_r	1.81E+04	7.38E+03	68.12	66.95	6.82	6.21	11.68	11.36
A.5.18	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	0	-	0	-	0	-	-	-
		$E_{r,10^2}$	0	-	0	-	0	-	-	-
		$E'_{r,10^2}$	3.41E+02	-	3.40E+02	-	3.19E+02	-	-	-
		Δ_r	3.41E+02	-	3.40E+02	-	3.19E+02	-	2.68E+02	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.60	1	2.20	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2	2	1.20	0	0	0	-	-
		$E_{r,10^2}$	9.73E+03	5.95E+03	8.90E+03	0	0	0	-	-
		$E'_{r,10^2}$	2.24E+03	2.23E+03	1.20E+03	7.95E+02	3.47E+02	3.47E+02	-	-
		Δ_r	1.02E+04	6.22E+03	9.32E+03	7.95E+02	3.47E+02	3.47E+02	5.14E+02	4.83E+02
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	2.40	2	3	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	2.40	3	1.60	0	0	0	-	-
		$E_{r,10^2}$	1.16E+04	7.13E+03	8.50E+04	0	0	0	-	-
		$E'_{r,10^2}$	2.16E+03	2.28E+03	1.80E+03	1.38E+03	3.10E+02	2.96E+02	-	-
		Δ_r	1.19E+04	7.48E+03	8.56E+04	1.38E+03	3.10E+02	2.96E+02	4.50E+02	4.16E+02

			$\epsilon = 10^{-2}$							
Function No.	*Sample Type*		$r_1(x)$		$r_2(x)$		$r_3(x)$		$p(x)$	
			AM	*MD*	*AM*	*MD*	*AM*	*MD*	*AM*	*MD*
A.5.19	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	7	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	136	-	0	-	0	-	-	-
		$E_{r,10^2}$	4.87E+05	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.24E+04	-	2.70E+03	-	1.17E+03	-	-	-
		Δ_r	4.88E+05	-	2.70E+03	-	1.17E+03	-	3.65E+02	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	2.25E+02	204	14.80	5	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.88E+02	163	36.60	22	0	0	-	-
		$E_{r,10^2}$	6.59E+06	1.10E+06	5.58E+04	4.09E+04	0	0	-	-
		$E'_{r,10^2}$	1.93E+04	1.69E+04	6.82E+03	5.81E+03	1.12E+03	1.13E+03	-	-
		Δ_r	6.59E+06	1.10E+06	5.64E+04	4.13E+04	1.12E+03	1.13E+03	2.43E+03	2.31E+03
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	1.36E+02	57	10.60	7	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	1.34E+02	117	44.80	35	0	0	-	-
		$E_{r,10^2}$	5.73E+05	3.46E+05	1.51E+06	6.83E+04	0	0	-	-
		$E'_{r,10^2}$	1.60E+04	1.51E+04	8.10E+03	6.62E+03	7.47E+02	7.42E+02	-	-
		Δ_r	5.73E+05	3.47E+05	1.51E+06	6.90E+04	7.47E+02	7.42E+02	4.00E+02	4.08E+02
A.5.20	SG	$\lvert W^{(fc)}_{r,10^2}\rvert$	3	-	0	-	0	-	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	4	-	0	-	0	-	-	-
		$E_{r,10^2}$	7.41E+03	-	0	-	0	-	-	-
		$E'_{r,10^2}$	1.40E+03	-	1.78E+02	-	2.60E+02	-	-	-
		Δ_r	7.54E+03	-	1.78E+02	-	2.60E+02	-	14.96	-
	LHS	$\lvert W^{(fc)}_{r,10^2}\rvert$	3.20	4	1.18E+02	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	11.80	12	23.80	0	0	0	-	-
		$E_{r,10^2}$	9.32E+03	8.21E+03	8.45E+04	0	0	0	-	-
		$E'_{r,10^2}$	2.08E+03	2.10E+03	4.25E+03	1.12E+03	46.80	48.87	-	-
		Δ_r	9.57E+03	8.49E+03	8.50E+04	1.12E+03	46.80	48.87	1.35E+02	1.15E+02
	d-LHD	$\lvert W^{(fc)}_{r,10^2}\rvert$	4.60	1	0	0	0	0	-	-
		$\lvert W^{(in)}_{r,10^2}\rvert$	13.20	10	0	0	0	0	-	-
		$E_{r,10^2}$	1.03E+04	8.13E+03	0	0	0	0	-	-
		$E'_{r,10^2}$	2.17E+03	2.12E+03	2.19E+02	2.10E+02	18.19	19.86	-	-
		Δ_r	1.05E+04	8.32E+03	2.19E+02	2.10E+02	18.19	19.86	32.87	32.29

SM2. CPU Times and Number of Iterations

In Fig. 9 we gave the total CPU time taken by the four approximation approaches for five typical test functions from Table A.5 when the interpolation data is sampled using d-LHD and is noise-free. Additionally, in Fig. 6 we showed the number of iterations taken by Algorithm 4.1 to approximate noise-free data of the same five functions sampled using LHS and d-LHD strategies. We showed summary results of the the CPU times and number of iterations over all functions from Table A.5 in Table 3 and Table 1, respectively. In this section, we present detailed results of the CPU time required to compute the four approximations of all functions from Table A.5 sampled using the three strategies of SG, LHS, D-LHD. Additionally, we also give detailed results of the number of iterations taken by Algorithm 4.1 to approximate these functions. These results are presented for noise-free data ($\epsilon = 0$) as well as for data with the relative noise of $\epsilon = 10^{-6}$ and $\epsilon = 10^{-2}$.

The timing and iteration results for approximating noise-free interpolation data of all functions from Table A.5 is shown in Table SM2.4. In this table, $p(x)$ is the polynomial approximation, (b) $r_1(x)$ is the rational approximation using Algorithm 3.1 without degree reduction, (c) $r_2(x)$ is the rational approximation using Algorithm 3.1 with the degree reduction described in Algorithm 3.3, and (d) $r_3(x)$ is the rational approximation using Algorithm 4.1. *Function No.* is the number of the function from Table A.5. *Sample Type* is the name of the sampling strategy used to pick interpolation points from the domain. For $r_3(x)$, the total time is given as its fit time and multistart time. The number of iterations taken by Algorithm 4.1 to obtain

$r_3(x)$ is also given. M is the arithmetic mean and SD is the standard deviation of the results from running the experiment over LHS and d-LHD samples with different random seeds. Since SG is deterministic in picking points from the domain, a "-" is placed for *SD* whenever the sampling strategy is SG. Similarly, the timing and iteration results for approximating noisy data of $\epsilon = 10^{-6}$ and $\epsilon = 10^{-2}$ is given in Table SM2.5 and Table SM2.6, respectively.

Table SM2.4: CPU times for all four approximations of functions from Table A.5. The interpolation data for these functions are noise-free, i.e., $\epsilon = 0$. *Function No.* is the number of the function from Table A.5. *Sample Type* is the name of the sampling strategy used to pick interpolation points from the domain. For $r_3(x)$, the total time is given as its fit time and multistart time. The number of iterations taken by Algorithm 4.1 to obtain $r_3(x)$ is also given. M and SD are the average and standard deviation, respectively. Since SG is deterministic in picking points from the domain, a "-" is placed for *SD* whenever the sampling strategy is SG.

		$\epsilon = 0$											
Function No.	*Sample Type*	$r_1(x)$		$r_2(x)$		$r_3(x)$ *fit time*		$r_3(x)$ *ms time*		$r_3(x)$ *iterations*		$p(x)$	
		M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
A.5.1	SG	2.12	-	2.55	-	0.04	-	15.85	-	1	-	0.41	-
	LHS	3.27	0.12	3.59	0.06	0.05	9.86E-04	15.85	2.81E-04	1	0	0.93	0.06
	d-LHD	3.24	0.11	4.02	0.03	0.05	0.02	15.85	1.19E-04	1	0	0.90	0.02
A.5.2	SG	2.25	-	2.55	-	0.05	-	15.85	-	1	-	0.52	-
	LHS	3.54	0.16	3.69	0.13	0.04	1.39E-03	15.85	1.92E-04	1	0	0.90	0.01
	d-LHD	3.36	0.13	4.02	0.02	0.04	7.90E-03	15.85	1.24E-04	1	0	0.93	0.03
A.5.3	SG	2.09	-	2.56	-	0.04	-	15.85	-	1	-	0.42	-
	LHS	3.67	0.10	3.98	0.03	0.04	2.21E-03	15.85	2.48E-04	1	0	0.93	0.02
	d-LHD	3.51	0.07	4.04	0.07	0.03	1.95E-03	15.85	2.40E-04	1	0	0.91	0.02
A.5.4	SG	2.07	-	2.55	-	0.04	-	15.85	-	1	-	0.42	-
	LHS	3.77	0.05	4.04	0.04	0.06	4.17E-03	15.85	1.03E-04	2	0	0.90	0.04
	d-LHD	3.75	0.08	4.05	0.09	0.03	4.01E-03	15.85	5.91E-05	1	0	0.92	0.02
A.5.5	SG	2.07	-	2.55	-	0.04	-	15.85	-	1	-	0.52	-
	LHS	3.80	0.02	4.17	0.15	0.03	1.34E-03	15.85	4.45E-04	1	0	0.86	0.01
	d-LHD	3.80	0.04	4.17	0.09	0.04	6.54E-03	15.85	1.05E-04	1	0	0.89	0.02
A.5.6	SG	2.07	-	2.56	-	0.04	-	15.85	-	1	-	0.52	-
	LHS	3.87	0.02	3.73	0.63	0.03	1.02E-03	15.85	2.84E-04	1	0	0.92	0.02
	d-LHD	3.82	0.03	4.12	0.03	0.03	6.44E-04	15.85	3.21E-04	1	0	0.86	0.05
A.5.7	SG	2.25	-	2.55	-	0.04	-	15.85	-	1	-	0.42	-
	LHS	2.46	0.14	3.32	0.52	0.38	0.27	15.86	7.98E-03	4	2.28	0.70	0.07
	d-LHD	2.93	0.61	4.03	0.06	0.69	0.37	15.86	9.83E-03	1.80	0.75	0.71	0.05
A.5.8	SG	2.24	-	2.55	-	0.04	-	15.85	-	1	-	0.42	-
	LHS	2.36	0.02	3.93	0.06	0.10	0.05	15.85	7.27E-04	2.20	0.75	0.68	8.45E-03
	d-LHD	2.40	0.02	3.87	0.37	0.10	0.06	15.85	6.48E-05	1	0	0.67	0.04
A.5.9	SG	7.79	-	7.42	-	3.00	-	3.73E+02	-	1	-	0.56	-
	LHS	3.09	0.03	4.79	0.43	4.68	2.43	3.73E+02	0.04	1.60	1.20	0.67	0.05
	d-LHD	3.16	0.02	4.91	0.01	9.32	8.35	3.73E+02	0.01	1.20	0.40	0.71	5.57E-03
A.5.10	SG	2.26	-	2.55	-	0.06	-	15.85	-	1	-	0.52	-
	LHS	2.30	0.01	2.31	8.92E-03	0.02	2.64E-03	15.85	1.03E-04	1	0	0.64	8.47E-03
	d-LHD	2.36	0.04	3.01	0.79	0.03	3.07E-03	15.85	5.89E-03	1	0	0.63	0.05
A.5.11	SG	2.07	-	2.56	-	0.07	-	15.85	-	1	-	0.52	-
	LHS	2.38	0.29	2.86	0.46	0.02	4.45E-03	15.85	1.37E-04	1	0	0.61	0.04
	d-LHD	2.33	0.01	2.26	0.04	0.03	5.63E-03	15.85	3.18E-04	1	0	0.62	0.02
A.5.12	SG	2.07	-	2.55	-	0.04	-	15.85	-	1	-	0.52	-
	LHS	2.25	6.90E-03	3.22	0.07	0.03	7.98E-03	15.85	3.05E-04	1.20	0.40	0.61	0.01
	d-LHD	2.24	0.01	2.64	0.71	0.04	1.89E-03	15.85	1.53E-04	1	0	0.58	0.05
A.5.13	SG	2.08	-	2.56	-	0.04	-	15.85	-	1	-	0.52	-
	LHS	2.23	7.86E-03	3.53	0.21	0.03	3.38E-03	15.85	1.22E-04	1	0	0.59	0.01
	d-LHD	2.35	0.25	4.00	0.02	0.04	1.52E-03	15.85	1.86E-04	1	0	0.52	0.04
A.5.14	SG	2.26	-	2.56	-	0.04	-	15.85	-	1	-	0.41	-
	LHS	2.53	0.30	3.54	0.02	0.03	0.01	15.85	5.00E-04	1.20	0.40	0.58	3.20E-03
	d-LHD	2.50	0.19	4.02	0.02	0.04	0.01	15.85	3.38E-04	1	0	0.48	0.05
A.5.15	SG	2.50	-	2.69	-	0.39	-	44.94	-	1	-	0.42	-
	LHS	3.06	0.12	3.97	0.13	1.26	0.68	45.08	0.15	5.80	3.54	0.53	0.06
	d-LHD	2.86	0.04	4.10	0.03	0.56	0.21	45.52	0.68	2.40	1.02	0.45	0.04
A.5.16	SG	7.89	-	9.87	-	2.03E+02	-	3.73E+02	-	1	-	0.57	-
	LHS	4.01	0.06	4.72	0.04	52.04	6.40	3.73E+02	0.10	27.60	3.77	0.50	0.02
	d-LHD	3.84	0.03	4.94	0.18	21.81	3.65	3.73E+02	0.27	9.20	1.17	0.49	0.06

Function No.	*Sample Type*	$\epsilon = 0$											
		$r_1(x)$		$r_2(x)$		$r_3(x)$ *fit time*		$r_3(x)$ *ms time*		$r_3(x)$ *iterations*		$p(x)$	
		M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
A.5.17	SG	7.76	-	9.11	-	4.01	-	3.73E+02	-	1	-	0.45	-
	LHS	4.18	0.03	4.79	0.29	2.71	0.36	3.73E+02	6.96E-04	1	0	0.48	9.14E-03
	d-LHD	4.00	0.13	5.07	0.08	2.43	0.20	3.73E+02	2.79E-03	1	0	0.49	0.05
A.5.18	SG	7.86	-	9.69	-	7.28	-	3.73E+02	-	1	-	0.46	-
	LHS	4.33	0.16	4.50	0.58	45.16	12.52	3.74E+02	0.49	26.40	8.01	0.48	7.19E-03
	d-LHD	4.00	0.35	4.82	0.43	1.36E+02	28.80	3.78E+02	2.29	80.40	30.55	0.46	0.02
A.5.19	SG	1.79	-	2.55	-	0.04	-	15.85	-	1	-	0.51	-
	LHS	3.50	0.03	3.58	0.22	0.04	0.01	15.86	0.02	1.20	0.40	0.55	0.03
	d-LHD	3.43	0.03	4.00	0.03	0.03	1.33E-03	15.85	6.93E-04	1	0	0.42	0.01
A.5.20	SG	1.99	-	3.64	-	0.05	-	15.85	-	1	-	0.50	-
	LHS	3.62	0.04	3.60	0.10	0.04	0.01	15.85	2.13E-03	1.20	0.40	0.64	0.04
	d-LHD	3.52	0.04	4.00	0.05	0.16	0.12	15.85	2.67E-04	1	0	0.50	0.03

Table SM2.5: CPU times for all four approximations of functions from Table A.5. The relative noise level in the interpolation data is $\epsilon = 10^{-6}$. *Function No.* is the number of the function from Table A.5. *Sample Type* is the name of the sampling strategy used to pick interpolation points from the domain. For $r_3(x)$, the total time is given as its fit time and multistart time. The number of iterations taken by Algorithm 4.1 to obtain $r_3(x)$ is also given. M and SD are the average and standard deviation, respectively. Since SG is deterministic in picking points from the domain, a "-" is placed for *SD* whenever the sampling strategy is SG.

Function No.	*Sample Type*	$\epsilon = 10^{-6}$											
		$r_1(x)$		$r_2(x)$		$r_3(x)$ *fit time*		$r_3(x)$ *ms time*		$r_3(x)$ *iterations*		$p(x)$	
		M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
A.5.1	SG	2.25	-	2.07	-	0.03	-	15.85	-	1	-	0.52	-
	LHS	3.28	0.16	3.34	0.13	0.04	2.97E-03	15.85	4.32E-04	1	0	0.91	0.01
	d-LHD	3.35	0.04	3.45	0.12	0.05	0.02	15.85	2.23E-04	1	0	1.19	0.05
A.5.2	SG	2.07	-	2.24	-	0.07	-	15.85	-	1	-	0.53	-
	LHS	3.55	0.11	3.59	0.08	0.04	6.98E-04	15.87	0.04	1	0	0.90	0.02
	d-LHD	3.57	0.10	3.67	0.17	0.04	6.35E-03	15.85	8.69E-05	1	0	1.18	0.03
A.5.3	SG	2.24	-	2.07	-	0.03	-	15.85	-	1	-	0.52	-
	LHS	3.68	0.03	3.67	0.06	0.04	1.75E-03	15.85	4.57E-04	1	0	0.83	6.95E-03
	d-LHD	3.60	0.04	3.65	0.04	0.04	3.16E-03	15.85	5.49E-04	1	0	1.19	0.04
A.5.4	SG	2.07	-	2.08	-	0.04	-	15.85	-	1	-	0.41	-
	LHS	3.75	0.05	3.89	0.11	0.06	0.01	15.85	3.07E-03	2.20	0.40	0.80	0.01
	d-LHD	3.74	0.06	3.74	0.11	0.03	1.22E-03	15.85	1.51E-04	1	0	1.15	0.13
A.5.5	SG	2.07	-	2.26	-	0.04	-	15.85	-	1	-	0.41	-
	LHS	3.75	0.01	4.05	0.08	0.04	5.35E-03	15.85	3.36E-04	1	0	0.77	2.08E-03
	d-LHD	3.70	0.02	4.05	0.12	0.05	0.02	15.85	7.20E-04	1	0	1.19	0.02
A.5.6	SG	2.25	-	2.08	-	0.03	-	15.85	-	1	-	0.53	-
	LHS	3.79	0.02	3.92	0.06	0.03	1.43E-03	15.85	3.54E-04	1	0	0.73	0.08
	d-LHD	3.76	0.02	4.07	0.10	0.03	2.91E-03	15.85	4.94E-04	1	0	1.05	0.11
A.5.7	SG	2.23	-	2.06	-	0.03	-	15.85	-	1	-	0.40	-
	LHS	2.68	0.24	3.81	0.03	0.17	0.16	15.86	0.02	2.40	1.02	0.58	0.05
	d-LHD	2.48	0.06	3.98	0.17	2.22	1.74	15.86	9.00E-03	2.40	1.02	0.94	9.60E-03
A.5.8	SG	2.06	-	2.07	-	0.03	-	15.85	-	1	-	0.41	-
	LHS	2.33	0.07	3.26	0.65	0.23	0.36	15.85	2.15E-03	1.80	1.17	0.55	3.25E-03
	d-LHD	2.37	0.06	3.24	0.63	0.06	0.03	15.85	6.15E-05	1	0	0.92	0.05
A.5.9	SG	7.30	-	7.55	-	6.28	-	3.73E+02	-	1	-	0.57	-
	LHS	3.10	0.02	3.40	0.13	5.45	1.86	3.73E+02	0.04	1.20	0.40	0.57	6.26E-03
	d-LHD	3.14	0.02	3.50	0.26	45.66	86.27	3.73E+02	8.23E-04	1	0	0.95	0.02
A.5.10	SG	2.07	-	2.26	-	0.05	-	15.85	-	1	-	0.53	-
	LHS	2.30	9.61E-03	2.43	0.28	0.03	4.17E-03	15.85	1.38E-04	1	0	0.57	0.07
	d-LHD	2.30	0.05	2.28	0.06	0.03	7.01E-03	15.85	1.19E-04	1	0	0.94	0.01
A.5.11	SG	2.22	-	2.08	-	0.08	-	15.85	-	1	-	0.40	-
	LHS	2.26	0.02	2.26	0.01	0.03	3.98E-03	15.87	0.04	1	0	0.55	4.11E-03
	d-LHD	2.26	0.02	2.38	0.34	0.05	0.04	15.85	1.67E-04	1	0	0.87	0.17
A.5.12	SG	2.24	-	2.07	-	0.05	-	15.85	-	1	-	0.41	-
	LHS	2.23	0.01	2.31	0.24	0.04	0.01	15.85	2.45E-04	1.20	0.40	0.53	0.05
	d-LHD	2.20	0.06	2.18	0.07	0.03	7.10E-03	15.85	6.37E-05	1	0	0.95	0.04

Function No.	*Sample Type*	$\epsilon = 10^{-6}$											
		$r_1(x)$		$r_2(x)$		$r_3(x)$ *fit time*		$r_3(x)$ *ms time*		$r_3(x)$ *iterations*		$p(x)$	
		M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
A.5.13	SG	2.07	-	2.07	-	0.07	-	15.85	-	1	-	0.41	-
	LHS	2.19	0.06	3.00	0.06	0.04	4.29E-03	15.85	1.74E-04	1	0	0.54	8.68E-03
	d-LHD	2.36	0.27	2.39	0.28	0.04	5.36E-03	15.85	1.81E-04	1	0	0.94	7.95E-03
A.5.14	SG	2.07	-	2.28	-	0.06	-	15.85	-	1	-	0.41	-
	LHS	2.30	0.24	2.97	0.28	0.04	0.01	15.85	5.76E-04	1.20	0.40	0.54	8.47E-03
	d-LHD	2.42	0.40	2.98	0.06	0.04	0.01	15.85	6.09E-05	1	0	0.93	9.30E-03
A.5.15	SG	2.49	-	2.64	-	0.90	-	44.94	-	1	-	0.41	-
	LHS	3.08	0.17	3.11	0.32	1.40	1.09	45.04	0.09	6.80	5.67	0.55	0.03
	d-LHD	3.04	0.03	3.14	0.08	0.49	0.23	44.98	0.05	2.20	0.98	1.12	0.15
A.5.16	SG	7.44	-	6.81	-	1.56E+02	-	3.73E+02	-	1	-	0.45	-
	LHS	4.05	0.08	4.08	0.67	52.50	9.45	3.73E+02	0.20	30	5.25	0.54	0.02
	d-LHD	4.02	0.06	3.94	0.03	18.10	2.65	3.73E+02	0.12	9.80	1.60	1.21	0.03
A.5.17	SG	7.46	-	7.59	-	3.28	-	3.73E+02	-	1	-	0.44	-
	LHS	4.17	0.03	4.59	0.20	2.72	0.37	3.73E+02	1.43E-03	1	0	0.54	4.34E-03
	d-LHD	4.08	0.05	4.55	0.37	2.53	0.19	3.73E+02	1.35E-03	1	0	1.23	0.05
A.5.18	SG	6.26	-	8.46	-	2.83E+02	-	3.73E+02	-	1	-	0.40	-
	LHS	4.15	0.08	4.63	0.11	41.62	12.70	3.74E+02	0.53	26	7.62	0.59	0.12
	d-LHD	4.11	0.08	4.53	0.07	1.00E+02	39.12	3.78E+02	2.39	79.40	30.93	1.22	0.04
A.5.19	SG	1.99	-	2.04	-	0.05	-	15.85	-	1	-	0.50	-
	LHS	3.40	0.02	3.53	0.10	0.04	0.01	15.85	5.81E-03	1.20	0.40	0.76	3.22E-03
	d-LHD	3.34	0.03	3.66	0.22	0.03	3.07E-03	15.85	4.13E-04	1	0	1.18	0.05
A.5.20	SG	1.74	-	1.81	-	0.06	-	15.85	-	1	-	0.34	-
	LHS	3.49	0.04	3.64	0.02	0.04	0.01	15.85	8.86E-03	1.20	0.40	0.77	4.33E-03
	d-LHD	3.47	0.03	3.44	0.02	0.08	0.05	15.85	2.33E-04	1	0	1.18	6.01E-03

Table SM2.6: CPU times for all four approximations of functions from Table A.5. The relative noise level in the interpolation data is $\epsilon = 10^{-2}$. *Function No.* is the number of the function from Table A.5. *Sample Type* is the name of the sampling strategy used to pick interpolation points from the domain. For $r_3(x)$, the total time is given as its fit time and multistart time. The number of iterations taken by Algorithm 4.1 to obtain $r_3(x)$ is also given. M and SD are the average and standard deviation, respectively. Since SG is deterministic in picking points from the domain, a "-" is placed for *SD* whenever the sampling strategy is SG.

Function No.	*Sample Type*	$\epsilon = 10^{-2}$											
		$r_1(x)$		$r_2(x)$		$r_3(x)$ *fit time*		$r_3(x)$ *ms time*		$r_3(x)$ *iterations*		$p(x)$	
		M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
A.5.1	SG	2.22	-	1.62	-	0.03	-	15.85	-	1	-	0.51	-
	LHS	3.35	0.09	3.76	0.03	0.07	0.03	15.85	3.72E-03	2	0.89	0.85	1.89E-03
	d-LHD	3.23	0.25	3.99	3.57E-03	0.05	0.02	15.85	2.65E-04	1	0	0.91	6.43E-03
A.5.2	SG	2.05	-	2.24	-	0.04	-	15.85	-	1	-	0.51	-
	LHS	3.64	0.08	3.86	0.05	0.04	2.23E-03	15.85	4.81E-04	1	0	0.84	4.30E-03
	d-LHD	3.65	0.12	4.00	0.04	0.04	8.95E-03	15.85	4.46E-04	1	0	0.93	0.04
A.5.3	SG	2.06	-	2.25	-	0.03	-	15.85	-	1	-	0.51	-
	LHS	3.65	0.07	3.99	0.11	0.09	0.01	15.86	4.93E-03	2.60	0.49	0.85	8.79E-03
	d-LHD	3.72	0.04	4.00	0.01	0.04	9.88E-03	15.85	7.30E-04	1	0	0.95	0.06
A.5.4	SG	2.06	-	2.25	-	0.04	-	15.85	-	1	-	0.51	-
	LHS	3.79	0.04	4.05	0.04	0.03	2.62E-03	15.85	7.38E-05	1	0	0.85	0.02
	d-LHD	3.78	0.06	3.96	0.07	0.04	2.03E-03	15.86	0.02	1	0	0.91	0.08
A.5.5	SG	2.07	-	2.24	-	0.03	-	15.85	-	1	-	0.51	-
	LHS	3.78	0.02	4.05	0.05	0.04	2.31E-03	15.85	4.10E-04	1	0	0.83	0.01
	d-LHD	3.74	0.03	4.09	0.16	0.06	0.02	15.85	4.24E-04	1	0	0.84	0.02
A.5.6	SG	2.05	-	2.25	-	0.04	-	15.85	-	1	-	0.51	-
	LHS	3.84	0.01	4.04	0.08	0.04	2.68E-03	15.85	2.23E-04	1	0	0.79	0.08
	d-LHD	3.77	5.32E-03	3.93	0.10	0.06	0.03	15.85	1.98E-04	1	0	0.83	0.10
A.5.7	SG	2.23	-	2.11	-	0.04	-	15.85	-	1	-	0.51	-
	LHS	2.70	0.30	3.98	0.05	0.08	0.01	15.86	7.47E-03	2.20	0.40	0.67	0.06
	d-LHD	2.66	0.30	3.25	0.65	0.05	0.02	15.85	0.01	1.20	0.40	0.72	9.13E-03
A.5.8	SG	2.24	-	2.24	-	0.04	-	15.85	-	1	-	0.51	-
	LHS	2.36	7.28E-03	4.02	0.02	0.14	0.04	15.86	9.77E-03	3.80	1.17	0.68	0.04
	d-LHD	2.34	0.06	3.34	0.57	0.05	6.60E-03	15.86	0.02	1	0	0.63	0.05

		$\epsilon = 10^{-2}$											
Function No.	*Sample Type*	$r_1(x)$		$r_2(x)$		$r_3(x)$ *fit time*		$r_3(x)$ *ms time*		$r_3(x)$ *iterations*		$p(x)$	
		M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
A.5.9	SG	7.19	-	7.89	-	4.10	-	3.73E+02	-	1	-	0.56	-
	LHS	3.09	0.01	5.08	0.04	8.71	3.56	3.73E+02	0.17	3.40	1.02	0.68	0.05
	d-LHD	3.10	8.19E-03	4.38	0.50	1.84	0.25	3.73E+02	1.98E-03	1	0	0.65	0.04
A.5.10	SG	2.06	-	2.24	-	0.06	-	15.85	-	1	-	0.51	-
	LHS	2.28	0.01	2.66	0.53	0.03	3.30E-03	15.85	3.44E-04	1	0	0.63	0.05
	d-LHD	2.33	8.95E-03	2.43	0.32	0.04	7.34E-03	15.85	3.09E-04	1	0	0.61	0.02
A.5.11	SG	2.06	-	2.24	-	0.08	-	15.85	-	1	-	0.51	-
	LHS	2.25	0.02	2.25	8.32E-03	0.04	3.71E-03	15.85	5.09E-04	1	0	0.61	0.05
	d-LHD	2.30	0.02	2.09	8.19E-03	0.07	0.03	15.85	3.56E-04	1	0	0.54	0.03
A.5.12	SG	2.06	-	2.23	-	0.05	-	15.85	-	1	-	0.51	-
	LHS	2.23	7.00E-03	3.25	0.02	0.04	1.74E-03	15.85	1.53E-04	1	0	0.63	0.05
	d-LHD	2.23	8.13E-03	2.43	0.71	0.06	0.03	15.85	8.47E-05	1	0	0.57	0.05
A.5.13	SG	2.07	-	2.23	-	0.05	-	15.85	-	1	-	0.51	-
	LHS	2.23	6.88E-03	3.15	0.39	0.03	2.41E-03	15.85	1.83E-04	1	0	0.69	0.06
	d-LHD	2.22	8.53E-03	3.86	0.02	0.07	0.04	15.85	1.79E-04	1	0	0.54	1.97E-03
A.5.14	SG	2.06	-	2.24	-	0.05	-	15.85	-	1	-	0.51	-
	LHS	2.38	0.29	3.42	0.02	0.04	6.43E-03	15.85	2.58E-04	1	0	0.64	0.05
	d-LHD	2.23	0.13	3.85	6.24E-03	0.03	7.63E-03	15.85	2.71E-04	1	0	0.54	0.01
A.5.15	SG	2.45	-	2.66	-	0.76	-	44.94	-	1	-	0.51	-
	LHS	3.02	0.07	3.19	0.69	1.02	0.54	45.07	0.12	4	2.10	0.85	4.82E-03
	d-LHD	3.06	0.05	3.89	0.16	0.33	0.10	44.94	3.64E-03	1.20	0.40	0.49	0.04
A.5.16	SG	6.97	-	8.20	-	2.90E+02	-	3.73E+02	-	1	-	0.56	-
	LHS	4.05	0.06	4.70	0.18	63.03	13.12	3.74E+02	0.57	34	7.51	0.89	0.02
	d-LHD	4.12	0.04	4.56	0.64	17.07	2.42	3.73E+02	0.06	9.20	1.33	0.51	0.05
A.5.17	SG	7.15	-	8.21	-	2.62	-	3.73E+02	-	1	-	0.56	-
	LHS	4.11	0.07	4.74	0.10	4.25	1.43	3.73E+02	0.30	1.60	0.49	0.88	5.68E-03
	d-LHD	4.18	0.03	4.82	0.12	2.61	0.18	3.73E+02	8.42E-04	1	0	0.50	0.08
A.5.18	SG	6.19	-	8.09	-	4.77	-	3.73E+02	-	1	-	0.55	-
	LHS	4.21	0.05	4.80	0.03	54.06	30.88	3.74E+02	0.74	25	10.73	0.89	0.02
	d-LHD	4.20	0.07	4.47	0.40	1.07E+02	33.93	3.76E+02	2.14	77.80	26.98	0.45	3.29E-03
A.5.19	SG	1.99	-	2.03	-	0.04	-	15.85	-	1	-	0.50	-
	LHS	3.42	0.04	3.95	0.06	0.04	0.02	15.85	2.88E-03	1.20	0.40	0.78	0.09
	d-LHD	3.21	0.41	3.65	0.02	0.05	0.01	15.85	6.27E-04	1	0	0.43	0.02
A.5.20	SG	1.98	-	2.02	-	0.04	-	15.85	-	1	-	0.50	-
	LHS	3.55	0.03	3.78	0.25	0.04	4.15E-03	15.85	3.33E-04	1	0	0.83	0.01
	d-LHD	3.50	0.03	3.76	0.10	0.05	0.02	15.85	4.52E-04	1	0	0.51	0.03