How do firm characteristics affect the corporate income tax revenue?∗

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ABSTRACT

We use a dynamic model of the firm to study the determinants of the Laffer tax rate (i.e., the corporate income tax rate that maximizes tax proceeds). Under a standard parameterization of the model, we find that the curvature of the production function, the market cost of capital, and the operating costs are among the main determinants of that revenue-maximizing rate. We also find that the Laffer tax rate is around 68% for a representative firm and it varies across U.S. industries in the range 64%–74%. Finally, our results show that the revenue-maximizing rate behaves procyclically over the business cycle.

1. Introduction

It is generally attributed to Arthur B. Laffer in 1974 the point that “there are always two tax rates that yield the same revenues” (see, e.g., Wanniski, 1978). While simple, this idea has important implications for the government agencies that define the fiscal policy. This work investigates how the different characteristics of the firm affect the corporate income tax rate that maximizes tax revenues (i.e., the Laffer tax rate). In turn, this analysis allows us to explain the variability observed in the revenue-maximizing tax rates across sectors or industries in the U.S. economy. These results may guide targeted fiscal policies that contemplate the specific situation of the different U.S. industries. Last, but not least, we analyze the behavior of the Laffer tax rate over the business cycle.

Using a dynamic model of the firm, we start computing the revenue-maximizing tax rate for a representative firm in the U.S. and find it to be around 68%. This result is consistent with Trabandt and Uhlig (2011), who find a Laffer tax rate of around 65% for the U.S. economy. We then study the effect of the different primitive features of the firm (e.g., the curvature of the production function, the persistence of profit shocks, etc.) on that Laffer rate. We find that the concavity of the production function is the main corporate determinant of the tax rate that maximizes tax revenues. The reason for such a large effect is that, as we describe below in detail, the curvature of the production function is the main driver of the flatness of the tax base (i.e., pre-tax income) with respect to the income tax rate, which defines the Laffer tax rate. Additionally, we find that the market cost of capital and the operating costs are firm features with a significant effect on the revenue-maximizing tax rate. Finally, we study the evolution of the Laffer tax rate over the business cycle and find that it has a procyclical behavior. That is, the revenue-maximizing rate increases during economic expansions, and vice versa. The reason for this behavior is that optimal capital and labor increase and become less sensitive to the income tax rate in periods of high

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profits, making the tax base a relatively flatter function of the income tax rate, which, in turn, induces an increment in the Laffer tax rate.

We next perform an analysis of the Laffer tax rate across firms in different SIC industries, aggregated at the division level (i.e., Construction, Manufacturing, Services, etc.) We find that those revenue-maximizing rates vary between 64% and 74%, with the Services industry in the lower end and the Mining industry in the upper end of that range. As expected, the structural reason of the observed variability is the fact that these industries differ from each other in all of their primitive characteristics, especially the ones described in the previous paragraph. These results also suggest that, given the current corporate income tax rates below 50% (federal plus state taxes), the individual U.S. industries seem to be on the upward sloping side of the Laffer curve. However, these results should be interpreted with some caution. First, we find that some individual firms in our dataset have fundamental features that make them be close to, if not already on, the slippery side of the Laffer curve. Second, our model does not capture the phenomenon of international firm mobility (see, e.g., Osmundsen, Hagen, & Guttorm Schjelderup, 1998, and Olsen & Osmundsen, 2011), which could counteract the effects of tax changes.

Finally, we perform robustness checks after we relax some of the initial assumptions. First, we analyze the impact on the Laffer tax rate of letting the fiscal depreciation be different from the economic depreciation. Second, we do an analogue study after allowing for the existence of investment tax credits.

By focusing on the corporate determinants of the Laffer tax rate as well as on the observed variability in that revenue-maximizing rate across U.S. industries, our work contributes to the large literature on structural models studying the different aspects of the so-called Laffer curves. The closest work is Trabandt and Uhlig (2011, 2012), who study Laffer rates for capital and labor Laffer curves for the U.S. and 14 European countries under different parameterizations of constant Frisch elasticity preferences. Using an endogenous growth model, Ireland (1994) shows that there exists a dynamic Laffer curve with respect to capital income taxes. However, using the same model, Tsuchiya (2016) suggests dynamic Laffer curves might not arise in the context of public debt. Schmitt-Grohé and Uribe (1997) find that there is a labor income Laffer curve in the steady state in the context of a neoclassical growth model. Bianconi (2000) studies alternative fiscal policies and finds cases where dynamic Laffer curves might not arise. Floden and Lindé (2001) do a thorough analysis of labor income Laffer curves for different calibrations of model parameters.1

Our work is also related to the large literature that analyzes the elasticity of taxable income as well as of income tax revenue with respect to income tax rates. Lindsey (1987) studies the response of taxpayers to the personal tax reductions occurred in the U.S. during 1982–1984 and analyzes their implications regarding tax revenue maximization. Feldstein (1995) uses the tax reform of 1986 to analyze the sensitivity of taxable income with respect to income tax rates. Contrary to the results of those authors, who study tax reforms occurred in the 1980s, Goosbee (1999) finds that the elasticities of taxable income with respect to tax changes in other time periods are much smaller. Slemrod (1998) provides a description of the relevant issues in both estimating the elasticity of taxable income and using it in the context of tax reform analysis. Giertz (2009) summarizes the empirical and theoretical developments regarding the elasticity of taxable income and analyzes them in the context of forthcoming tax challenges for the U.S. Saez, Joel Slemrod, and Gertz (2012) also survey the empirical literature on the elasticity of taxable income, and develop a theoretical framework that determines the conditions under which that elasticity is a sufficient statistic for the analysis of efficiency and optimal taxes. Creedy (2015) contains a technical primer on how to perform welfare and optimal tax analysis with the elasticity of taxable income. Relatedly, Creedy and Gemmell (2015) derive analytical forms for the revenue-maximizing elasticity of taxable income, and show that the latter can take quite different values both within and across income tax brackets. Devereux, Liu, and Loretz (2014) provide empirical estimates of the elasticity of taxable income for corporations in the U.K., and calculate the marginal deadweight cost of the levy. Sanz (2016a) derives novel explicit expressions for the elasticity of tax revenue in the presence of non-standard allowances and characterizes the implicit Laffer curve. Sanz (2016b) studies Laffer tax rates in the context of both personal income and consumption taxes. He finds that not considering the effect of personal income taxes on consumption yields revenue-maximizing tax rates that are systematically overestimated. Following this literature, in Appendix 3 we study the elasticity of income tax revenue with respect to the income tax rate for both the representative U.S. firm and the different U.S. industries.

The paper is organized as follows. In Section 2, we solve the dynamic model of the firm in closed-form and define the value of corporate income tax revenue. In Section 3, we study how the different corporate determinants affect the Laffer tax rate. The computation of the revenue-maximizing tax rate for each of the different SIC industries is in Section 4. In Section 5, we include some robustness checks on the Laffer tax rate. Section 6 concludes. Appendix 1 contains the proofs, Appendix 2 describes the calibration of model parameters, and Appendix 3 describes the mechanical and behavioral components of a revenue change.

2. A dynamic model of the firm

In this section we derive an analytic solution of the dynamic model of the firm and define the value of the corporate income tax revenue.

Our dynamic firm model is based on the separation principle.2 According to this principle, assuming perfect capital markets, the firm manager maximizes the lifetime expected utility of all current shareholders by maximizing the current price of the stock, independently of their individual subjective preferences. It follows that the model does not require any assumption about equity-holders’ utility

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1 The list of important papers in the literature studying the Laffer curves is large, including, but not limited to, Bender (1984), Malcomson (1986), Pecorino (1995), Agell and Persson (2001), Novales and Ruiz (2002), Jonsson and Klein (2003), Strulik and Trimborn (2012), and Fu-Sheng Hung (2017).

2 See, for example, Copeland, Weston, and Shastri (2005) for a more complete discussion of the separation principle.
functions, as long as we discount future cash flows with an appropriately risk-adjusted discount rate.

The CEO of the firm selects the level of capital, labor, and debt in every period (e.g., quarter, year, etc.) such that they maximize the market value of equity. We write a tilde on variable \( X \) (i.e., \( \tilde{X} \)) to indicate that it is growing over time. The firm uses capital \( \tilde{K}_t \) and labor \( \tilde{L}_t \) for production in period \( t \), and the former depreciates at constant rate \( \delta \in [0, 1] \) in each period. The debt of the firm is denoted by variable \( \tilde{D}_t \) and we assume it expires in one period. For simplicity, we let the coupon rate \( \tilde{c}_\beta \) be equal to the market cost of debt \( \tilde{r}_B \), which makes the book value of debt \( \tilde{D}_t \) equal the market value of debt \( \tilde{B}_t \). Following DeAngelo, DeAngelo, and Whited (2011), we assume the firm issues risk-free debt. Then, the market cost of debt, \( \tilde{r}_B \), equals the risk-free interest rate, \( \tilde{r}_f \).³

We introduce randomness into the model through the profit shock, \( z_t \), which follows an AR(1) process in logs

\[
\ln(z_t) = \ln(c) + \rho \ln(z_{t-1}) + \epsilon_t \tag{1}
\]

where \( c > 0 \) is the constant of the log-process and scales the moments of the distribution of \( z_t \). This parameter has a direct impact on expected profits and, thus, defines the size of the firm. Parameter \( \rho \in (0, 1) \) is the auto-regressive coefficient and determines the persistence of profit shocks. For instance, a high value of \( \rho \) makes periods of high profit innovations (e.g., economic expansions) and low profit shocks (e.g., recessions) last more on average, and vice versa. Finally, \( \epsilon_t \) is an iid normal term with mean 0 and variance \( \sigma^2 \).

In each period, the net profits of the firm are given by the following function

\[
\tilde{N}_t = [(1 + g)^{(1-(\alpha_0+\alpha_1))}z_t\tilde{K}_t^{\alpha_0}\tilde{L}_t^{\alpha_1} - f\tilde{K}_t - \delta\tilde{K}_t - \omega\tilde{L}_t - r_B\tilde{B}_t](1 - \tau) \tag{2}
\]

where \([1 + g]^{(1-(\alpha_0+\alpha_1))}\) represents the level of technology in period \( t \) and implies the firm grows at constant rate \( g \geq 0 \) in each period. The elasticity of capital and labor are given by parameters \( \alpha_0 \) and \( \alpha_1 \), respectively, and we assume \( \alpha_0 \in (0, 1) \), \( \alpha_1 \in (0, 1) \) and \( \alpha_0 + \alpha_1 < 1 \). The operating costs are denoted by \( f > 0 \), labor wages are given by \( \omega > 0 \), and the corporate income tax rate is indicated by \( \tau \in [0, 1] \).

Equation (2) suggests that the production function (the first term) is of the Cobb-Douglas form with decreasing returns to scale in capital and labor inputs. Finally, the dividend paid by the firm to shareholders in period \( t \) is

\[
\tilde{Y}_t = \tilde{N}_t - [(\tilde{K}_{t+1} - \tilde{K}_t) - (\tilde{B}_{t+1} - \tilde{B}_t)]. \tag{3}
\]

Equation (3) says that the dividend equals net profits minus the change in equity. We let \( \tilde{r}_K \) indicate the market cost of equity and \( \tilde{r}_B \) denote the market cost of capital. In addition, to guarantee the existence of the market value of equity, we assume that the market cost of capital exceeds the growth rate (i.e., \( \tilde{r}_K > g \)).

At the beginning of the firm’s life (i.e., at \( t = 0 \)), the CEO chooses the infinite stream of future assets, labor, and debt, \( \{\tilde{K}_{t+1}, \tilde{L}_{t+1}, \tilde{B}_{t+1}\}_{t=0}^{\infty} \), such that the market value of equity is maximized. Letting \( E_0 \) indicate the expectation operator given information at \( t = 0 \) (i.e., \( \tilde{K}_0, \tilde{L}_0, \tilde{B}_0, z_0 \)), the stock price can therefore be written as

\[
\tilde{S}_0(\tilde{K}_0, \tilde{L}_0, \tilde{B}_0, z_0) = \max_{(\tilde{K}_{t+1}, \tilde{L}_{t+1}, \tilde{B}_{t+1})_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{1}{(1 + \tilde{r}_K)} \tilde{Y}_t \tag{4}
\]

subject to the restriction of risk-free debt. Formally, we say debt is risk-free if, in every period, the after-shock book value of equity is weakly positive (i.e., if net profits plus assets, \( \tilde{N}_t + \tilde{K}_t \), are sufficient to cover debt, \( \tilde{B}_t \)). This condition represents a weakly positive networth covenant, which is a common restriction in short-term debt contracts (see, e.g., Leland, 1994), and is consistent with the one-period debt in our model.

We solve equation (4) in closed-form and find the following expression for the stock price.

**Proposition 1.** The optimal decisions of the firm are given by

\[
\tilde{K}_{t+1}(z_t) = (1 + g)^{1+1}E[z_{t+1} | z_t]^{-\frac{1+\rho}{1+1}}\Phi_1, \quad \tilde{L}_{t+1}(z_t) = (1 + g)^{1+1}E[z_{t+1} | z_t]^{-\frac{1+\rho}{1+1}}\Phi_2, \quad \text{and} \quad \tilde{B}_{t+1}(z_t) = \tilde{c}^z \tilde{K}_{t+1} \tag{5}
\]

where \( E[z_{t+1} | z_t] = cz_t^z \tilde{e}^z \). The time-invariant parts of optimal capital and labor take the form

³ Lazzati and Menichini (2015) show that these assumptions about debt generate a leverage behavior that is consistent with several important results described by the corporate finance literature, such as the negative relation between profitability and leverage, as well as the existence of zero-debt firms and their observed characteristics. Furthermore, in untabulated results, we find that letting debt be risky does not change qualitatively any of our results. However, the model cannot be solved in closed-form, which is essential for the purpose of explaining our results.
The resulting model is more realistic in the sense that variable \( \Phi \) dominates the effect on optimal leverage for most parameter values and, thus, optimal debt is a decreasing function of the income tax rate. Optimal leverage increases with the income tax rate, optimal capital decreases with the same rate. It turns out that the effect on optimal capital

\[
\phi(t) = \frac{1}{1 - \tau}
\]

Finally, variable \( P^* \) takes the form

\[
P^* = \left( \Phi_1^{\alpha} \Phi_2^{\alpha} - f \Phi_1^{\alpha} - \delta \Phi_1^{\alpha} - \omega \Phi_1^{\alpha} \right) (1 - \tau) - r_B \Phi_1^{\alpha} + \left( \frac{1 + \alpha}{1 + \beta} \right) r_B \phi(t) \phi(t).
\]

The market value of equity is

\[
S_t(\bar{K}_t, \bar{L}_t, \bar{B}_t, \bar{z}_t) = \left( 1 + g \right)^{(1 - \alpha x + \alpha g)} \left( \bar{K}_t - \bar{B}_t + M_1(\bar{z}_t) \right) (1 - \tau) + \bar{K}_t - \bar{B}_t + M_1(\bar{z}_t) P^*
\]

Finally, the corporate income tax revenue can be defined as the present value of the current and future corporate income tax.

\[\text{This model extends Lazzati and Menichini (2015) by adding the labor decision (\( L_t \)) as well as the elasticity of labor (\( \alpha_t \)) and labor wages (\( \omega \)). The resulting model is more realistic in the sense that firms’ output depends on both capital and labor, as opposed to only capital.}\]

\[\text{We also assume } \left( f + \delta + \omega \frac{\alpha g}{\alpha + \beta} \right) (1 - \tau) \leq 1. \text{ To guarantee that the stock price is weakly positive.}\]

\[\text{This particular effect of } \tau \text{ on debt is due to the following: optimal debt is the product of optimal leverage, } \phi^*, \text{ and optimal capital, } \bar{K}^*(\bar{z}_t). \text{ While optimal leverage increases with the income tax rate, optimal capital decreases with the same rate. It turns out that the effect on optimal capital dominates the effect on optimal leverage for most parameter values and, thus, optimal debt is a decreasing function of the income tax rate.}\]
payments. Given the results in Proposition 1, we can express the value of the corporate income tax base (i.e., pre-tax income) and tax proceeds in the following way.

Corollary 2. The value of the corporate income tax base is given by

\[
\tilde{T}(\tilde{K}, \tilde{L}, \tilde{B}, z_t) = (1 + g)^{(\alpha - \omega) z_t} K_t^{\alpha} L_t^{\alpha} - f \tilde{K}_t - \delta \tilde{K}_t - \omega \tilde{L}_t - r_B \tilde{B}_t + \\
M(z_t) \left[ \Phi_1^{\omega} \Phi_2^{\omega} - f \Phi_1 + \delta \Phi_1 - \omega \Phi_2 - \left( \frac{1 + r_B}{1 + r_B} \right) \Phi^c \right]
\]

and the value of the corporate income tax revenue takes the form

\[
\tilde{R}(\tilde{K}, \tilde{L}, \tilde{B}, z_t) = r \tilde{T}(\tilde{K}, \tilde{L}, \tilde{B}, z_t).
\]

The first term in equation (13) represents the income tax base in the current period, while the second term indicates the present value of the expected income tax base in all the future periods. The latter is a function of the future stream of optimal decisions and, as Proposition 1 shows, it depends explicitly on all the fundamental features of the firm (i.e., the curvature of the production function, the persistence of profit shocks, etc.) We then define the Laffer tax rate \( (\tau^*) \) as the value of the income tax rate \( (\tau) \) that maximizes the corporate income tax proceeds

\[
\tau^* = \arg \max \tilde{R}(\tilde{K}, \tilde{L}, \tilde{B}, z_t).
\]

Unfortunately, there is not an analytic expression for \( \tau^* \), so we compute it numerically.

The income tax base shown in equation (13) is consistent with a tax schedule having the same flat rate applied to any level of taxable income.\(^7\) In untabulated results, we also analyze the effects of expanding the tax structure from the single-rate schedule to a progressive multi-rate schedule on the revenue-maximizing top marginal tax rate (keeping all other marginal rates constant). We find that the resulting Laffer tax rate increases, as the lower income brackets are unaffected by the top marginal rate in the maximization problem described by equation (15). Finally, Appendix 3 analyzes the mechanical and behavioral effects of changing the tax rate on the tax revenue shown in equation (14) for a representative U.S. firm and the different SIC industries.

In the next section, we evaluate how each of the primitive firm characteristics affect the revenue-maximizing tax rate. \( (\tau^*) \).

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\( ^7 \) This context is similar to the current corporate income tax structure implemented by the Tax Cuts and Jobs Act of 2017 at the federal level in the U.S.
Table 2
Corporate Income Tax and the Laffer Tax Rate. The table exhibits the base case ($\tau$) and Laffer ($\tau^*$) tax rates. It also shows the corporate income tax base, the tax proceeds, and the stock price corresponding to both tax rates.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Base Case</th>
<th>Laffer</th>
<th>Laffer/Base Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Rate</td>
<td>31.57%</td>
<td>68.32%</td>
<td>215.37%</td>
</tr>
<tr>
<td>Tax Base</td>
<td>5.66</td>
<td>4.45</td>
<td>78.64%</td>
</tr>
<tr>
<td>Tax Revenue</td>
<td>1.79</td>
<td>3.039</td>
<td>169.36%</td>
</tr>
<tr>
<td>Stock Price</td>
<td>3.13</td>
<td>1.22</td>
<td>38.86%</td>
</tr>
</tbody>
</table>

Fig. 1. Corporate income tax revenue, tax base, and share price.

3. The determinants of the corporate income tax revenue

In this section, we study the determinants of the corporate income tax revenue. We first construct the Laffer curve for a representative firm in the U.S. and then analyze the impact of the different fundamental features of the firm on the Laffer tax rate. Finally, we explore how the revenue-maximizing tax rate evolves over the business cycle.

We begin our analysis defining the base case parameterization of the model, which aims to represent the average firm in the U.S. We use Compustat data to compute the model parameters and describe the calibration procedure in Appendix 2. Compustat is a database of company fundamental and market data published by Standard and Poor’s. It provides annual and quarterly balance sheet, income statement, flow of funds, and supplemental data items on thousands of publicly traded companies in the U.S. and Canada (specifically, on more than 10,000 active and 9,700 inactive firms) dating back to the 1950s.

The initial parameterization refers to a yearly period and is summarized in Table 1. Note that the calibrated corporate income tax rate ($\tau$) turns out to be 0.3157, which is below the statutory value of 0.35.8

To simplify the analysis, we initialize the state of the model by assuming that the firm is at the outset of its life (i.e., $t = 0$) and the state $(\tilde{K}_0, \tilde{L}_0, \tilde{B}_0, z_0)$ is at the mean of the stationary distribution of profit shocks

$$z_0 = c^{'11} e^{'1i z_0^{'1} / \tau}$$

$$\tilde{K}_0 = \left[ c^{'11} e^{'1i z_0^{'1} / \tau} \right] ^{\tau / (\alpha K + \alpha L^{'2} / \tau)} \Phi_1^{'1}$$

$$\tilde{L}_0 = \left[ c^{'11} e^{'1i z_0^{'1} / \tau} \right] ^{\tau / (\alpha K + \alpha L^{'2} / \tau)} \Phi_2^{'1} \text{ and } \tilde{B}_0 = \ell^{'1} \tilde{K}_0.$$

With the parameterization described above, the initial state turns out to be $z_0 = 1.06, \tilde{K}_0 = 0.80, \tilde{L}_0 = 0.29$, and $\tilde{B}_0 = 0.52$.

Table 2 shows the income tax base (i.e., pre-tax income), the tax proceeds, and the stock price for the base case income tax rate ($\tau = 0.3157$) as well as for the Laffer tax rate ($\tau^*$). As we defined in Section 2, the latter is the one that maximizes the income tax revenue

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8 This result might be capturing the phenomenon of tax planning aggressiveness (see, e.g., Frank, Lynch, & Rego, 2009, and Armstrong, Blouin, & Larcker, 2012).
for the government and turns out to be 68.32% for our representative firm. The income tax revenue is 1.79 with the base case tax rate and increases to 3.03 with the revenue-maximizing rate, roughly a 69% increment. However, the growth in the income tax proceeds comes at the cost of a substantial reduction in the tax base and share price. With a tax rate of 31.57% the tax base is 5.66 and the market value of equity is 3.13, while with the Laffer tax rate of 68.32% those values fall to 4.45 and 1.22, respectively. These changes imply a reduction of approximately 21% in the tax base and 61% in the stock price.

Fig. 1 shows the Laffer curve associated with the corporate income tax, as well as the tax base and share price for the representative firm. The three lines are normalized to 100% at the base case (i.e., $\tau = 0.3157$). As expected, the income tax revenue increases monotonically as the tax rate goes up, until the latter equals the revenue-maximizing rate (i.e., $\tau^* = 0.6832$). At that point the income tax revenue is around 69% more than in the base case. Above the Laffer rate the tax revenue falls steadily. Additionally, the figure shows clearly that the tax base and the stock price fall monotonically as the tax rate rises. Overall, these results are in line with the evidence reported by Trabandt and Uhlig (2011), who find Laffer capital income tax rates close to 65% for the U.S. economy.

The figure exhibits the income tax proceeds (solid line), the tax base (dotted line), and the stock price (dashed line) for different values of the income tax rate ($\tau$). The three lines are normalized to 100% at the base case (i.e., $\tau = 0.3157$).

We next explore the corporate determinants of the revenue-maximizing tax rate, $\tau^*$. To understand how the different parameters affect that Laffer rate, we need first to clarify how the tax base depends on those parameters. We showed in equation (13) that the tax
base is a function of the optimal capital and labor chosen for the current and all the future periods. Then, expanding optimal assets in equation (5), we get

$$\tilde{K}_{t+1}(z) = (1 + g)^{-1} \left( c_{t} \rho_{t} \right)^{\alpha_{K}} \left[ \left( \frac{\alpha_{K}}{\alpha_{L}} \right) \left( \frac{\alpha_{L}}{\alpha_{a}} \right) \right]^{\frac{1 - \alpha_{K}}{1 - (\alpha_{K} + \alpha_{L})}} (4)$$

It becomes evident that the exponent $\frac{1}{1 - (\alpha_{K} + \alpha_{L})}$ in the last two factors will define the impact of the primitive parameters on optimal capital. When the elasticity of capital and labor $\alpha_{K} + \alpha_{L}$ is low (i.e., close to 0), that exponent is close to 1 and changes in the parameters will have a very small effect on optimal capital. The opposite is true when $\alpha_{K} + \alpha_{L}$ is high (i.e., close to 1), which makes the exponent a large positive number. As consequence, the impact of changes in the fundamental parameters (e.g., $\tau$) on the tax base and, ultimately, on the Laffer tax rate ($\tau^{*}$) will depend largely on $\alpha_{K} + \alpha_{L}$. We can clearly observe this phenomenon in Fig. 2. Panel A shows the tax base and the Laffer curve associated with the corporate income tax ($\tau$) for the representative firm described in Table 1, except that we assume $\alpha_{K}$ takes on the lower value of 0.45. Accordingly, the tax base is quite insensitive to changes in the income tax rate for most values of $\tau$, falling more strongly only when $\tau$ is high. That is, a low value of $\alpha_{K}$ makes the tax base a relatively flat function of $\tau$, and the revenue-maximizing tax rate turns out to be very high at $\tau^{*} = 0.7850$. On the contrary, Panel B shows the corresponding results for the same firm, but with a higher value of $\alpha_{K}$ at 0.75. The tax base is now much more sensitive to the income tax rate $\tau$, falling more sharply as $\tau$ increases than when $\alpha_{K} = 0.45$. As consequence, the Laffer tax rate is much lower at $\tau^{*} = 0.4888$. Finally, the figures also suggest that the proportional increment in tax revenue from an increase in $\tau$ is potentially larger (smaller) when $\alpha_{K}$ is low (high).

The figure exhibits the income tax proceeds (solid line) and the tax base (dotted line) for different values of the income tax rate $\tau$. The two lines are normalized to 100% at the base case (i.e., $\tau = 0.3157$).

We can now perform a comparative statics analysis that shows how that revenue-maximizing rate changes as we vary the fundamental features of the firm. This type of analysis allows us to understand the directional effect of those primitive characteristics as well as to pinpoint those with the greatest impact on the Laffer tax rate. Accordingly, we start with the base case parameterization outlined above and change those values by up to ±20%.

Table 3 presents the results of this sensitivity analysis. It is clear that the elasticity of capital ($\alpha_{K}$) is the main determinant of the revenue-maximizing tax rate. For instance, for the base case parameter value of $\alpha_{K} = 0.6025$, the Laffer rate is 68.32%, while that rate falls to 53.85% when $\alpha_{K}$ goes up 20% to 0.7230. In other words, $\alpha_{K}$ has a strongly negative effect on the revenue-maximizing rate $\tau^{*}$. As we described above, the reason for such a large impact is that the elasticity of capital plays a fundamental role defining the flatness of the tax base with respect to $\tau$ and, thus, has a very good effect on $\tau^{*}$.

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9 The same results are true for optimal labor.
The operating costs \( f \) and the market cost of capital \( r_A \) are also important parameters. A 20% increment in \( f \) from 0.4501 to 0.5401 augments the Laffer tax rate from 68.32% to 70.39%, while a similar proportional increment in \( r_A \) (i.e., from 0.0886 to 0.1063) reduces that revenue-maximizing rate from 68.32% to 66.22%. As before, the impact of these parameters on the Laffer tax rate depends on whether the tax base becomes flatter as a function of \( \tau \). For instance, increasing \( f \) in the denominator of equation (17) reduces the relative dependence of optimal capital on \( \tau \), which makes the tax base a flatter function of \( \tau \), and the revenue-maximizing tax rate increases accordingly. An analogous argument holds for \( r_A \). The other parameters play a smaller role regarding the Laffer tax rate.

We end the section studying the time-series behavior of the Laffer tax rate. For this analysis, we simulate the model over 100 periods with the base case parameterization described in Table 1. Fig. 3 exhibits the stochastic evolution of the profit shock, \( z_t \), and the revenue-maximizing tax rate over time. It is evident that the revenue-maximizing rate (solid line) is positively related to the profit shock (dashed line). This result suggests that the Laffer tax rate is procyclical over the business cycle, that is, periods of high profit shocks are followed by high Laffer tax rates, and vice versa. The structural reason behind this behavior is that, during economic expansions, optimal assets and labor increase and become less sensitive to changes in the income tax rate. As consequence, the tax base becomes a flatter function of \( \tau \), inducing an increase in the revenue-maximizing tax rate (see, e.g., the role of \( z_t \) in equation (17)). This finding could help the government to adjust the corporate income tax rates during economic booms and depressions.

The figure exhibits the evolution over time of the Laffer tax rate (solid line, right Y-axis) as well as the profit shocks (dashed line, left Y-axis). The model is simulated over 100 periods with the parameterization described in Table 1.

### 3.1. Cross-industry analysis

In the previous section, we showed that the revenue-maximizing tax rate for a representative firm in the U.S. is around 68% and evaluated its main corporate determinants. We now extend the previous analysis to representative firms in different SIC industries, aggregated at the division level, such as Mining, Construction, Manufacturing, etc. These firms differ from each other with respect to all of their primitive characteristics, especially the elasticity of capital \( (\alpha_K) \), the operating costs \( f \), and the market cost of capital \( r_A \). Accordingly, they turn out to have dissimilar Laffer tax rates. Appendix 2 describes the procedure followed to calibrate the model parameters for each industry using Compustat data, and we show their values in Table 4.

The last column in Table 4 shows the results from the cross-sectional comparison of Laffer tax rates. It is clear that the revenue-maximizing tax rate exhibits some variability across industries and is in the range of 64%–74%. The Mining industry exhibits the highest Laffer tax rate at 74.01% mainly because its concavity of the production function with respect to capital is smaller than in all the other industries, while at the same time its market cost of capital is lower than in most other industries. As we saw in the previous section, this parameter combination tends to yield a higher Laffer tax rate. On the other extreme is the Services industry, which has a revenue-maximizing tax rate of 64.30%. In this case, the low Laffer tax rate is mainly due to the high elasticity of capital plus the high market cost of capital.

Regarding the location of the individual U.S. industries on the corporate income tax Laffer curve, our results seem to suggest that they are on the upward sloping side of that curve, given that the current income tax rates (including federal plus state taxes) are below 50%. However, while those representative firms seem not to be on the slippery side of the Laffer curve, it could happen that a number of individual firms are currently on that side. We find some evidence in this direction as we observe that some firms in our Compustat dataset have primitive characteristics that yield low Laffer tax rates (e.g., high values of \( \alpha_K \)). For instance, it is not uncommon to find firms with parameter values that are close to those of the base case described in Table 1, but with values of \( \alpha_K \) above 0.85, which would...
Table 4
Cross-Industry Parameter Values and the Laffer Tax Rate. The table presents the values used to parameterize the dynamic model of the firm for the different SIC industries. The parameters are the drift in logs ($c$), the persistence of profit shocks ($\rho$), the standard deviation of the innovation term ($\sigma$), the concavity of the production function with respect to capital ($\alpha_K$) and labor ($\alpha_L$), the operating costs ($f$), the capital depreciation rate ($\delta$), labor wages ($\omega$), the corporate income tax rate ($\tau$), the market cost of debt ($r_B$), the market cost of capital ($r_A$), and the growth rate ($g$). The last column exhibits the Laffer tax rate ($\tau^*$) for each industry.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Parameters</th>
<th>Laffer Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c$</td>
<td>$\rho$</td>
</tr>
<tr>
<td>Agriculture, Forestry, And Fishing</td>
<td>1.0000</td>
<td>0.5922</td>
</tr>
<tr>
<td>Mining</td>
<td>1.0000</td>
<td>0.5446</td>
</tr>
<tr>
<td>Construction</td>
<td>1.0000</td>
<td>0.5609</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.0000</td>
<td>0.6332</td>
</tr>
<tr>
<td>Transportation and Public Utilities</td>
<td>1.0000</td>
<td>0.6964</td>
</tr>
<tr>
<td>Wholesale Trade</td>
<td>1.0000</td>
<td>0.5898</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>1.0000</td>
<td>0.6642</td>
</tr>
<tr>
<td>Finance, Insurance, And Real Estate</td>
<td>1.0000</td>
<td>0.5752</td>
</tr>
<tr>
<td>Services</td>
<td>1.0000</td>
<td>0.6028</td>
</tr>
</tbody>
</table>
Table 5
Fiscal Depreciation vs. Economic Depreciation. The table presents the Laffer tax rate for different values of economic depreciation (δ_e) for the different SIC industries as well as for the representative U.S. firm. The value of the fiscal depreciation (δ_f) is taken from Tables 1 and 4.

<table>
<thead>
<tr>
<th>Industry</th>
<th>δ_f</th>
<th>Laffer Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>δ_f = 0.02</td>
<td>δ_f = 0.05</td>
</tr>
<tr>
<td>Agriculture, Forestry, And Fishing</td>
<td>0.0411</td>
<td>73.67%</td>
</tr>
<tr>
<td>Mining</td>
<td>0.0827</td>
<td>90.42%</td>
</tr>
<tr>
<td>Construction</td>
<td>0.0272</td>
<td>66.90%</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.0461</td>
<td>75.10%</td>
</tr>
<tr>
<td>Transportation and Public Utilities</td>
<td>0.0495</td>
<td>76.37%</td>
</tr>
<tr>
<td>Wholesale Trade</td>
<td>0.0329</td>
<td>68.78%</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>0.0465</td>
<td>75.67%</td>
</tr>
<tr>
<td>Finance, Insurance, And Real Estate</td>
<td>0.0303</td>
<td>69.64%</td>
</tr>
<tr>
<td>Services</td>
<td>0.0651</td>
<td>72.38%</td>
</tr>
<tr>
<td>Representative U.S. Firm</td>
<td>0.0503</td>
<td>73.59%</td>
</tr>
</tbody>
</table>

yield a revenue-maximizing tax rate (τ*) below 0.35.

4. Robustness checks

In this section, we relax some of the initial assumptions and perform robustness checks on the Laffer tax rate. We start studying the effect of letting the fiscal depreciation be different from the economic depreciation on the revenue-maximizing tax rate. We then analyze the impact on that rate of the existence of investment tax credits.

4.1. Fiscal depreciation vs. economic depreciation

Equation (13) shows the value of the corporate income tax base assuming that the fiscal depreciation is equal to the economic depreciation. In this subsection, we study the impact on the Laffer tax rate of relaxing this assumption. That is, we analyze how the revenue-maximizing tax rate changes when we allow the fiscal depreciation (δ_f) to be different from the economic depreciation (δ_e). After solving the problem of the firm in equation (4) in this new context, the value of the income tax base, \( T_t(K, L, B, z_t) \), changes only through the time-invariant parts of optimal capital, \( \Phi_1 \), and labor, \( \Phi_2 \), which now take the form

\[
\Phi_1 = \left( \frac{\alpha_k}{\alpha_k + \delta_f + \delta_e} \right) \left( \frac{\alpha_l}{\alpha_l + \delta_f + \delta_e} \right) \left( \frac{\alpha_m}{\alpha_m + \delta_f} \right) \left( \frac{\alpha_z}{\alpha_z + \delta_e} \right) \right]^{\frac{1}{1-\alpha_e}}
\]

and

\[
\Phi_2 = \left( \frac{\alpha_k}{\alpha_k + \delta_f + \delta_e} \right) \left( \frac{\alpha_l}{\alpha_l + \delta_f + \delta_e} \right) \left( \frac{\alpha_m}{\alpha_m + \delta_f} \right) \left( \frac{\alpha_z}{\alpha_z + \delta_e} \right) \right]^{\frac{1}{1-\alpha_f}}
\]

It is clear from equations (18) and (19) that, given the fiscal depreciation, \( \delta_f \), a higher economic depreciation, \( \delta_e \), diminishes optimal capital and labor. Accordingly, the value of the income tax base and the Laffer tax rate also fall. Table 5 shows the corresponding results for the different SIC industries as well as for the representative firm in the U.S. In all cases, the revenue-maximizing tax rate diminishes as the economic depreciation rate increases from 2% (column 3) to 5% (column 4) and then to 10% (column 5), given the fiscal depreciation (column 2).\(^{10}\) As benchmark, in column 6 we exhibit the Laffer tax rates when the fiscal depreciation and the economic depreciation are equal, which coincide with the results reported in Sections 3 and 4. As Table 5 shows, for the representative U.S. firm, assuming the economic depreciation equals the fiscal depreciation (at 5.03%) yields a Laffer tax rate of 68.32% (column 6). But that rate decreases to 62.51% if we assume an economic depreciation of 10% (column 5), given the fiscal depreciation of 5.03%.

4.2. Investment tax credits

Finally, the value of the tax base shown in equation (13) does not consider the existence of investment tax credits, which is a common tool used by policy-makers with the objective of stimulating investment and, ultimately, long-run economic growth. In this subsection, we analyze how the revenue-maximizing tax rate changes when we introduce investment tax credits. We let \( \tau_t \) denote the fraction of capital investment, \( K_{t+1} - (1 - \delta)K_t \), that firms are allowed to claim as a tax credit. Thus, in every period, the investment tax credit is

\(^{10}\) These values of economic depreciation are in line with those reported by Hulten and Wykoff (1980 and 1981) and Kim and Moore (1988).
Proof We say debt is risk-free if, in every period, the following inequality is true for all \( \tau \):

\[
\phi_1 [\tilde{K}_{t+1} - (1 - \delta)\tilde{K}_t], \quad \text{where } \phi_1 \text{ is an indicator function equal to 1 if investment in the period is positive (i.e., } \tilde{K}_{t+1} - (1 - \delta)\tilde{K}_t > 0) \text{, and 0 otherwise.}
\]

With this addition, in period \( t \), the dividend that the firm pays to shareholders is

\[
\bar{y}_t = \bar{N}_t - (\tilde{K}_{t+1} - \tilde{K}_t) - (\tilde{B}_{t+1} - \tilde{B}_t) + \phi_1 [\tilde{K}_{t+1} - (1 - \delta)\tilde{K}_t].
\]

(20)

Using this expression, we proceed to solve the problem of the firm in equation (4) and calculate the Laffer tax rate. Unfortunately, the model now does not have closed-form solution and we solve it numerically. The results are exhibited in Table 6. Column 2 shows the Laffer tax rate when there are no investment tax credits (i.e., \( \tau_1 = 0 \)). These results coincide with those described in Sections 3 and 4. Columns 3, 4, and 5 display the revenue-maximizing tax rates when the investment tax credits are 5%, 10%, and 15% of capital investment, respectively.\(^{11}\) It is clear that, for all SIC industries and the representative U.S. firm, as \( \tau_1 \) increases, the Laffer tax rate also increases. The reason behind this behavior is that the tax credit augments the marginal productivity of capital and labor and, thus, increases the size of optimal assets and labor. This, in turn, ends up incrementing the tax base and the resulting revenue-maximizing tax rate. For example, for the representative U.S. firm, when there are no investment tax credits the Laffer tax rate is 68.32% (column 2). If we let the investment tax credit be 10% (column 4), then the revenue-maximizing tax rate increases to 70.33%.

5. Conclusion

We use a dynamic model of the firm to ascertain the main determinants of the corporate income tax rate that maximizes tax revenues (i.e., the Laffer tax rate). For a reasonable parameterization of the dynamic model, we find that the elasticity of capital, the operating costs, and the market cost of capital are the primitive characteristics of the firm with the greatest impact on the revenue-maximizing tax rate. In addition, we find that the Laffer tax rate evolves procyclically over the business cycle.

Regarding the value of the Laffer tax rate, we find that it is around 68% for a representative firm in the U.S. Furthermore, we calculate the revenue-maximizing tax rate for the different U.S. industries and find that it varies within the range 64%–74%, with the Services and Mining industries at the lower and upper ends of that range, respectively. This result has interesting implications for fiscal policy as those industries seem to be on the upward sloping side of the Laffer curve, given the current income tax rates.

Appendix 1. Proofs The Proof of Proposition 1 requires an intermediate result that we present next.

Lemma 3 Restricting debt to be risk-free, the maximum level of book leverage in each period is given by

\[
L^* = \frac{1 - \left( f + \delta + \omega \frac{\phi_1}{\phi_2} \right) (1 - \tau)}{1 + r_g (1 - \tau)}
\]

(21)

Proof We say debt is risk-free if, in every period, the following inequality is true for all \( z' \):

\[
(z' K^{\tau_1} L^{\tau_1} - fK' - \delta K' - \omega L' - r_g \ell K') (1 - \tau) + K' - \ell K' \geq 0.
\]

That is, risk-free debt implies that next-period, after-shock book value of equity must be weakly positive for all \( z' \).\(^2\) In other words, net profits, \((z' K^{\tau_1} L^{\tau_1} - fK' - \delta K' - \omega L' - r_g \ell K') (1 - \tau)\), plus the value of assets, \( K' \), must be sufficient to cover debt, \( \ell K' \).

Given that the worst-case scenario is \( z' = 0 \), the maximum book leverage ratio consistent with risk-free debt, \( L^* \), satisfies

\[\text{Table 6}

<table>
<thead>
<tr>
<th>Industry</th>
<th>( \tau_1 = 0 )</th>
<th>( \tau_1 = 0.05 )</th>
<th>( \tau_1 = 0.10 )</th>
<th>( \tau_1 = 0.15 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apiculture, Forestry, And Fishing</td>
<td>69.81%</td>
<td>70.76%</td>
<td>71.78%</td>
<td>72.89%</td>
</tr>
<tr>
<td>Mining</td>
<td>74.01%</td>
<td>75.13%</td>
<td>76.45%</td>
<td>77.93%</td>
</tr>
<tr>
<td>Construction</td>
<td>65.81%</td>
<td>66.68%</td>
<td>67.61%</td>
<td>68.62%</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>70.76%</td>
<td>71.68%</td>
<td>72.69%</td>
<td>73.78%</td>
</tr>
<tr>
<td>Transportation and Public Utilities</td>
<td>69.79%</td>
<td>70.82%</td>
<td>71.98%</td>
<td>73.27%</td>
</tr>
<tr>
<td>Wholesale Trade</td>
<td>66.61%</td>
<td>67.52%</td>
<td>68.50%</td>
<td>69.56%</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>71.31%</td>
<td>72.21%</td>
<td>73.19%</td>
<td>74.26%</td>
</tr>
<tr>
<td>Finance, Insurance, And Real Estate</td>
<td>66.09%</td>
<td>67.52%</td>
<td>69.70%</td>
<td>70.62%</td>
</tr>
<tr>
<td>Services</td>
<td>64.30%</td>
<td>65.42%</td>
<td>66.65%</td>
<td>68.02%</td>
</tr>
<tr>
<td>Representative U.S. Firm</td>
<td>68.32%</td>
<td>69.28%</td>
<td>70.33%</td>
<td>71.49%</td>
</tr>
</tbody>
</table>

\(\Phi_1 [\tilde{K}_{t+1} - (1 - \delta)\tilde{K}_t] \), where \( \Phi_1 \) is an indicator function equal to 1 if investment in the period is positive (i.e., \( \tilde{K}_{t+1} - (1 - \delta)\tilde{K}_t > 0 \), and 0 otherwise. With this addition, in period \( t \), the dividend that the firm pays to shareholders is

\[
\bar{y}_t = \bar{N}_t - (\tilde{K}_{t+1} - \tilde{K}_t) - (\tilde{B}_{t+1} - \tilde{B}_t) + \phi_1 [\tilde{K}_{t+1} - (1 - \delta)\tilde{K}_t].
\]

(20)

\[^{11}\text{We use these parameter values for } \tau_1 \text{ following Bradford (1978) and Goolsbee (1998).}\]

\[^{12}\text{The same result is true if we define risk-free debt using the market value of equity as opposed to the book value of equity. That is, in both cases we arrive at equation (21) as the maximum level of book leverage consistent with risk-free debt. To simplify notation, we use the book value of equity.}\]
\[-fK' - \delta K' - \omega L' - r_B E_r' K'(1 - \tau) + K' - E_r' K' = 0.\]  

(23)

Working on the previous expression, we can derive the maximum level of book leverage as

\[
\ell^* = \frac{1 - \left(f + \delta + \omega \frac{\Phi}{\Phi'}\right)(1 - \tau)}{1 + r_B (1 - \tau)}
\]

(24)

which completes the Proof.\(^{13}\)

Proof of Proposition 1

The maximization in equation (4) requires a normalization of growing variables that keeps the expectation of the payoff function in the future periods bounded. This normalization is equivalent to the one used to find the solution of the canonical Gordon Growth Model. Let vector \(\tilde{X}_t = \{K_t, L_t, B_t, \tilde{N}_t, \tilde{Y}_t, \tilde{S}_t\}\) contain the growing variables of the model. We then transform vector \(\tilde{X}_t\) in the following way: \(X_t = \tilde{X}_t/(1 + g)^t\). Using the normalized variables and modifying the payoff function accordingly, the market value of equity can be expressed as

\[
S_0(K_0, \tilde{L}_0, \tilde{S}_0) = \max_{(K_{t+1}, L_{t+1}, S_{t+1})} E_0 \sum_{t=0}^{\infty} \frac{(1 + g)^t}{(1 + r_B)} Y_t
\]

(25)

subject to keeping debt risk-free. Because we use the Adjusted Present Value method of firm valuation, we solve the problem of the firm in equation (25) in three steps. First, we determine the value of the unlevered firm, \(S_u(K_0, \tilde{L}_0, \tilde{S}_0)\). Second, we solve for optimal debt and compute the present value of the financing side effects. Finally, we obtain the value of the levered firm in equation (25).

The market value of equity for the unlevered firm can be expressed as

\[
S_u(K_0, \tilde{L}_0, \tilde{S}_0) = \max_{(K_{t+1}, L_{t+1}, S_{t+1})} E_0 \sum_{t=0}^{\infty} \frac{(1 + g)^t}{(1 + r_A)} Y_t
\]

(26)

where \(Y_t = N_t - (K_{t+1} - K_t)\) and \(N_t = (\alpha_t K_t^{\eta} L_t^{\eta} - f K_t - \delta K_t - \omega L_t)(1 - \tau)\). We let normalized variables with primes indicate values in the next period and normalized variables with no primes denote current values. Then, the Bellman equation for the problem of the firm in equation (26) is given by

\[
S_u(K, L, z) = \max_{K, L} \{(z K^m L^m - f K - \delta K - \omega L)(1 - \tau) - (1 + g) K^* + K + \frac{(1 + g)}{(1 + r_A)} E \left[ S_u(K^*, L^*, z^*) \right]\}
\]

(27)

We use the guess and verify method as the Proof strategy. Thus, we start by guessing that the solution is given by

\[
S_u(K, L, z) = (z K^m L^m - f K - \delta K - \omega L)(1 - \tau) + K + M(z)P_u
\]

(28)

where

\[
M(z) = e^{-\frac{1}{1 + r_A}} \sum_{n=1}^{\infty} \left\{ \frac{1 + g}{1 + r_A} \left( \frac{1 + \alpha^\eta}{1 - (\alpha^{1+\eta} r_A)} \right)^n \left( \frac{1}{1 - (\alpha^{1+\eta} r_A)} \right)^{\frac{1}{1 + \alpha^\eta}} \right\}
\]

(29)

\[
P_u = \left( \Phi_1^{\infty} \Phi_2^{\infty} - f \Phi_1^* - \delta \Phi_2^* - \omega \Phi_2^* \right)(1 - \tau) - r_A \Phi_1^*
\]

(30)

\[
\Phi_1^* = \left[ \frac{\alpha_K}{\frac{\alpha_K}{1 + \delta} + \frac{\alpha_L}{\omega}} - \left( \frac{\alpha_K}{\omega} \right) \right]^{\frac{1}{1 + \alpha^\eta}}, \text{ and }
\]

(31)

\(^{13}\) The restriction \(f + \delta + \omega \frac{\Phi}{\Phi'}(1 - \tau) \leq 1\) described in Section 2 also guarantees that \(\ell^* \geq 0\).
We obtained this initial guess as the solution of equation (27) by the backward induction method. We now verify our guess. To this end, let us write

\[
S_u(K, L, z) = \max_{K', L'} \{ F(K', L', K, L, z) \}
\]

with \( F \) defined as the objective function in equation (27).

The FOC for this problem is

\[
\frac{\partial F(K', L', K, L, z)}{\partial K'} = -(1 + g) + \frac{(1 + g)}{(1 + r_a)} \left( (E[z']|z|K^{\alpha_k}L^{\rho_k} - f - \delta)(1 - \tau) + 1 \right) = 0
\]

and optimal capital and labor turn out to be

\[
K^* = E[z']|z|^{1/(\alpha_k + \rho_k)} \Phi_1^* \quad \text{and} \quad L^* = E[z']|z|^{1/(\alpha_k + \rho_k)} \Phi_2^*
\]

where \( \Phi_1^* \) and \( \Phi_2^* \) are as in equations (31) and (32), respectively.

Finally, the market value of equity for the unlevered firm becomes

\[
S_u(K, L, z) = (zK^{\alpha_k}L^{\rho_k} - fK - \delta K - \omega L)(1 - \tau) - (1 + g)K^* + K + \frac{1 + g}{1 + r_a} \left( (E[z']|z|K^{\alpha_k}L^{\rho_k} - fK^* - \delta K^* - \omega L^*)(1 - \tau) + K^* + E[M(z')|z|P_u^*] \right)
\]

\[
= (zK^{\alpha_k}L^{\rho_k} - fK - \delta K - \omega L)(1 - \tau) + K + (1 + g)E[z'|z|^{1/(\alpha_k + \rho_k)} \Phi_1^*] + \frac{1 + g}{1 + r_a} \left( (E[z']|z|^{1/(\alpha_k + \rho_k)} \Phi_1^* - f \Phi_1^* - \delta \Phi_1^* - \omega \Phi_1^*)(1 - \tau) + \Phi_1^* \right) + \frac{1 + g}{1 + r_a} \left( E[M(z')|z|P_u^*] \right)
\]

\[
= (zK^{\alpha_k}L^{\rho_k} - fK - \delta K - \omega L)(1 - \tau) + K + \frac{1 + g}{(1 + r_a)} \left( e^{1 + \frac{\rho_k}{\alpha_k + \rho_k} \tau} E[z'|z|^{1/(\alpha_k + \rho_k)}] + E[M(z')|z|] \right) P_u^*
\]

which is equivalent to our initial guess in equation (28).

Next, we obtain optimal debt. In each period, the firm solves the following problem

\[
\max_{B} \left\{ B^* - \frac{1}{1 + r_a} B^*[1 + r_a(1 - \tau)] \right\}
\]

subject to the restriction of risk-free debt. Because \( \tau > 0 \), the firm increases debt as much as possible (as long as it remains risk-free) to maximize the tax benefits of debt. Then, optimal debt is \( B^* = \ell^* K^* \) where

\[
\ell^* = \frac{1 - \left( f + \delta + \omega \frac{\Phi_1^*}{\Phi_2^*} \right)(1 - \tau)}{1 + r_a(1 - \tau)},
\]

as shown in Lemma 3. The present value of the financing side effects turns out to be
\[
Q(z) = \left( \frac{1 + g}{1 + r_g} \right) \left\{ \left( \frac{1 + r_A}{1 + r_B} \right) r_B \Phi^* + E[Q(z)|z] \right\} \\
= M(z) \left( \frac{1 + r_A}{1 + r_B} \right) r_B \Phi^*
\]

where \( M(z) \) is as in equation (29). Under this financial policy, the amount of debt and interest payments will vary with the future asset cash flows (i.e., they depend on future firm performance). Then, because future interest tax shields will have a level of risk in line with that of the firm cash flows, we use the cost of capital, \( r_A \), as the discount rate.

The third step consists in obtaining the market value of equity for the levered firm. If we assume the firm used debt \( B \) in the previous period, and now has to pay interest \( r_B B(1 - \tau) \), then the stock price for the levered firm is

\[
S(K, L, B, z) = S_e(K, L, z) + M(z) \left( \frac{1 + r_A}{1 + r_B} \right) r_B \Phi^* - B - r_B B(1 - \tau) \\
= (zK^{\alpha} L^{\beta} - fK - \delta K - \omega L - r_B B)(1 - \tau) + K - B + M(z) P
\]

where variable \( P^* \) takes the form

\[
P^* = \Phi^* \Phi^{u*} - f\Phi^* - \delta\Phi^* - \omega\Phi^* (1 - \tau) - r_i \Phi^* + \left( \frac{1 + r_A}{1 + r_B} \right) r_B \Phi^*.
\]

The last part of the Proof consists in transforming normalized variables back into growing variables. For this step, we return to the initial notation with growing variables, where next-period assets are \( \dot{K}_{t+1} \) and current-period assets are \( K_t \). Then, the required transformation is: \( \dot{X}_t = (1 + g)^t X_t \), where vector \( \dot{X}_t = \{K_t, L_t, N_t, Y_t, S_t\} \) contains the normalized variables of the model. Finally, the optimal decisions of the firm with growing variables are given by

\[
\dot{K}_{t+1}(z_t) = (1 + g)^{t+1} E[z_{t+1}|z_t]^{-1} \Phi^*;
\]

\[
\dot{L}_{t+1}(z_t) = (1 + g)^{t+1} E[z_{t+1}|z_t]^{-1} \Phi^{u*}, \text{ and}
\]

\[
\dot{B}_{t+1}(z_t) = \ell^c \dot{K}_{t+1}(z_t)
\]

while the growing market value of equity is

\[
\dot{S}_t(\dot{K}_t, \dot{L}_t, \dot{B}_t, \dot{z}_t) = \left[ (1 + g)^{1 - (\alpha + \alpha_u)} \dot{z}_t \dot{K}_{t}^{\alpha} \dot{L}_{t}^{\beta} - f\dot{K}_t - \delta \dot{K}_t - \omega L_t - r_B \dot{B}_t \right] (1 - \tau) + \\
\dot{K}_t - \dot{B}_t + \dot{H}_t(z_t) \dot{P}^*
\]

as shown in Proposition 1.

Appendix 2. Calibration of Model Parameters

We need to find parameter values for \( c, \rho, \sigma, \alpha_k, \alpha_l, f, \delta, \omega, \tau, r_A, r_B, \) and \( g \) for a representative firm in the U.S. and for each SIC industry at the division level. We do the calibration of model parameters using Compustat annual data. The sample covers the period 1950–2015 and includes a total of 469,242 firm-year observations. For the representative firm in the U.S. we use all firms in the sample while for the representative firm in each industry we use only the firms in the corresponding industry.

In order to obtain parameter \( f \), we average the ratio Selling, General, and Administrative Expense (XSGA)/Assets - Total (AT) for all firm-years. We follow the same procedure to get \( \delta \) as the ratio of Depreciation and Amortization (DP) over Assets - Total (AT), \( \omega \) as the fraction Staff Expense - Total (XLR)/Number of Employees (EMP), and \( \tau \) as the ratio of Income Taxes - Total (TXT) over Pretax Income (PI). We trim these ratios at the lower and upper one-percentiles to reduce the effect of outliers and errors in the data.

Following Moyen (2004), we use the firm’s autoregressive profit shock process of equation (1) and the gross profits function in equation (2), \( (1 + g)^{1 - (\alpha_k + \alpha_l)} z_t \dot{K}_t^{\alpha_k} \dot{L}_t^{\alpha_l} \), to obtain parameters \( \rho, \sigma, \alpha_k, \) and \( \alpha_l \). The data we use with these equations are Gross Profit (GP), Assets - Total (AT), and Number of Employees (EMP). Given that we are interested in evaluating representative firms, we normalize parameter \( c \) to 1.\(^1\)\(^4\) We assume that the risk-free interest rate \( (\tau_f = r_f) \) is 0.02. We derive \( r_A \) using CAPM with the corresponding (unlevered) industry betas estimated by Fama and French (1997) and assuming an expected market return \( (\tau_M) \) of 0.08. Finally, we obtain \( g \) for each industry following Jorgenson and Stiroh (2000).

\(^{14}\) This normalization is common in the corporate finance literature. See, e.g., Moyen (2004), Hennessy and Whited (2005, 2007), DeAngelo et al. (2011).
Table 7
Elasticity of Tax Revenue. The table presents the elasticity of tax revenue with respect to the income tax rate for the different SIC industries as well as for the representative U.S. firm.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Elasticity of Tax Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, Forestry, and Fishing</td>
<td>0.9306</td>
</tr>
<tr>
<td>Mining</td>
<td>0.9648</td>
</tr>
<tr>
<td>Construction</td>
<td>0.8835</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.9113</td>
</tr>
<tr>
<td>Transportation and Public Utilities</td>
<td>0.9081</td>
</tr>
<tr>
<td>Wholesale Trade</td>
<td>0.8787</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>0.9088</td>
</tr>
<tr>
<td>Finance, Insurance, and Real Estate</td>
<td>0.9032</td>
</tr>
<tr>
<td>Services</td>
<td>0.8710</td>
</tr>
<tr>
<td>Representative U.S. Firm</td>
<td>0.9083</td>
</tr>
</tbody>
</table>

Appendix 3. Mechanical and Behavioral Effects of Tax Rate Changes

In this appendix, we analyze the elasticity of tax revenue with respect to the tax rate and separate the mechanical and behavioral effects. Then, we parameterize that elasticity for the different SIC industries as well as for the representative firm in the U.S., and assess the results.

Following Creedy (2015), the (total) elasticity of tax revenue with respect to the income tax rate can be written as

$$\eta_{R,t} = \eta'_{R,t} + (\eta'_{R,t}).(\eta_{T,t})$$

(44)

where $\eta'_{R,t}$ denotes the (partial) elasticity of tax revenue with respect to the income tax rate, $\eta'_{R,t}$ indicates the (partial) elasticity of tax revenue with respect to taxable income, and $\eta_{T,t}$ reflects the (total) elasticity of taxable income with respect to the income tax rate. The first term of equation (44), $\eta'_{R,t}$, is the pure tax rate effect and is usually called the mechanical effect of the change in the tax rate. The second term, ($\eta'_{R,t})(\eta_{T,t})$, reflects the combined effect of both the change in the tax rate on taxable income, $\eta_{T,t}$, and the change in taxable income on tax revenue, $\eta'_{R,t}$. This second term is called the behavioral effect of the tax rate change.

Given that we can obtain the tax revenue in equation (14) in closed-form, we can rewrite equation (44) as

$$\eta_{R,t} = 1 + \tau \frac{dT_t}{d\tau}$$

(45)

where

$$\frac{dT_t}{d\tau} = M_z(\tau)\left[\alpha_k\Phi_{1}^{\alpha_k-1}\Phi_{2}^{\alpha_k} + \Phi_{1}^{\alpha_k}\alpha_k\Phi_{2}^{\alpha_k-1}\Phi_{2}^{\alpha_k} - f\frac{d\Phi_{1}}{d\tau} - \delta \frac{d\Phi_{2}}{d\tau} - \omega \frac{d\Phi_{3}}{d\tau}\right]$$

(46)

$$\frac{d\Phi_{1}}{d\tau} = \frac{\alpha_k r_A (1 - a_k)}{1 - (\alpha_k + a_k)} \left[ \frac{\alpha_k}{\alpha_k + f + \delta} \left( \frac{\alpha_k}{\omega} \right)^{1-a_k} \right]^{\alpha_k/(\alpha_k + a_k)} \left[ \frac{\alpha_k}{\alpha_k + f + \delta} \left( \frac{\alpha_k}{\omega} \right)^{1-a_k} \right]^{a_k} \left[ \frac{\alpha_k}{\alpha_k + f + \delta} \left( \frac{\alpha_k}{\omega} \right)^{1-a_k} \right]^{a_k} \left( \frac{\alpha_k}{\alpha_k + f + \delta} \left( \frac{\alpha_k}{\omega} \right)^{1-a_k} \right)^{a_k}$$

(47)

and

$$\frac{d\Phi_{2}}{d\tau} = \frac{\alpha_k r_A^2}{1 - (\alpha_k + a_k)} \left[ \frac{\alpha_k}{\alpha_k + f + \delta} \left( \frac{\alpha_k}{\omega} \right)^{1-a_k} \right]^{\alpha_k/(\alpha_k + a_k)} \left[ \frac{\alpha_k}{\alpha_k + f + \delta} \left( \frac{\alpha_k}{\omega} \right)^{1-a_k} \right]^{a_k} \left( \frac{\alpha_k}{\alpha_k + f + \delta} \left( \frac{\alpha_k}{\omega} \right)^{1-a_k} \right)^{a_k} \left( \frac{\alpha_k}{\alpha_k + f + \delta} \left( \frac{\alpha_k}{\omega} \right)^{1-a_k} \right)^{a_k}$$

(48)
Finally, we calibrate the elasticity of tax revenue with respect to the income tax rate, $\eta_{k,r}$, for the different SIC industries and the representative U.S. firm. The results are shown in Table 7. Regarding the individual industries, Mining exhibits the highest elasticity of tax revenue at $\eta_{k,r} = 0.9648$, while the Services industry is on the lower end with an elasticity of tax revenue of $\eta_{k,r} = 0.8710$. For the representative U.S. firm, we find that elasticity to be $\eta_{k,r} = 0.9083$, which is positive and below 1. In other words, a corporate income tax rate increase of 1% leads to an increment of 0.9083% in tax revenue.

References


